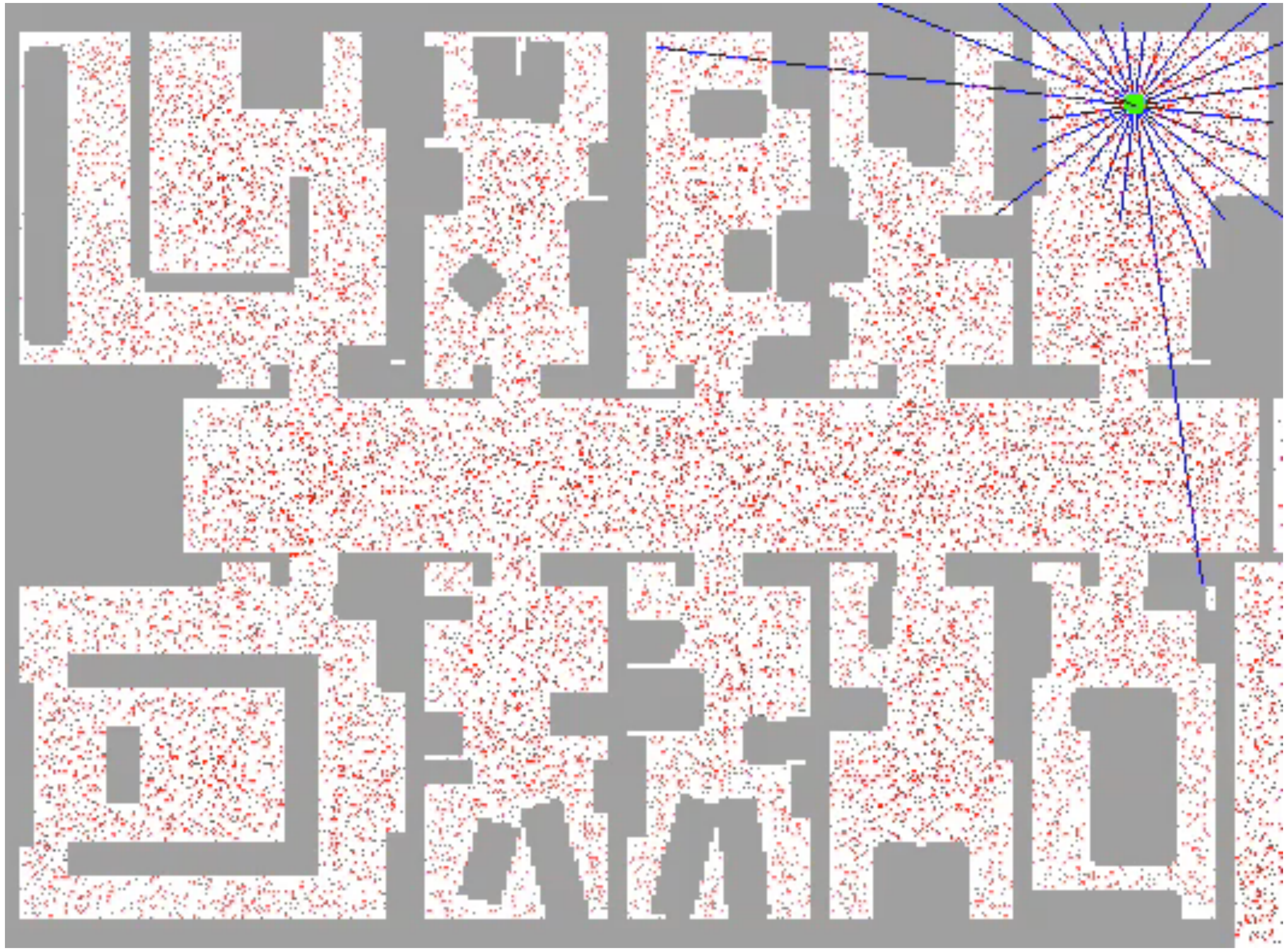
Lecture 19
Mobile Robotics - IV - Particle Filter

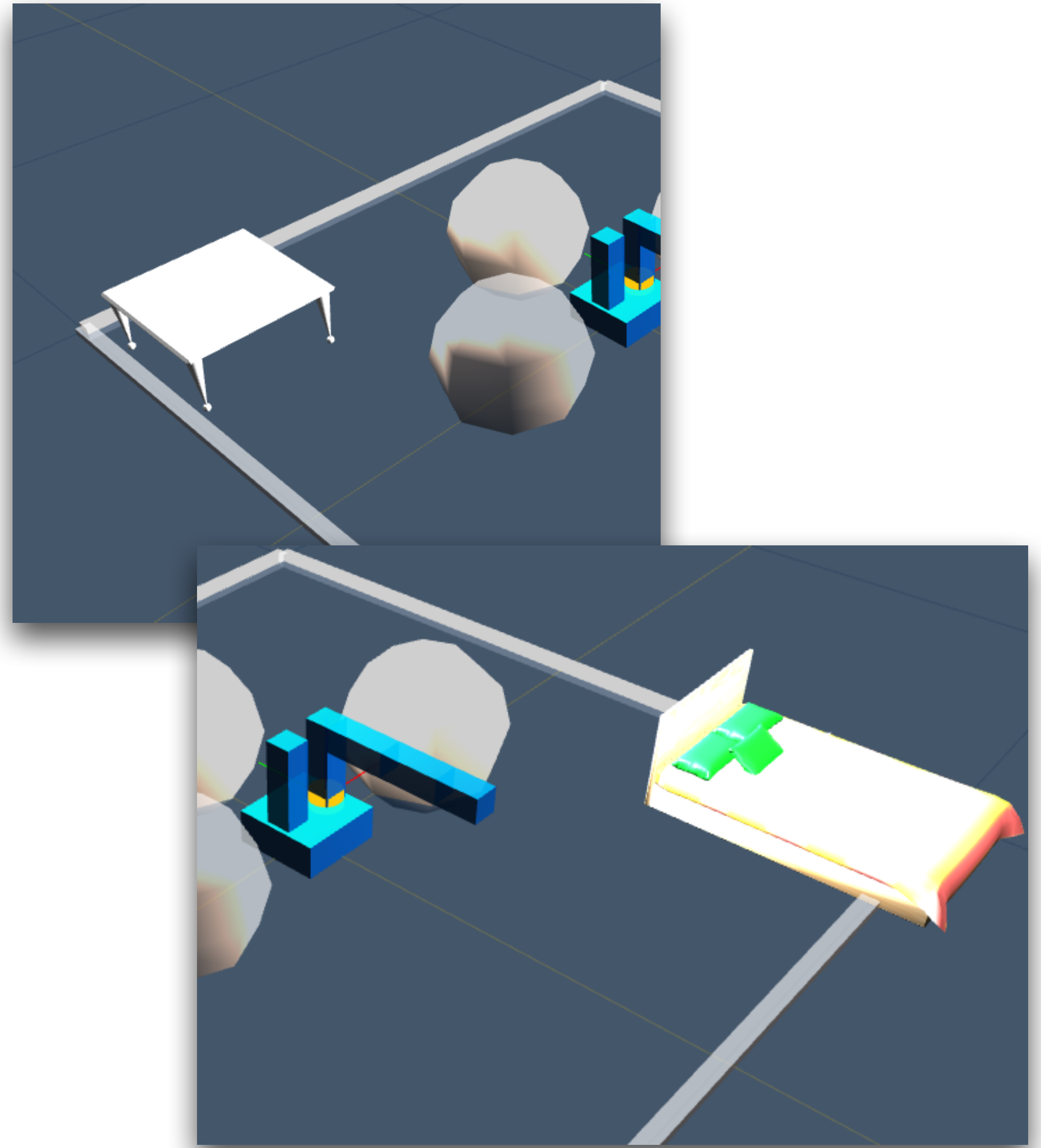


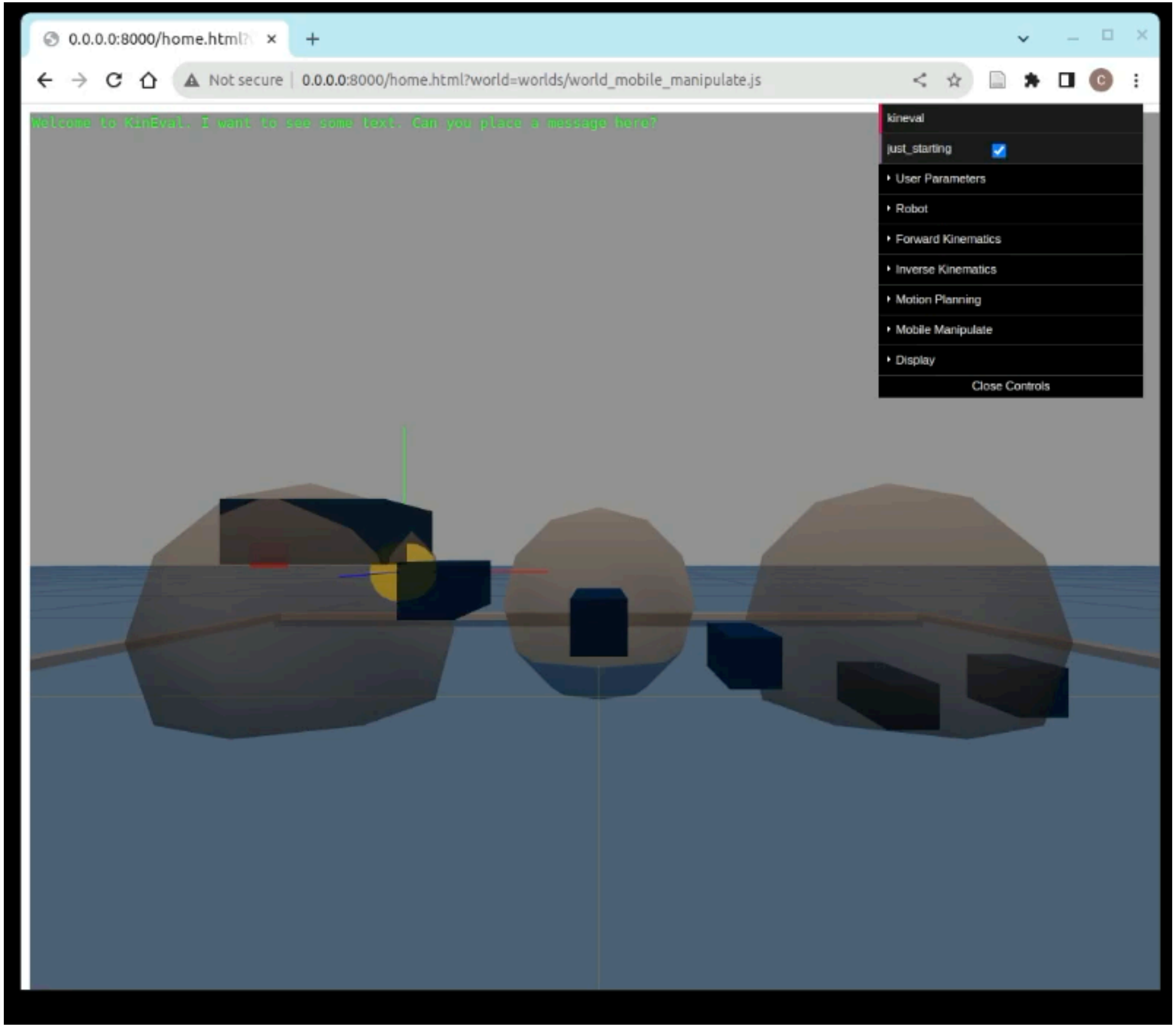
from Probabilistic Robotics

Course logistics

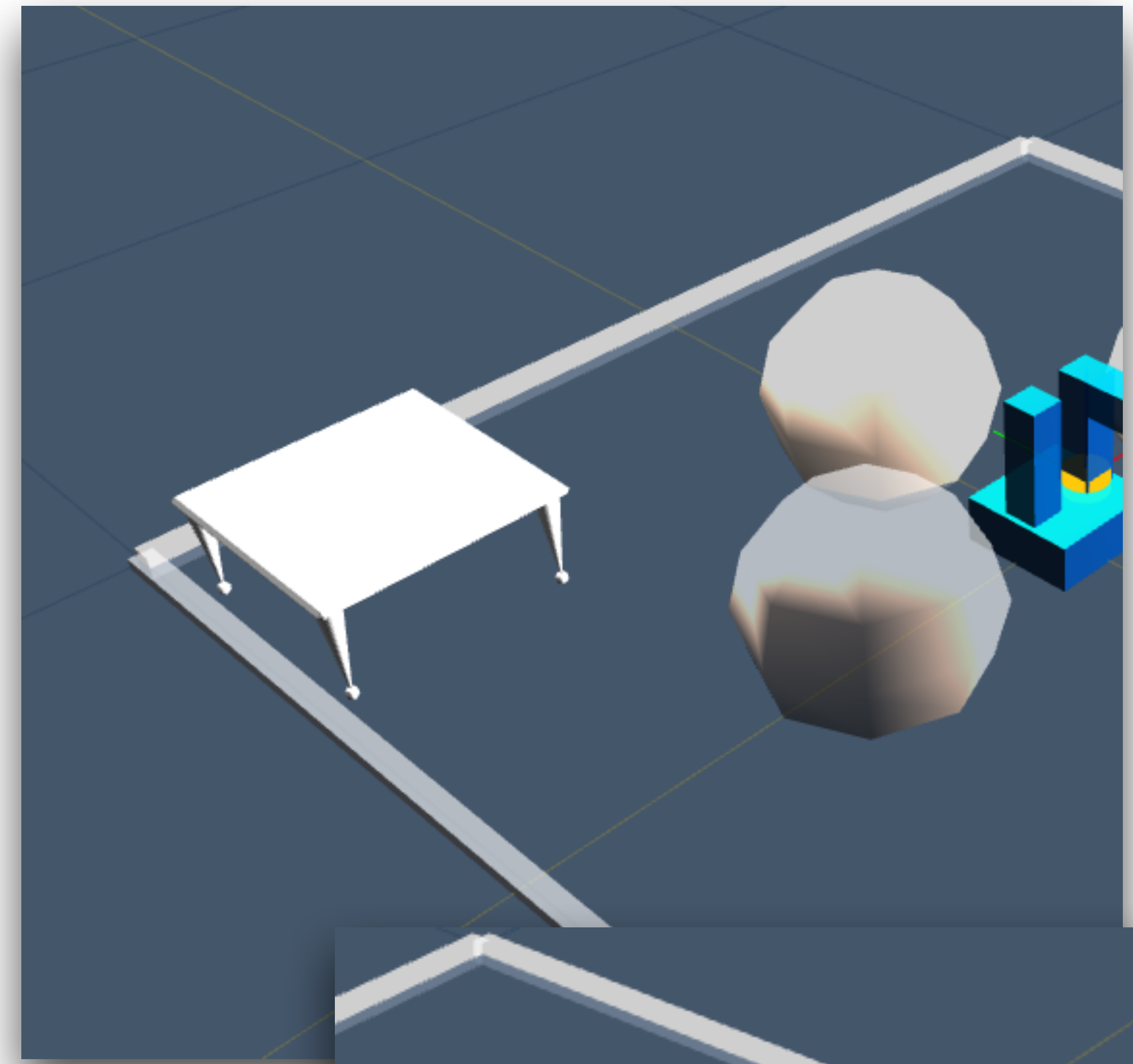
- **Quiz 9 was posted yesterday and was due today at noon.**
- Project 6 was posted on 03/24 and is due **04/02 (today)**
- P1-6 Grades and quiz grades will be posted on Canvas by Monday.
- Project 7:
  - Groups are formed.
  - Scheduler will be shared with the class later today.
    - Lab sessions to be completed by 04/23.
- Final Project:
  - Proposal slides are due 04/14.
- No TA OHs between 04/07 and 04/23.
  - They will be available on demand.
  - Karthik's OH will be available to discuss final projects.
- **Final Poster Session: 05/05/2025 - Monday - 12:30pm - 2:30pm, Shepherd Labs 164 - mark your calendars**

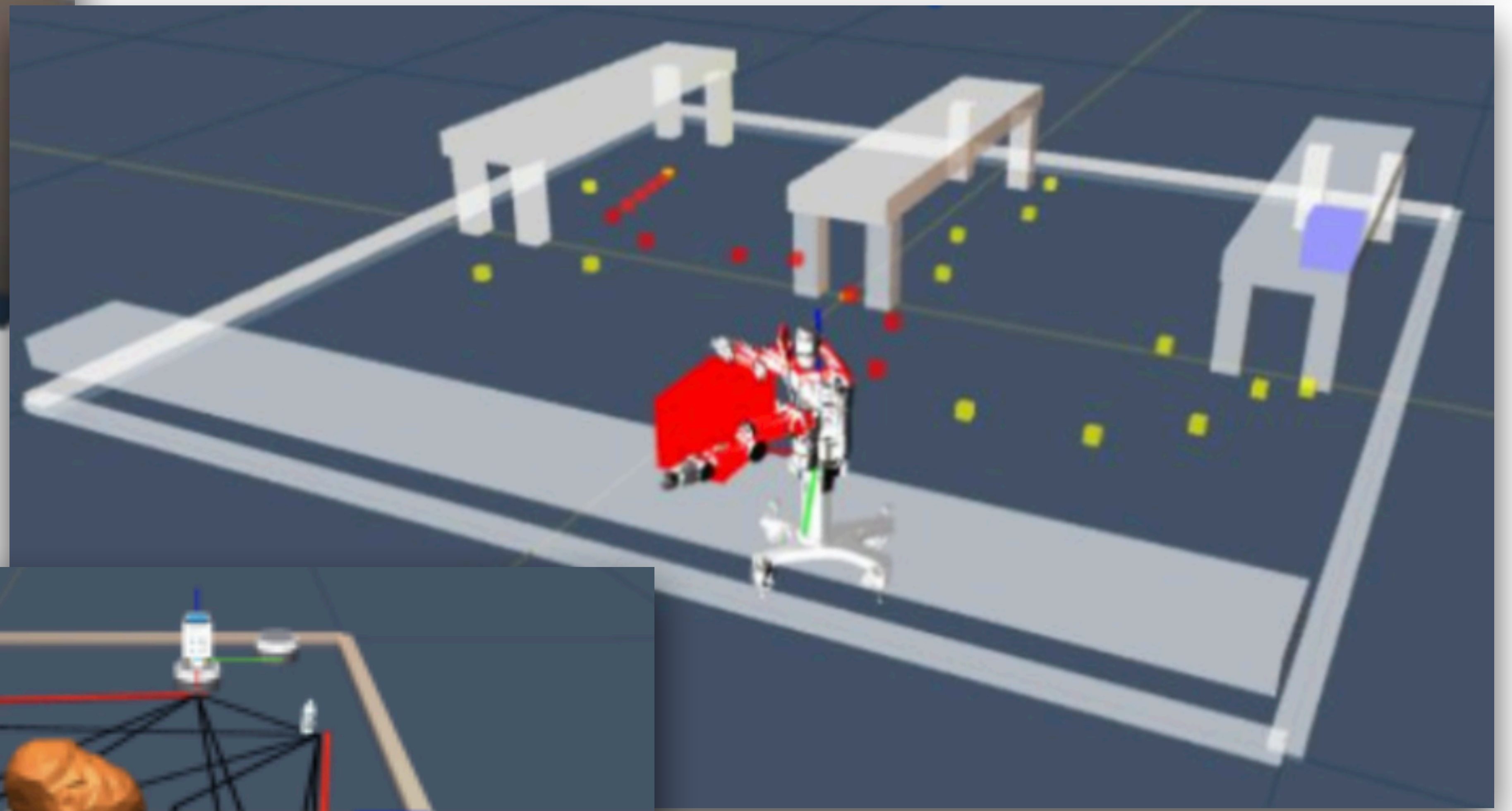
Final Project (Open ended)

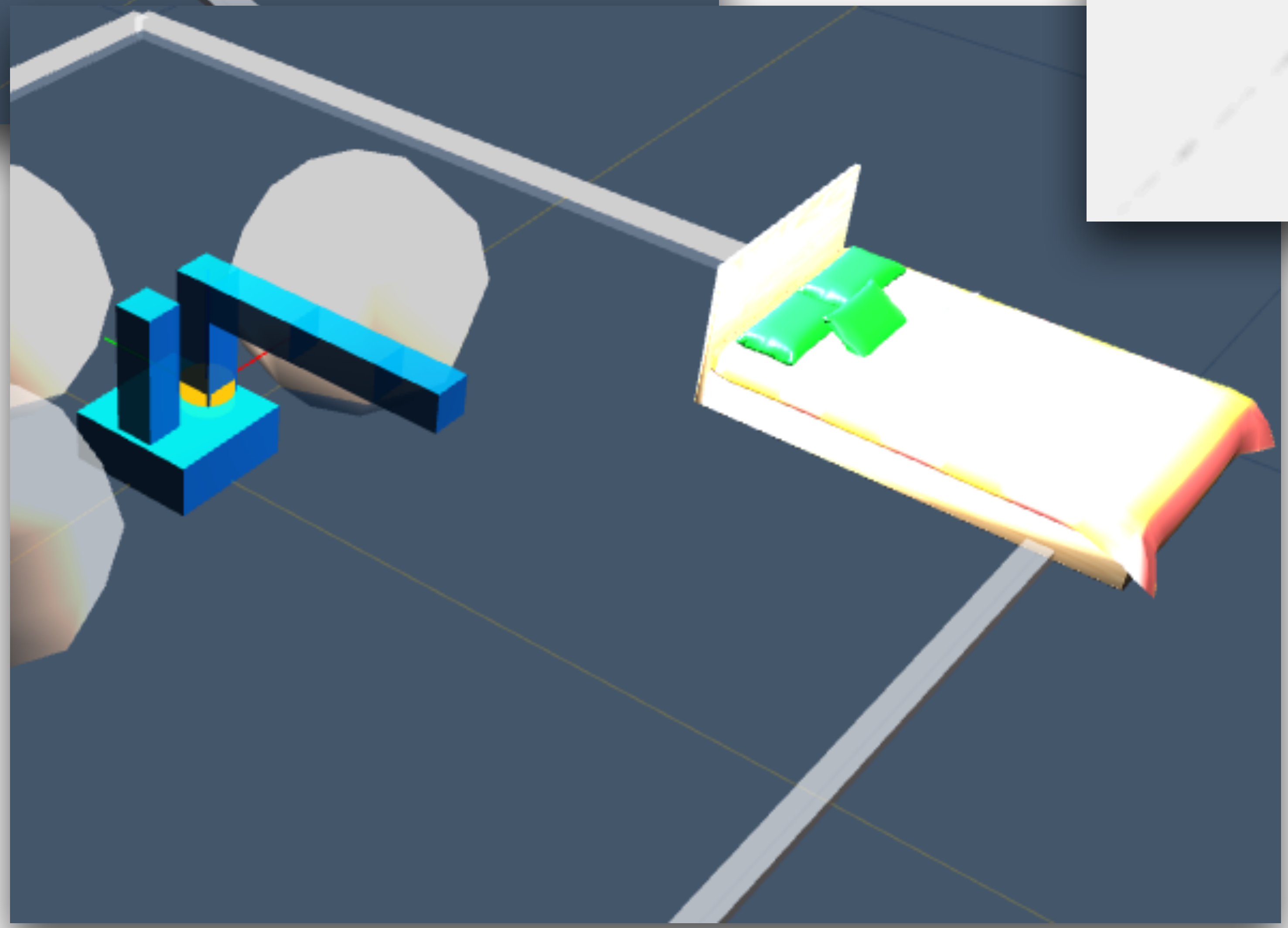


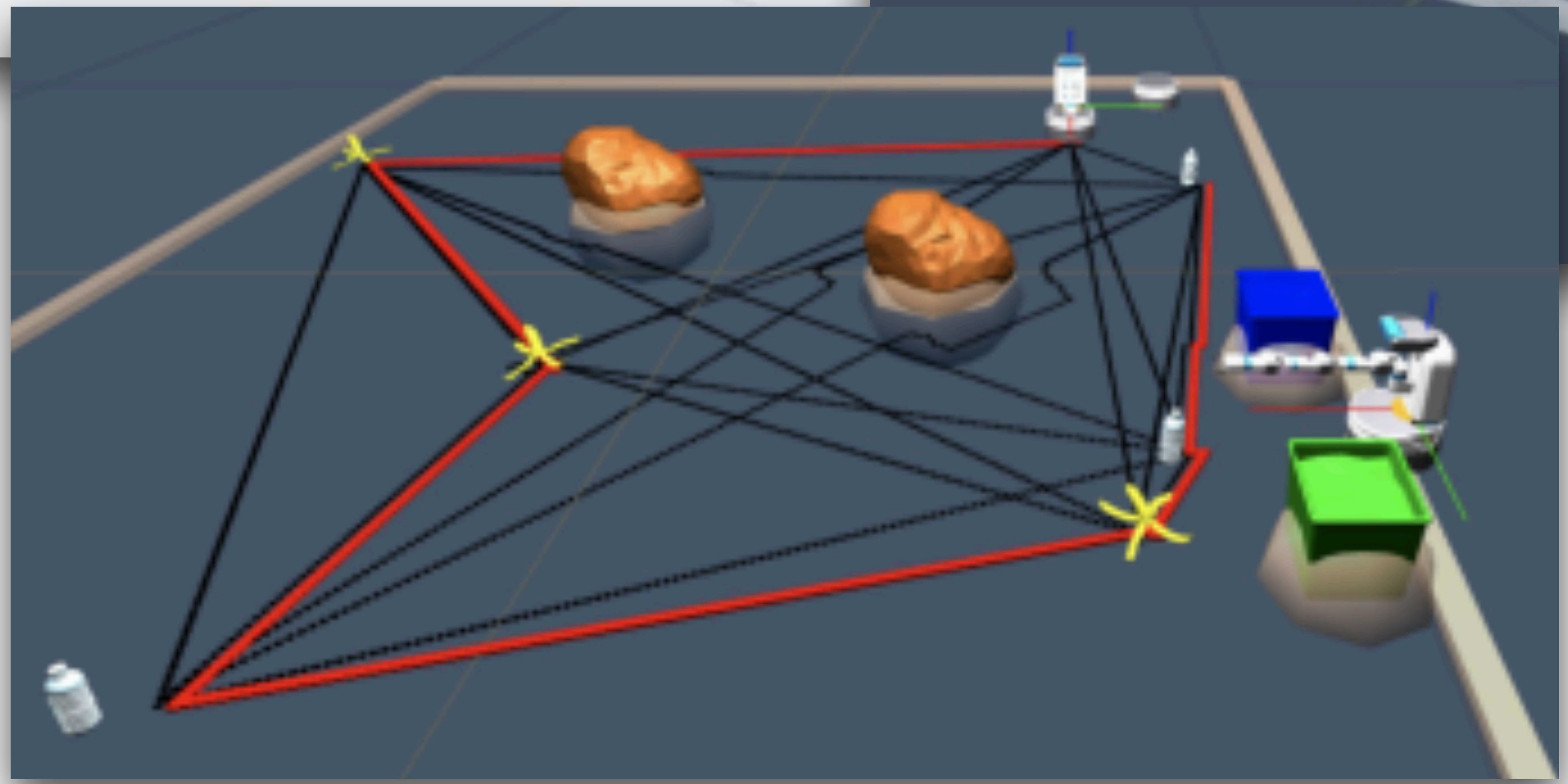


Final Project (Open ended)





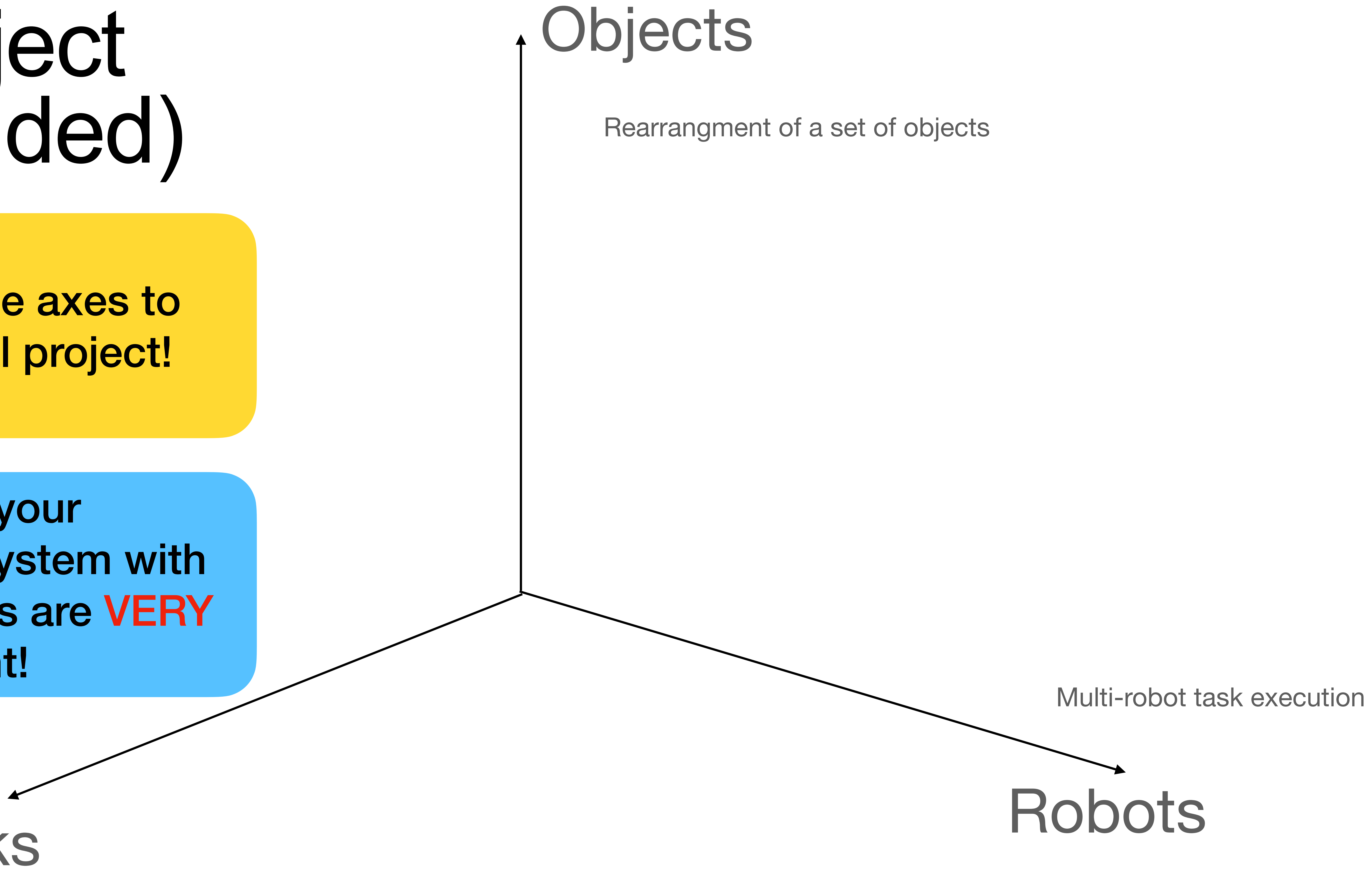




Final Project (Open ended)

Think along these axes to decide your final project!

Evaluating your implementation/system with quantitative results are VERY important!



Final Project (Open ended)
For inspiration!

Yang, Zhutian, Caelan Reed Garrett, Tomás Lozano-Pérez, Leslie Kaelbling, and Dieter Fox. "Sequence-based plan feasibility prediction for efficient task and motion planning." *arXiv preprint arXiv:2211.01576* (2022).

Final Project (Open ended)

Think along these axes to decide your final project!

Evaluating your implementation/system with quantitative results are VERY important!

Objects

Rearrangment of a set of objects

You may use:

- Kineval codebase
- Other sim environments (pybullet, Gazebo, DRAKE, Isaac sim)
- Turtlebot3 (provided only upon compelling proposal, only 5 are available)
- Other robots you may have access to.

Multi-robot task execution

Robots

Long horizon tasks

Tasks

During the P7 sessions we will show other robotic platforms and sensors that are accessible for the Final Projects

Continuing previous Lecture
KF and EKF

Discrete Kalman Filter

Estimates the state x of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_t = A_t x_{t-1} + B_t u_t + \varepsilon_t$$

with a measurement

$$z_t = C_t x_t + \delta_t$$

Components of a Kalman Filter

A_t Matrix (nxn) that describes how the state evolves from $t\text{-}1$ to t without controls or noise.

B_t Matrix (nxl) that describes how the control u_t changes the state from $t\text{-}1$ to t.

C_t Matrix (kxn) that describes how to map the state x_t to an observation z_t.

ε_t

δ_t

Random variables representing the process and measurement noise that are assumed to be independent and normally distributed with covariance R_t and Q_t respectively.

Kalman Filter Algorithm

1. Algorithm **Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

2. Prediction:
3. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

5. Correction:
6. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

Non-linear Dynamic Systems

- Most realistic problems involve nonlinear functions

$$x_t = A_t x_{t-1} + B_t u_t + \epsilon_t$$

$$\Downarrow$$

$$x_t = g(u_t, x_{t-1}) + \epsilon_t$$

$$z_t = C_t x_t + \delta_t$$

$$\Downarrow$$

$$z_t = h(x_t) + \delta_t$$

Linearization

$$x_t = A_t x_{t-1} + B_t u_t + \epsilon_t$$

↓

$$x_t = g(u_t, x_{t-1}) + \epsilon_t$$

$$x_t = g(u_t, x_{t-1}) + \varepsilon_t$$

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + \underbrace{g'(u_t, \mu_{t-1})}_{=: G_t} (x_{t-1} - \mu_{t-1})$$

$$= g(u_t, \mu_{t-1}) + G_t (x_{t-1} - \mu_{t-1})$$

$$z_t = C_t x_t + \delta_t$$

↓

$$z_t = h(x_t) + \delta_t$$

$$z_t = h(x_t) + \delta_t$$

$$h(x_t) \approx h(\bar{\mu}_t) + \underbrace{\frac{\partial h(\bar{\mu}_t)}{\partial x_t}}_{=: H_t} (x_t - \bar{\mu}_t)$$

Slide borrowed from Dieter Fox

EKF Algorithm

1. **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

2. Prediction:

3. $\bar{\mu}_t = g(u_t, \mu_{t-1})$ ⟵ $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$

4. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$ ⟵ $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

5. Correction:

6. $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$ ⟵ $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$

7. $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$ ⟵ $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$

8. $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$ ⟵ $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

$$H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t} \qquad G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}$$

Localization

> "Using sensory information to locate the robot in its environment is the most fundamental problem to providing a mobile robot with autonomous capabilities." [Cox '91]

- **Given**
  - Map of the environment.
  - Sequence of sensor measurements.
- **Wanted**
  - Estimate of the robot's position.
- **Problem classes**
  - Position tracking
  - Global localization
  - Kidnapped robot problem (recovery)

EKF Summary

- Highly efficient: Polynomial in measurement dimensionality k and state dimensionality n:

$$O(k^{2.376} + n^2)$$

- Not optimal!
- Can diverge if nonlinearities are large!
- Works surprisingly well even when all assumptions are violated!

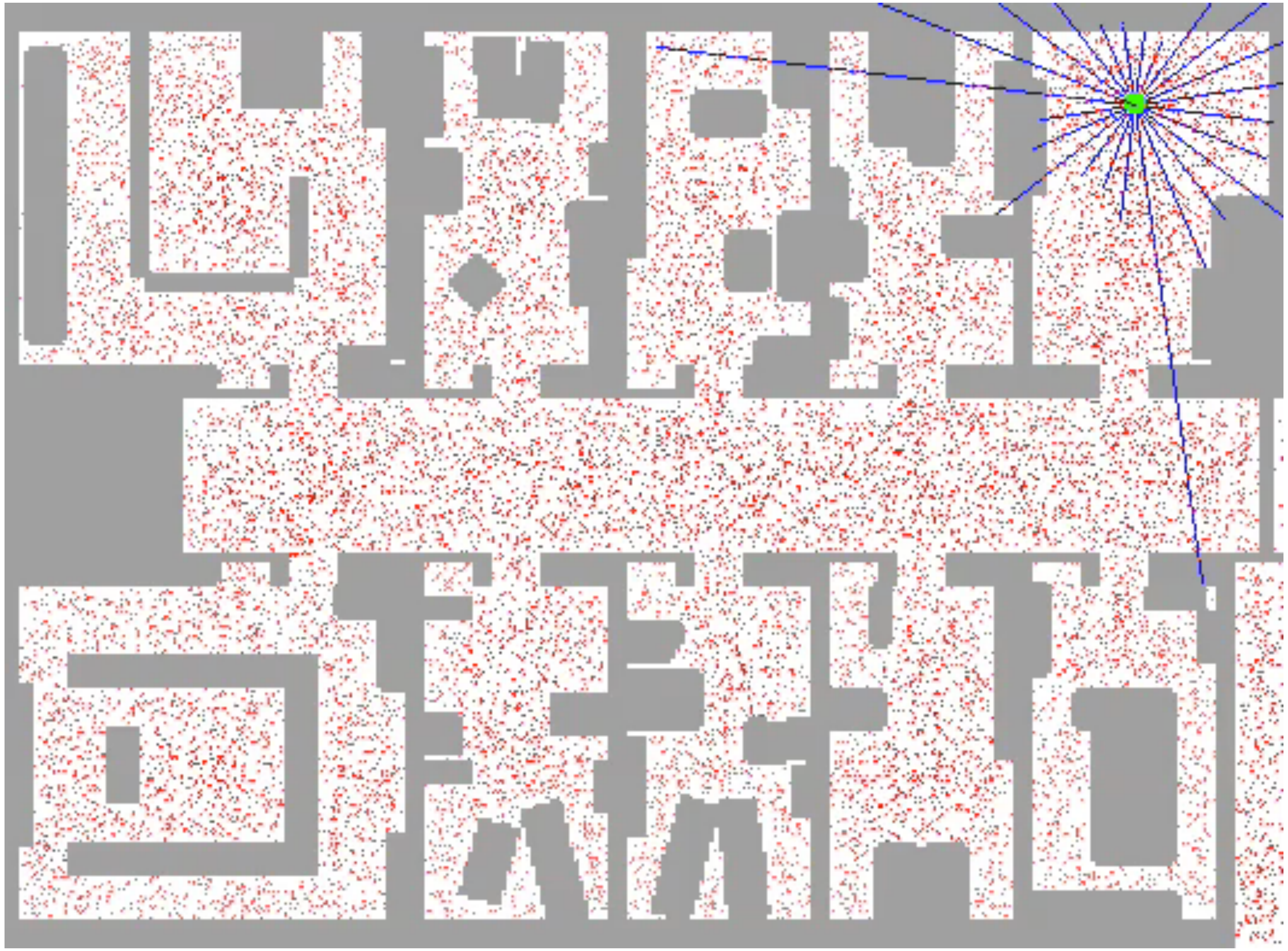
Particle Filter

A Bayesian Filter Implementation

Motivation

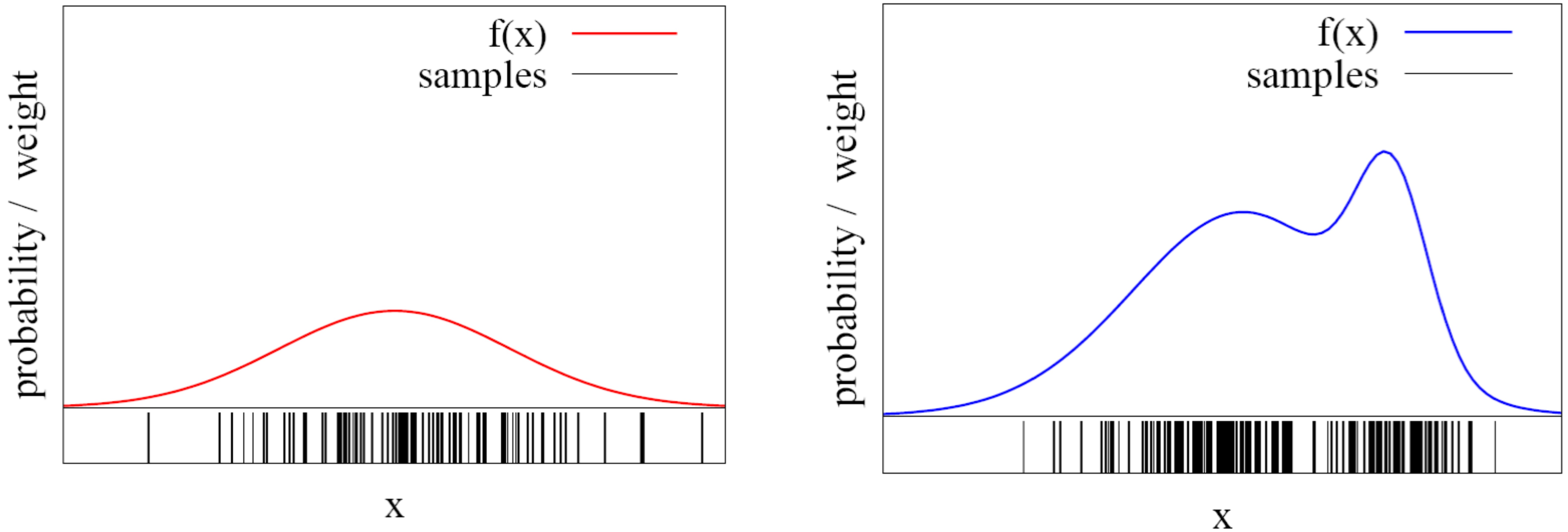
- So far, we discussed the
  - Kalman filter: Gaussian, linearization problems, multi-modal beliefs
- Particle filters are a way to **efficiently** represent **non-Gaussian distributions**
- Basic principle
  - Set of state hypotheses (“particles”)
  - Survival-of-the-fittest

Sample-based Localization (sonar)



Density Approximation

- Particle sets can be used to approximate densities



- The more particles fall into an interval, the higher the probability of that interval

- How to draw samples from a function/distribution?

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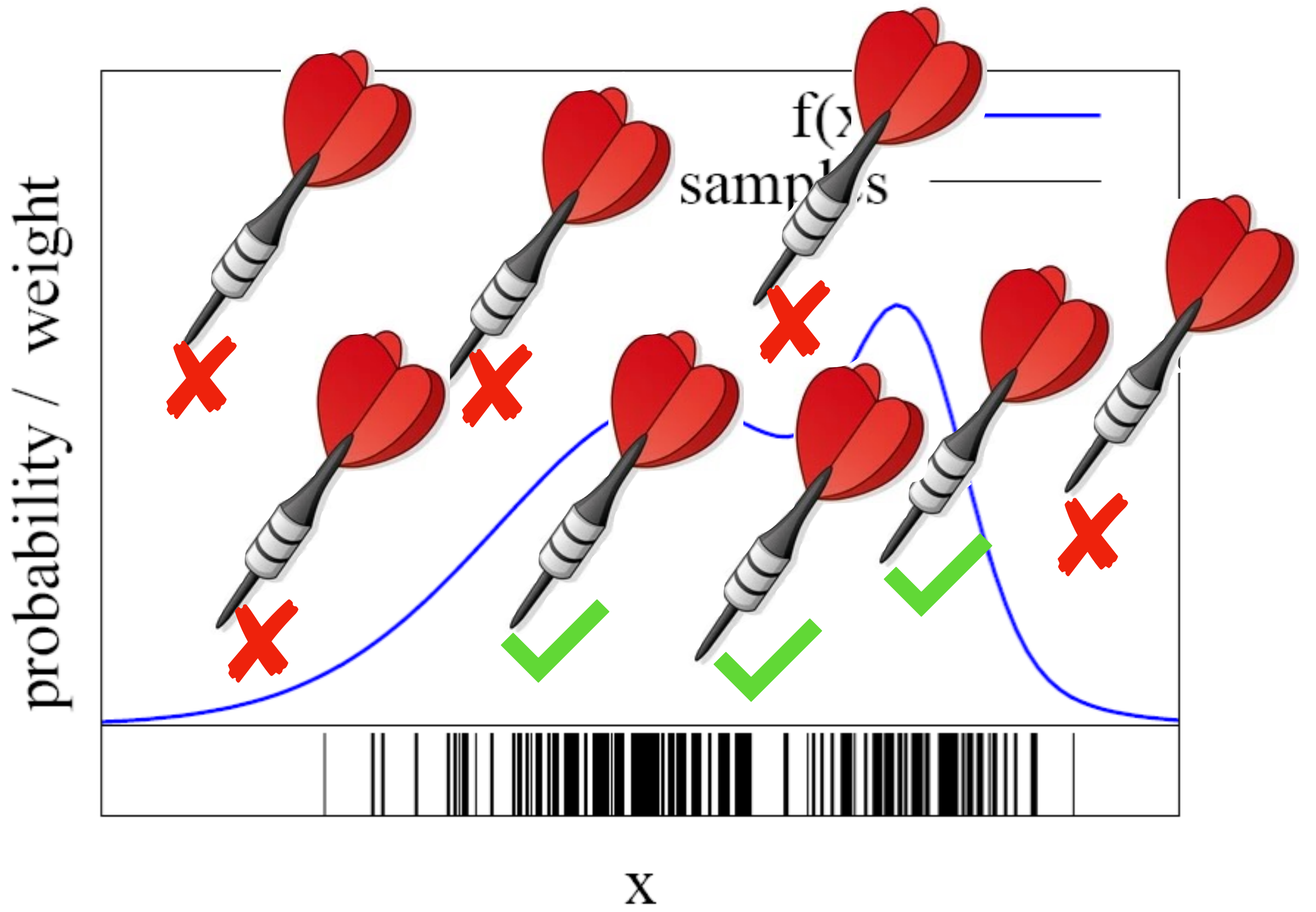
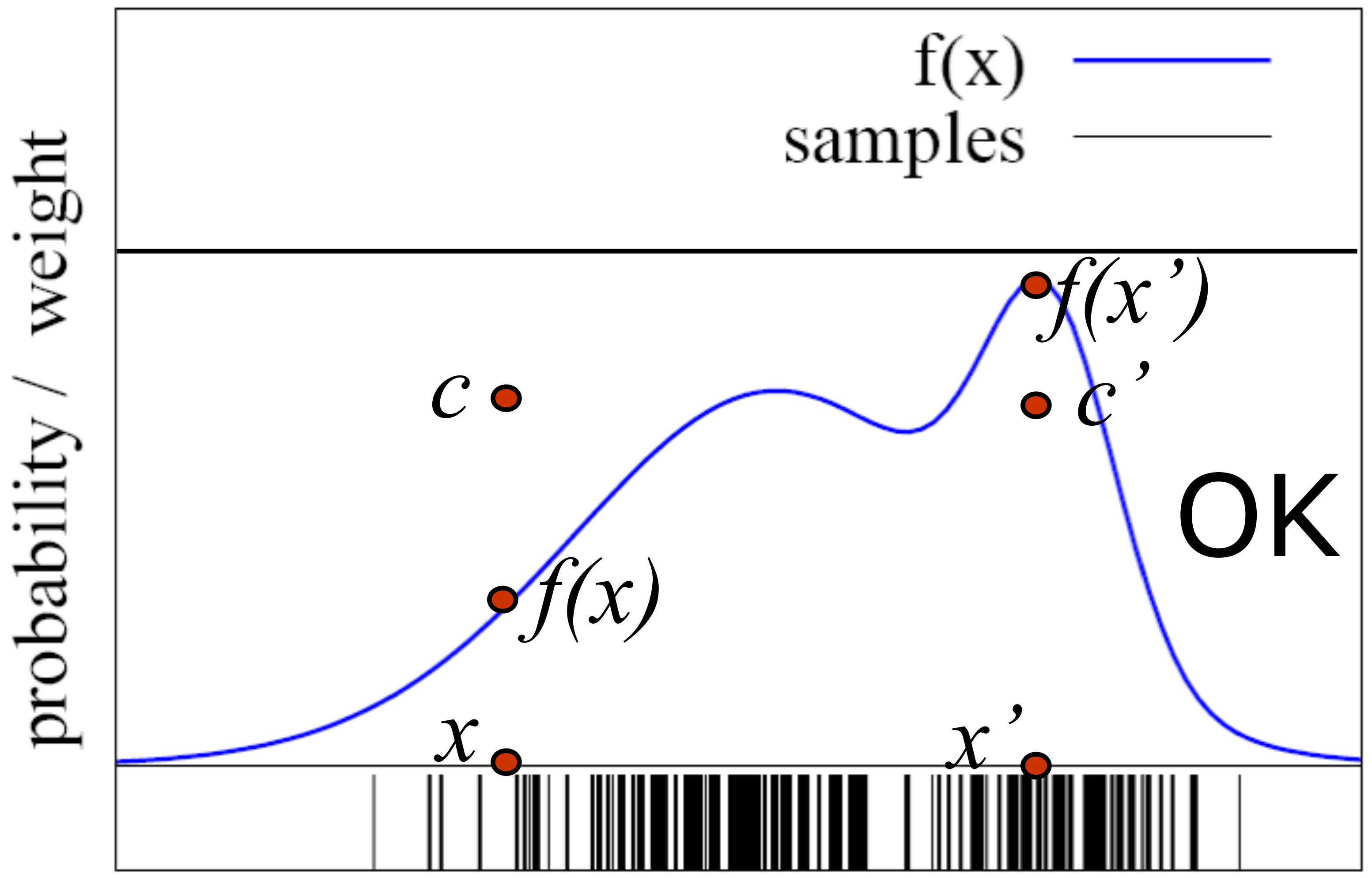
Rejection Sampling

- Let us assume that $f(x) <= 1$ for all x
- Sample x from a uniform distribution
- Sample c from [0,1]
- if $f(x) > c$ keep the sample
  otherwise reject the sampe





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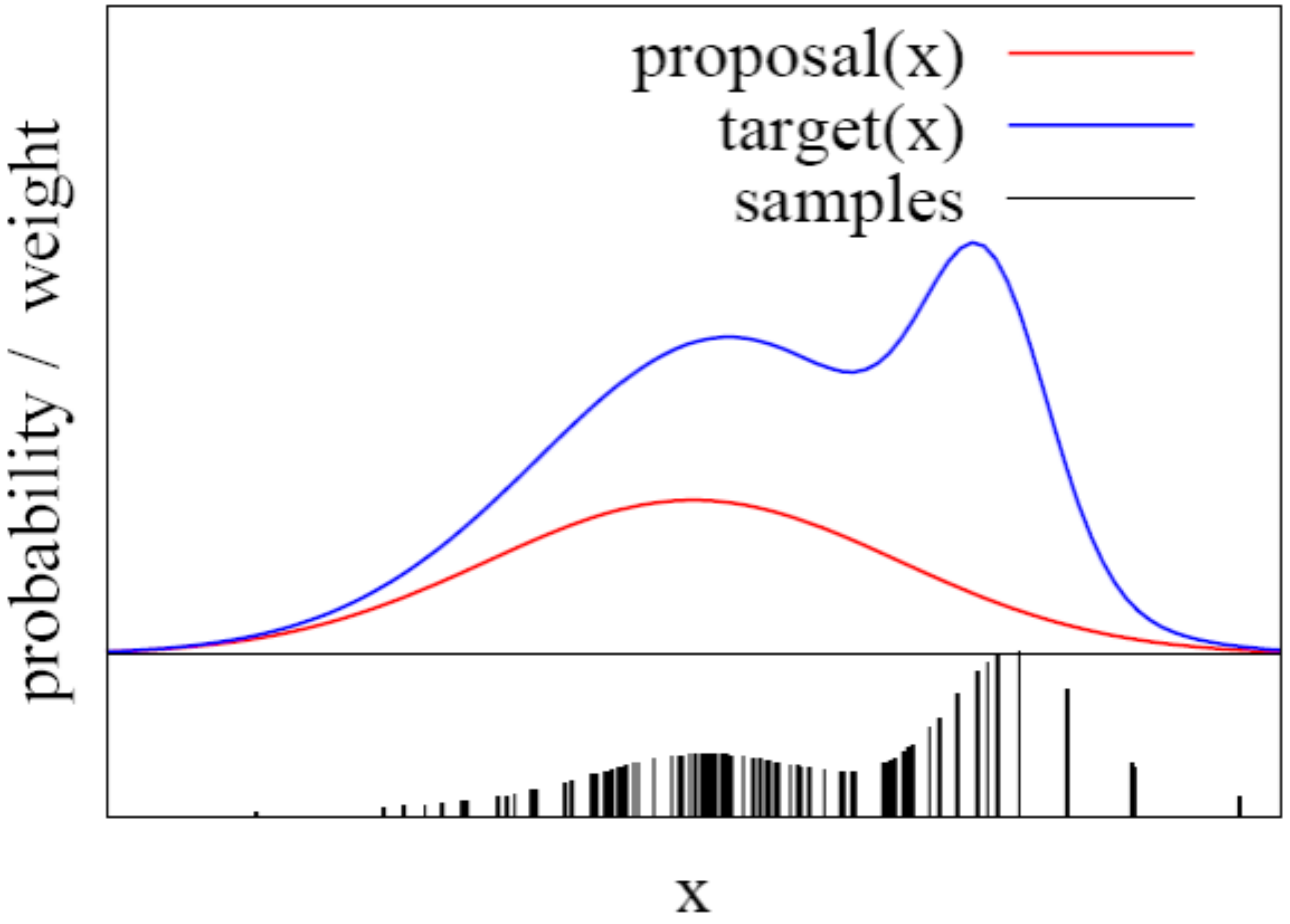
Importance Sampling Principle

- We can even use a different distribution g to generate samples from f
- By introducing an importance weight w, we can account for the "differences between g and f"
- $w = f / g$
- f is often called target
- g is often called proposal



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Particle Filter for State estimation

- Non-parametric approach
- Recursive Bayes Filter
- Models the distribution by samples
- **Prediction:** draw from the proposal g
- **Correction:** weighting by the ratio of the target f and the proposal g

The more samples we use, the better is the estimate

Particle Filter Algorithm

1. Sample the particles using the proposal distribution.

$$x_t^{[j]} \sim \text{proposal}(x_t \mid \ldots)$$

2. Compute the importance weights

$$w_t^{[j]} = \frac{\text{target}(x_t^{[j]})}{\text{proposal}(x_t^{[j]})}$$

3. Resampling: Draw samples i with probability $w_t^{[i]}$ and repeat J times

Particle Filter Algorithm

Particle_filter$(\mathcal{X}_{t-1}, u_t, z_t)$**:**

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

for $j = 1$ to J do

  sample $x_t^{[j]} \sim \pi(x_t)$

  $w_t^{[j]} = \frac{p(x_t^{[j]})}{\pi(x_t^{[j]})}$

  $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[j]}, w_t^{[j]} \rangle$

endfor

for $j = 1$ to J do

  draw $i \in 1, \ldots, J$ with probability $\propto w_t^{[i]}$

  add $x_t^{[i]}$ to $\mathcal{X}_t$

endfor

return $\mathcal{X}_t$

Particle Filter Algorithm

$$Bel(x_t) = \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})\, dx_{t-1}$$

draw x^i_{t-1} from $Bel(x_{t-1})$

draw x^i_t from $p(x_t \mid x^i_{t-1}, u_{t-1})$

Importance factor for x^i_t:

$$w^i_t = \frac{\text{target distribution}}{\text{proposal distribution}}$$

$$= \frac{\eta\, p(z_t \mid x_t)\, p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})}$$

$$\propto p(z_t \mid x_t)$$

Particle Filter

Particle_filter($\mathcal{X}_{t-1}, u_t, z_t$):

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

$\quad$ *for* $j = 1$ *to* J *do*

$\quad\quad$ *sample* $x_t^{[j]} \sim \pi(x_t)$

$\quad\quad$ $w_t^{[j]} = \frac{p(x_t^{[j]})}{\pi(x_t^{[j]})}$

$\quad\quad$ $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[j]}, w_t^{[j]} \rangle$

$\quad$ *endfor*

$\quad$ *for* $j = 1$ *to* J *do*

$\quad\quad$ *draw* $i \in 1, \ldots, J$ *with probability* $\propto w_t^{[i]}$

$\quad\quad$ *add* $x_t^{[i]}$ *to* $\mathcal{X}_t$

$\quad$ *endfor*

$\quad$ *return* $\mathcal{X}_t$

Particle Filter for Localization

Particle_filter($\mathcal{X}_{t-1}, u_t, z_t$):

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

$\quad$ *for* $j = 1$ *to* J *do*

$\quad\quad$ *sample* $x_t^{[j]} \sim p(x_t \mid u_t, x_{t-1}^{[j]})$

$\quad\quad$ $w_t^{[j]} = p(z_t \mid x_t^{[j]})$

$\quad\quad$ $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[j]}, w_t^{[j]} \rangle$

$\quad$ *endfor*

$\quad$ *for* $j = 1$ *to* J *do*

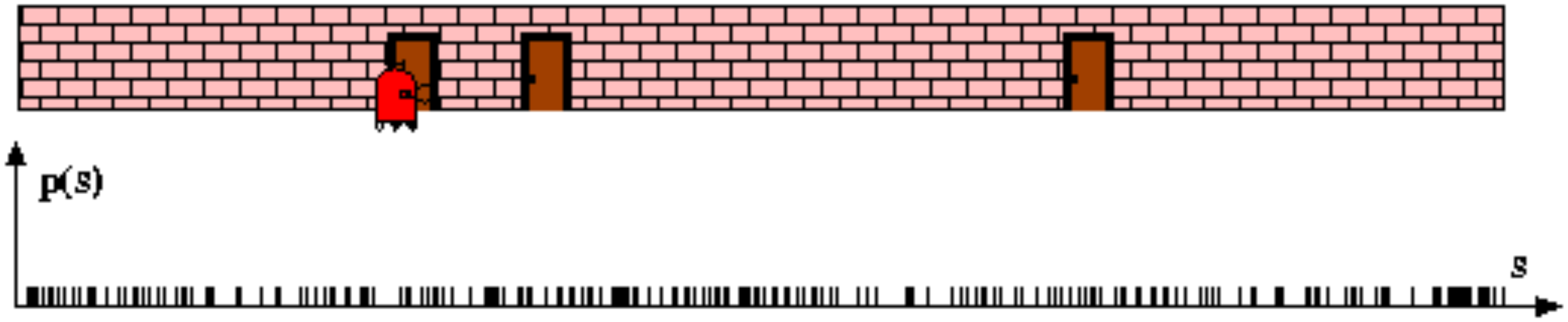
$\quad\quad$ *draw* $i \in 1, \ldots, J$ *with probability* $\propto w_t^{[i]}$

$\quad\quad$ *add* $x_t^{[i]}$ *to* $\mathcal{X}_t$

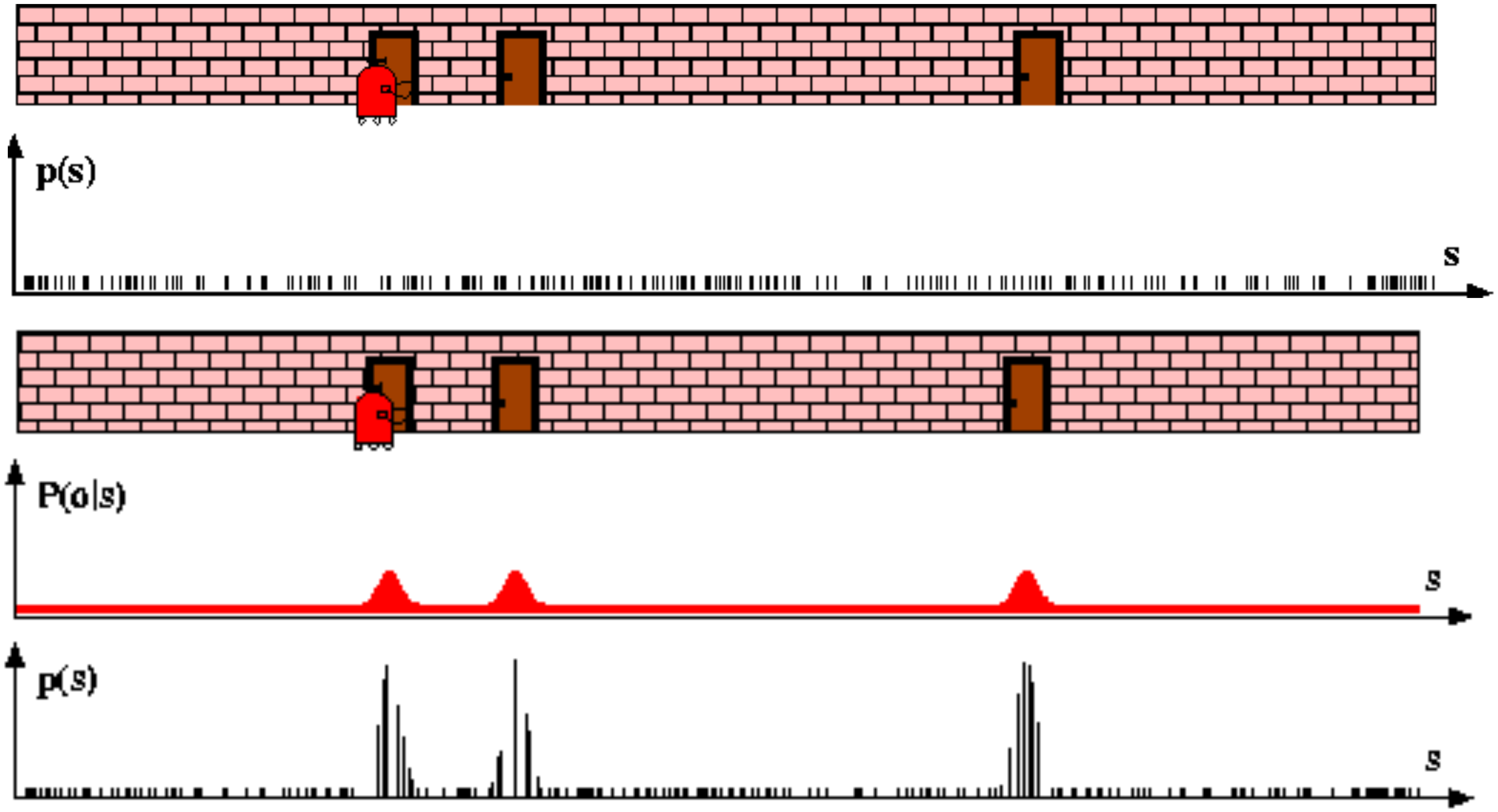
$\quad$ *endfor*

$\quad$ *return* $\mathcal{X}_t$

Particle Filters



Sensor Information: Importance Sampling



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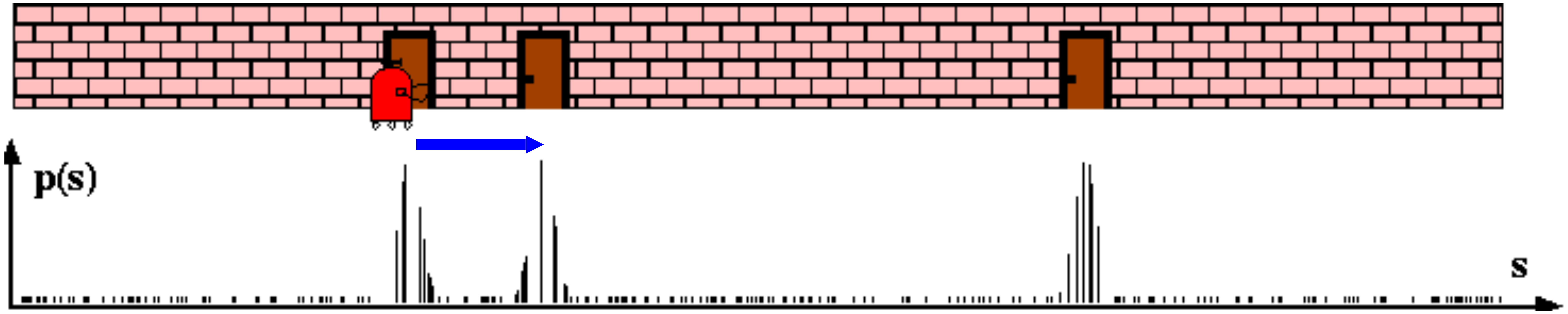
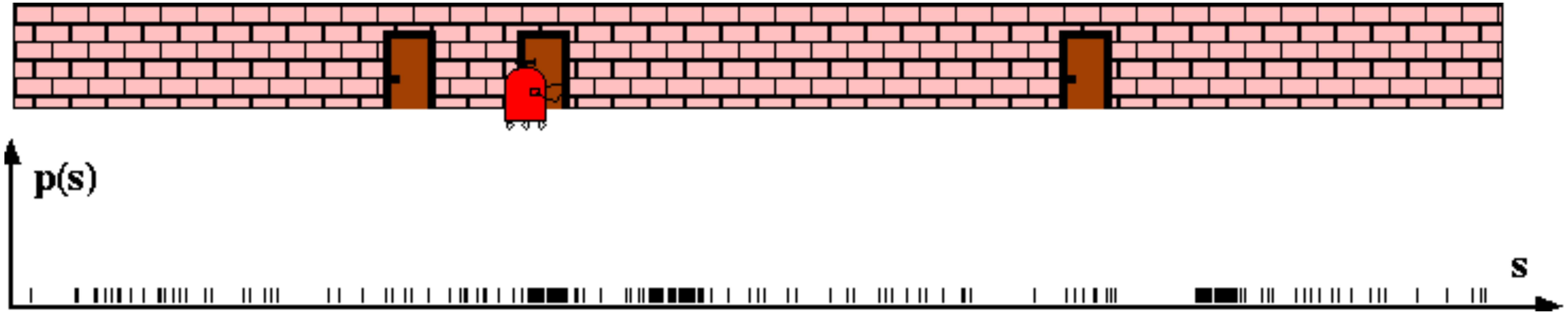
Robot Motion



Resampling Step +
Control Input



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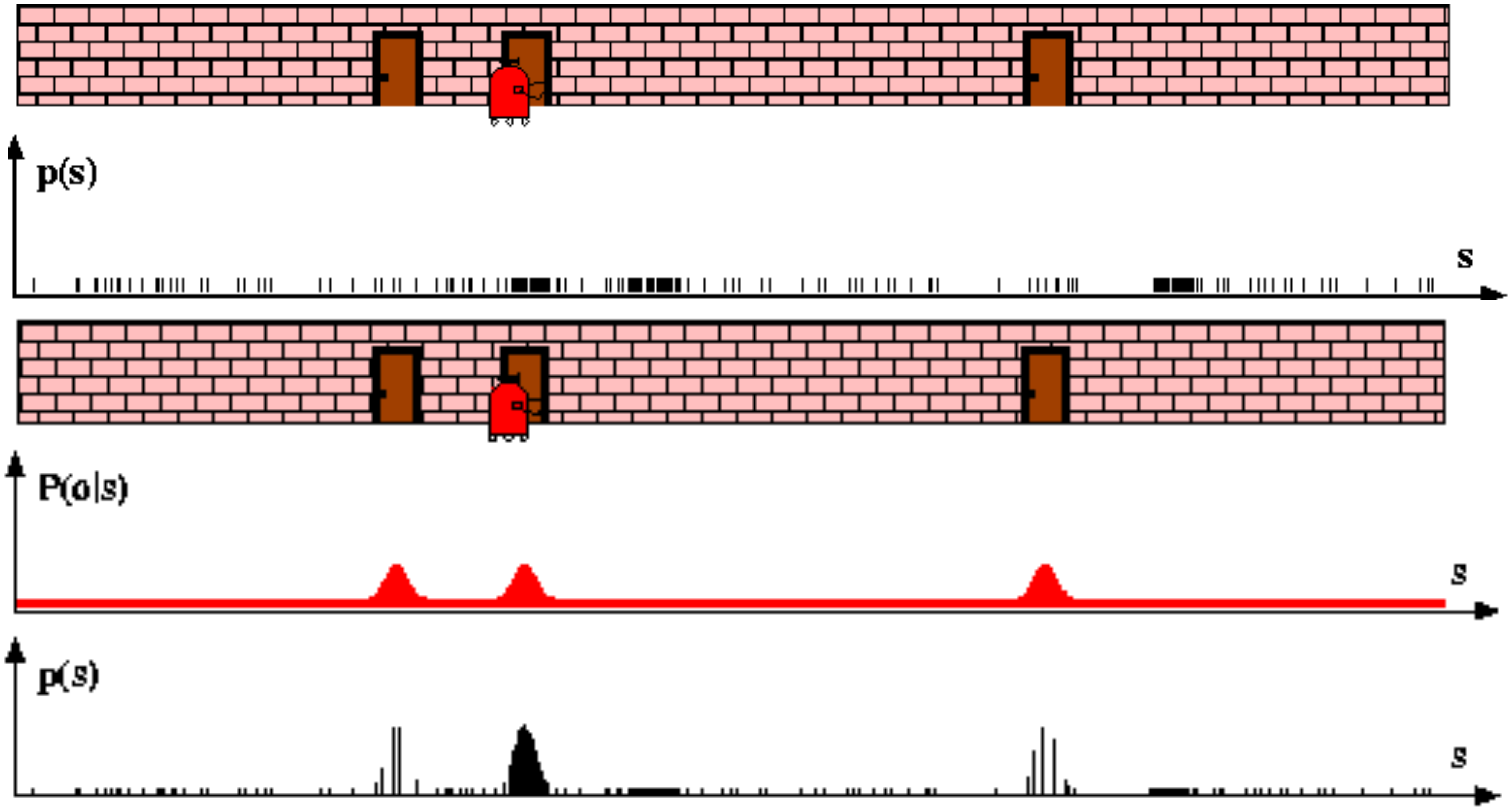
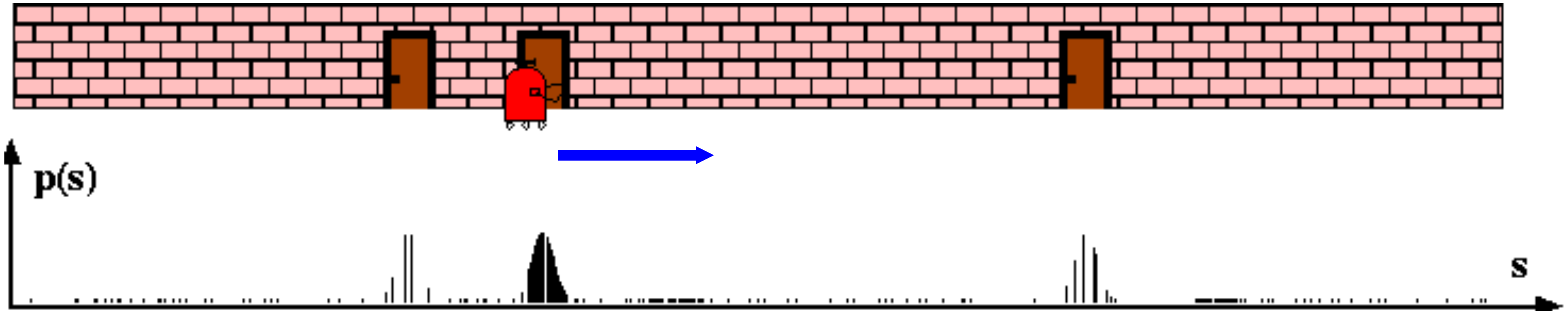


Robot Motion



Resampling Step +
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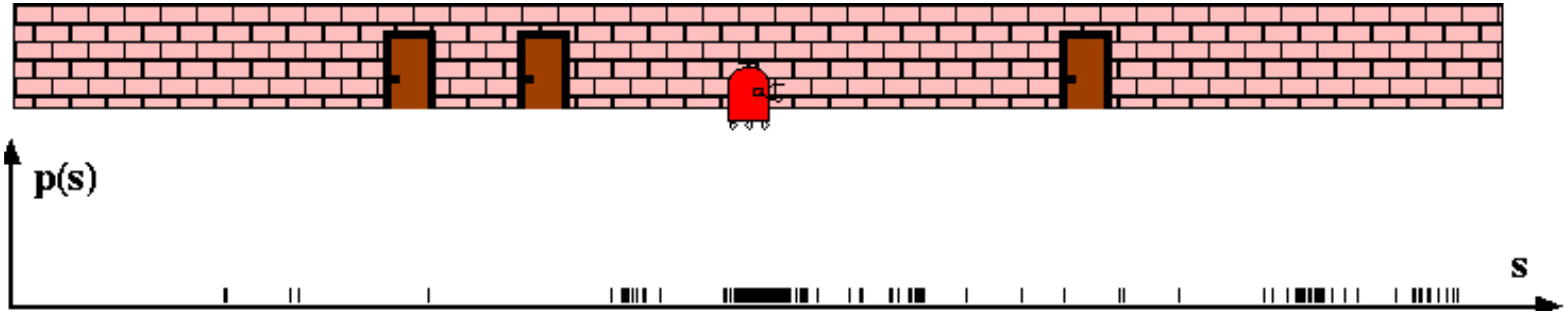

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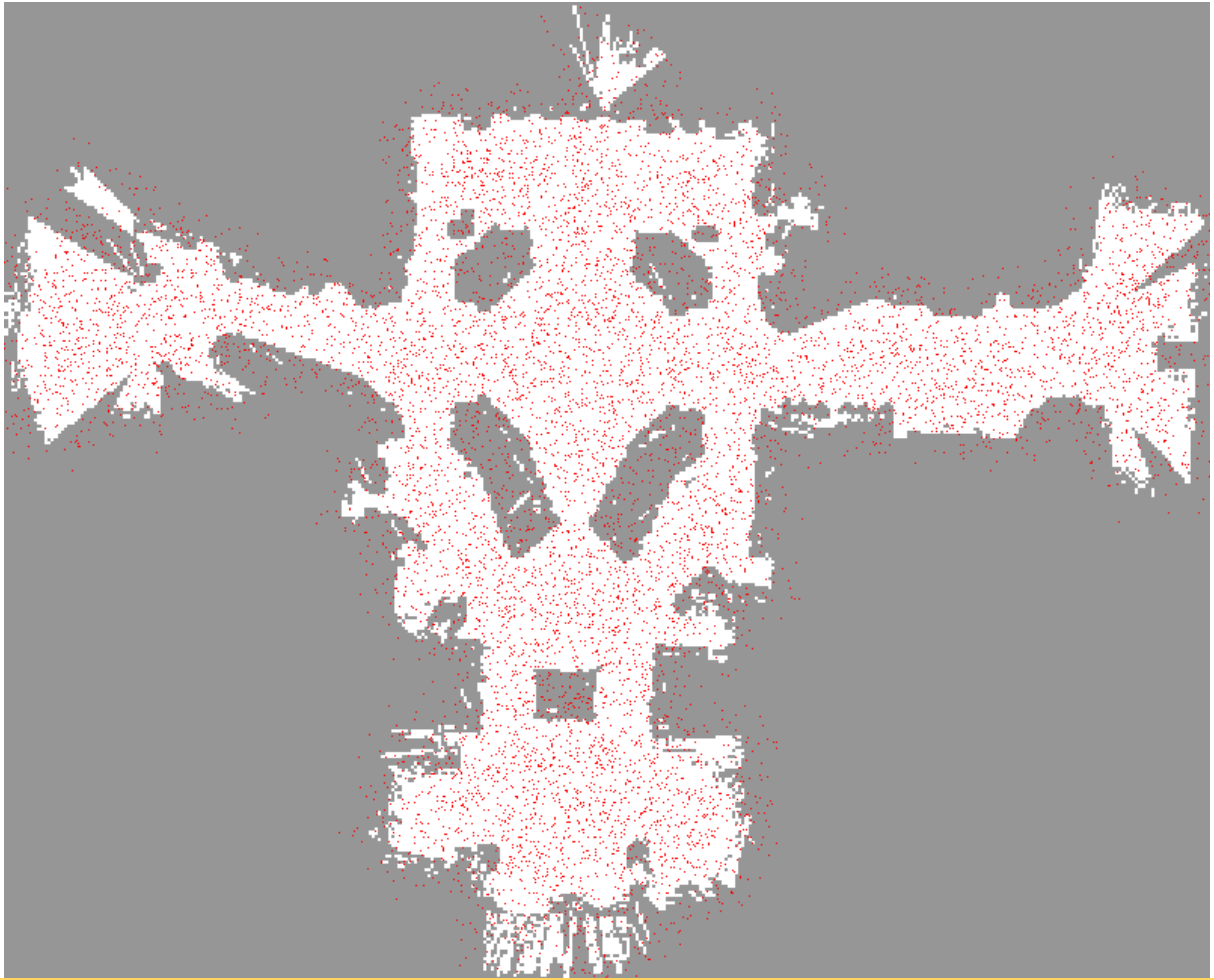
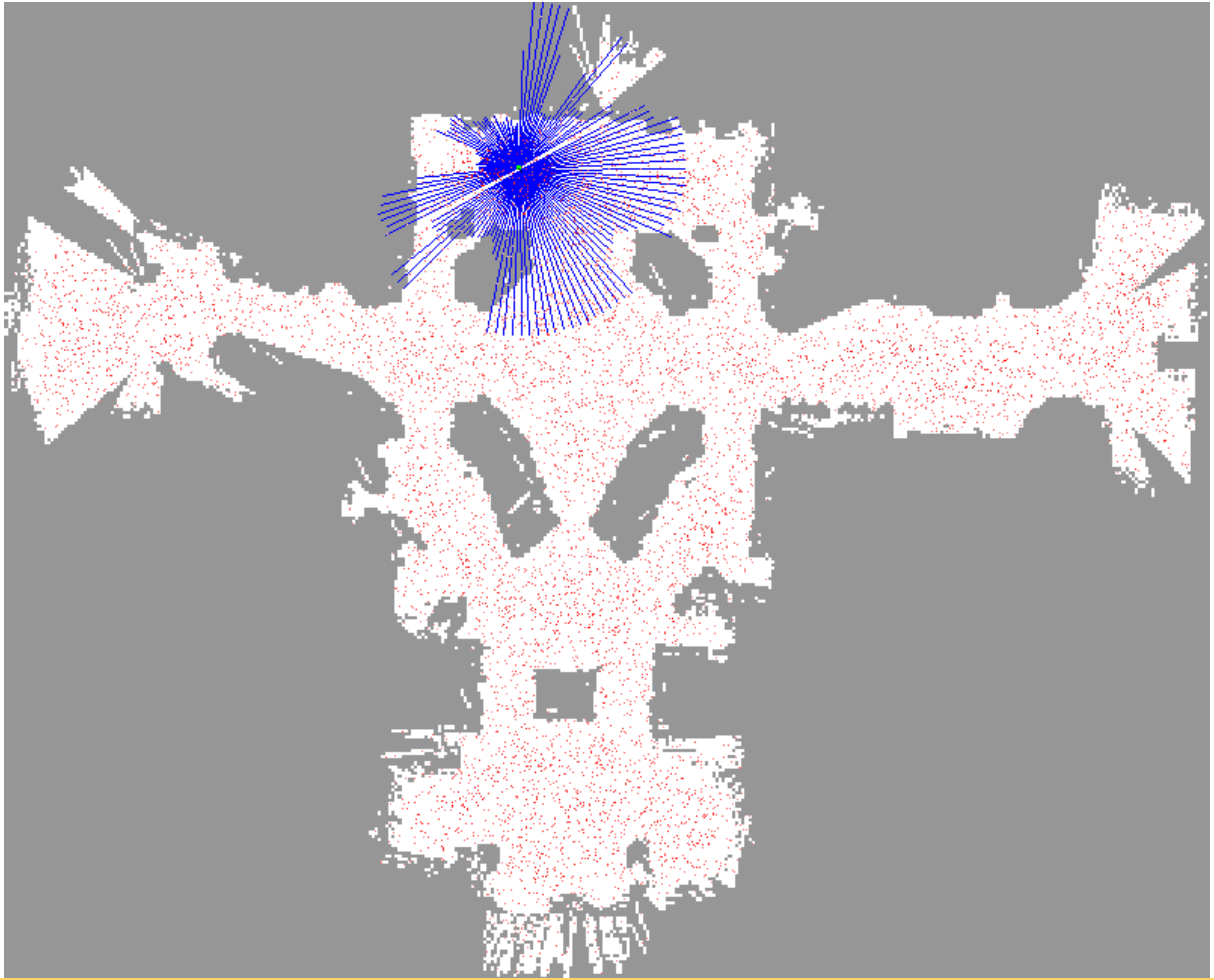


Observation Taken



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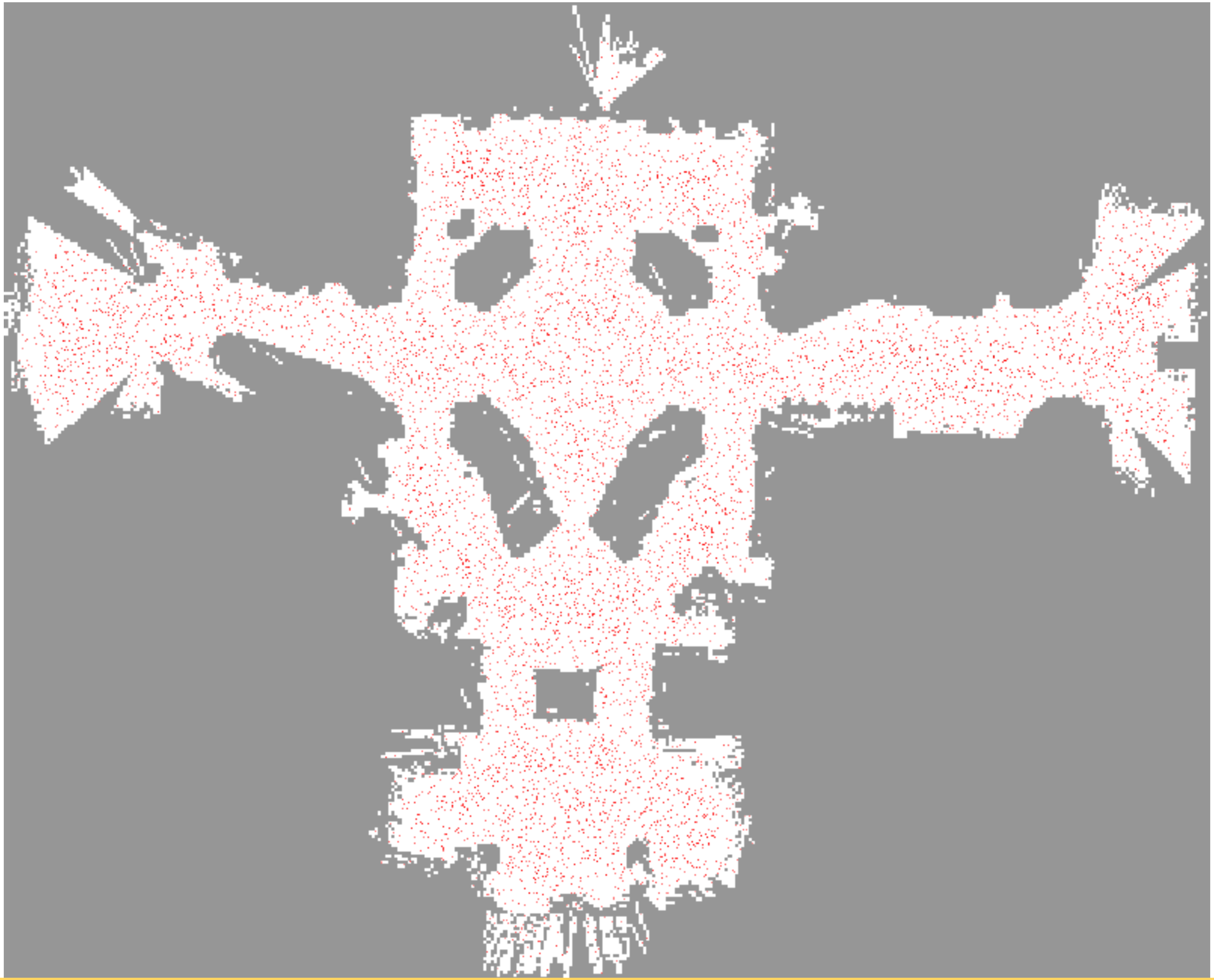
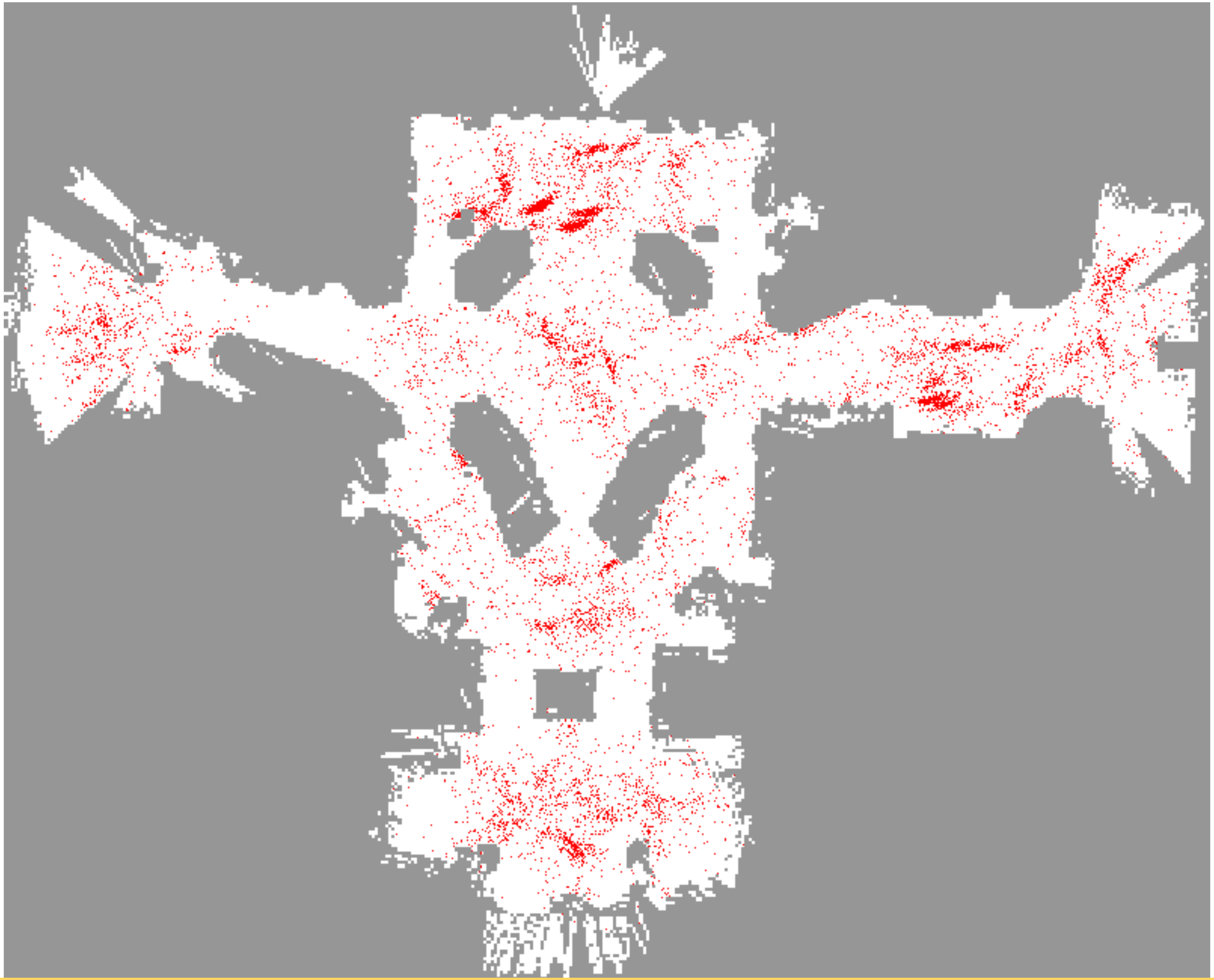


Measurement Update



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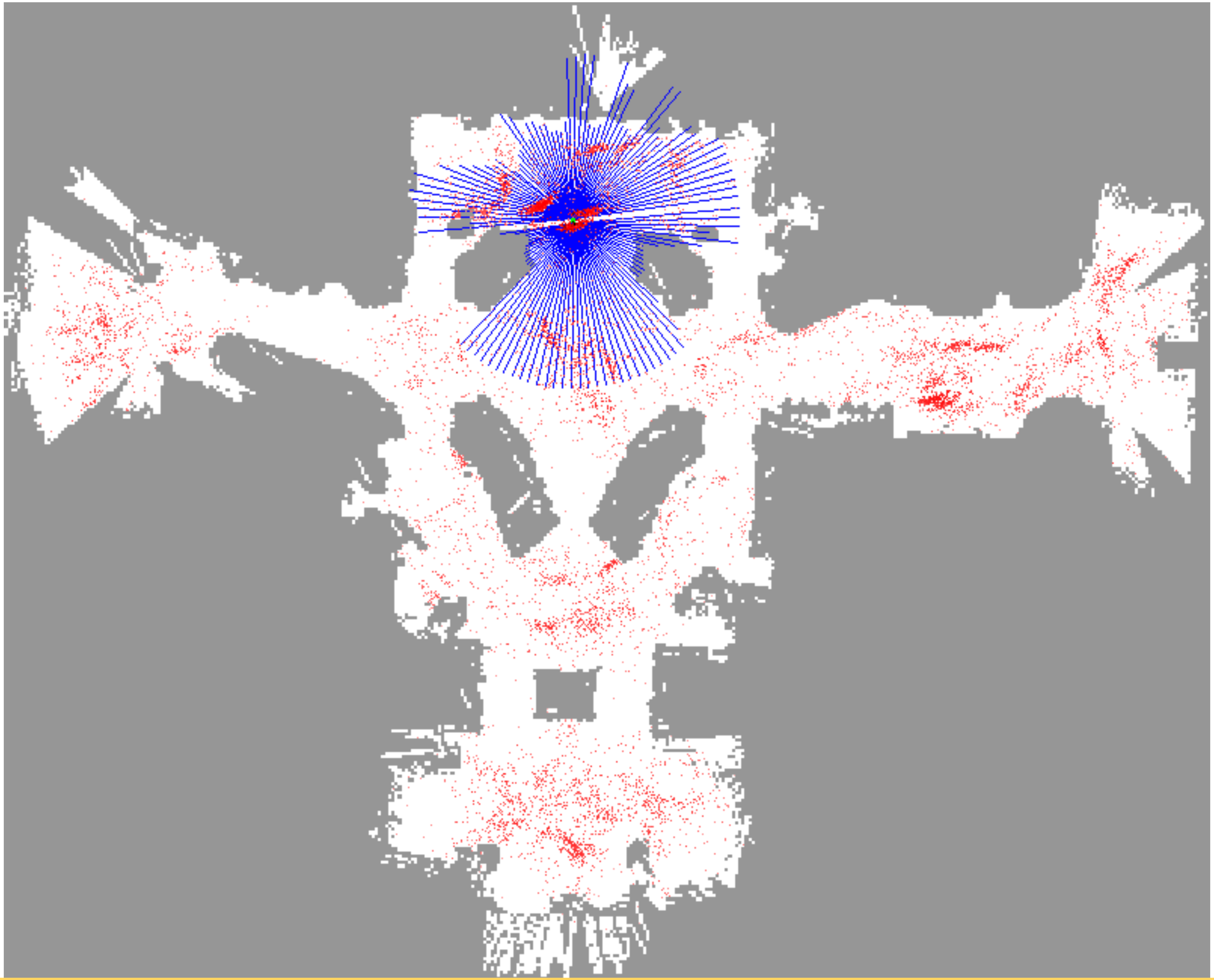

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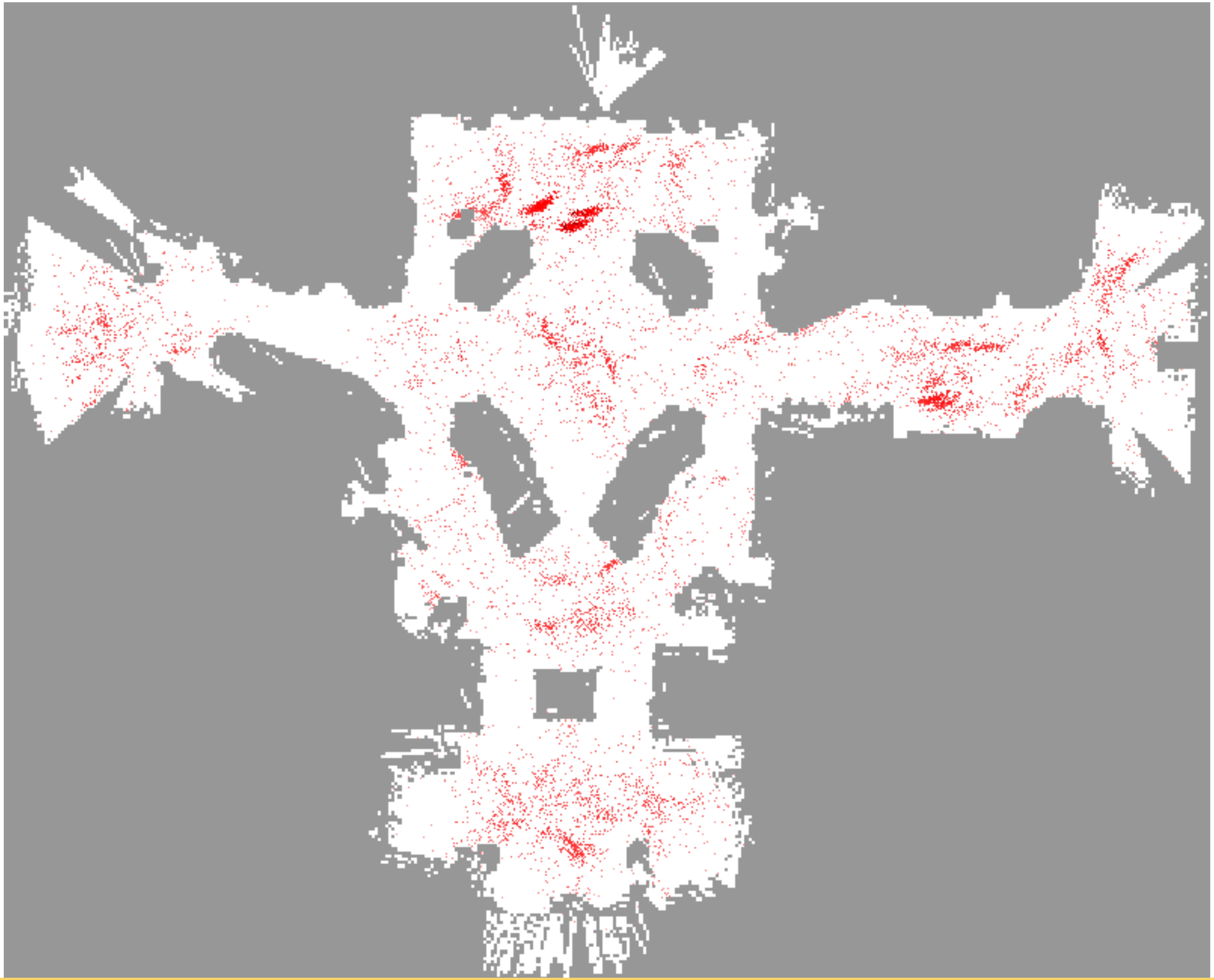

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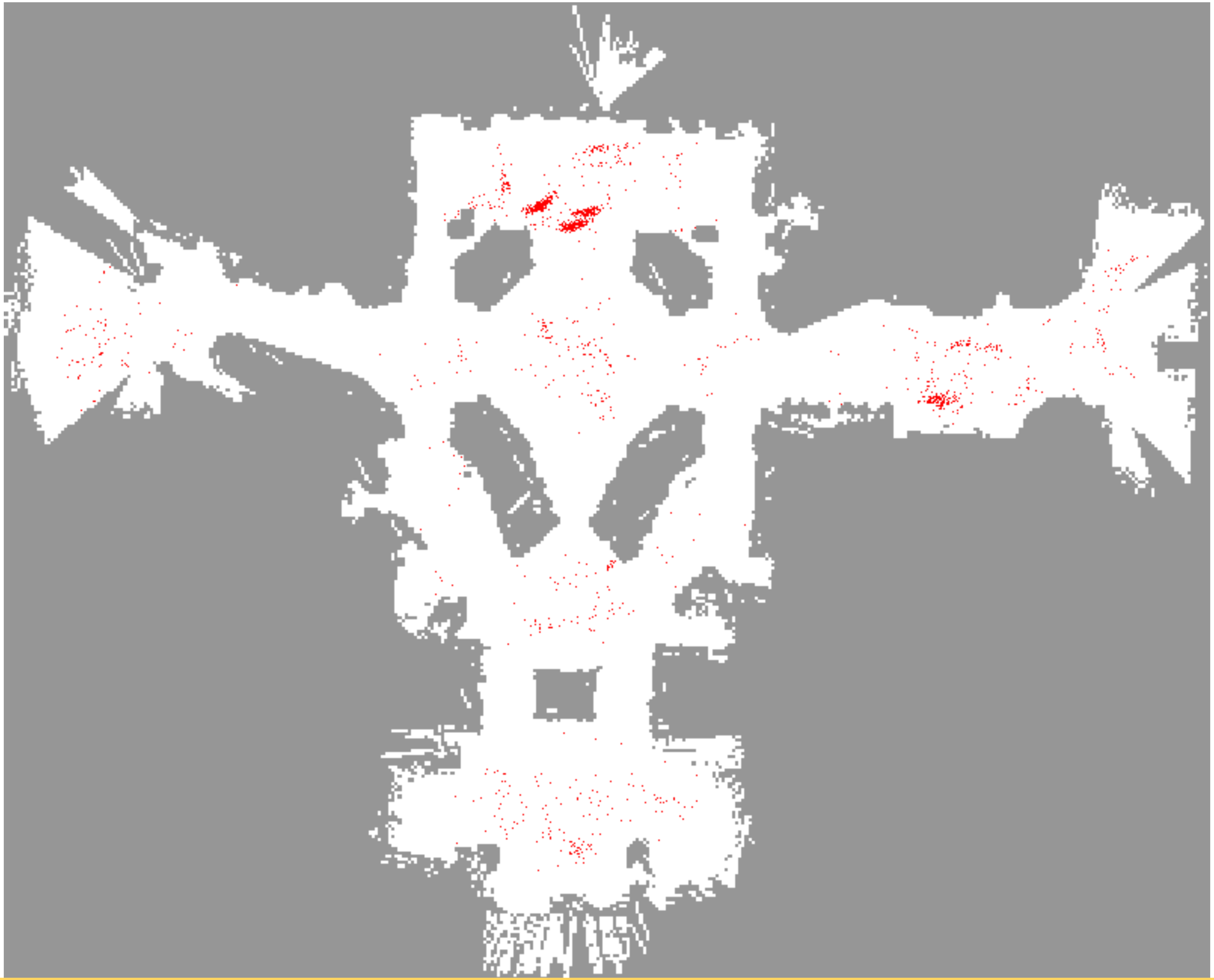

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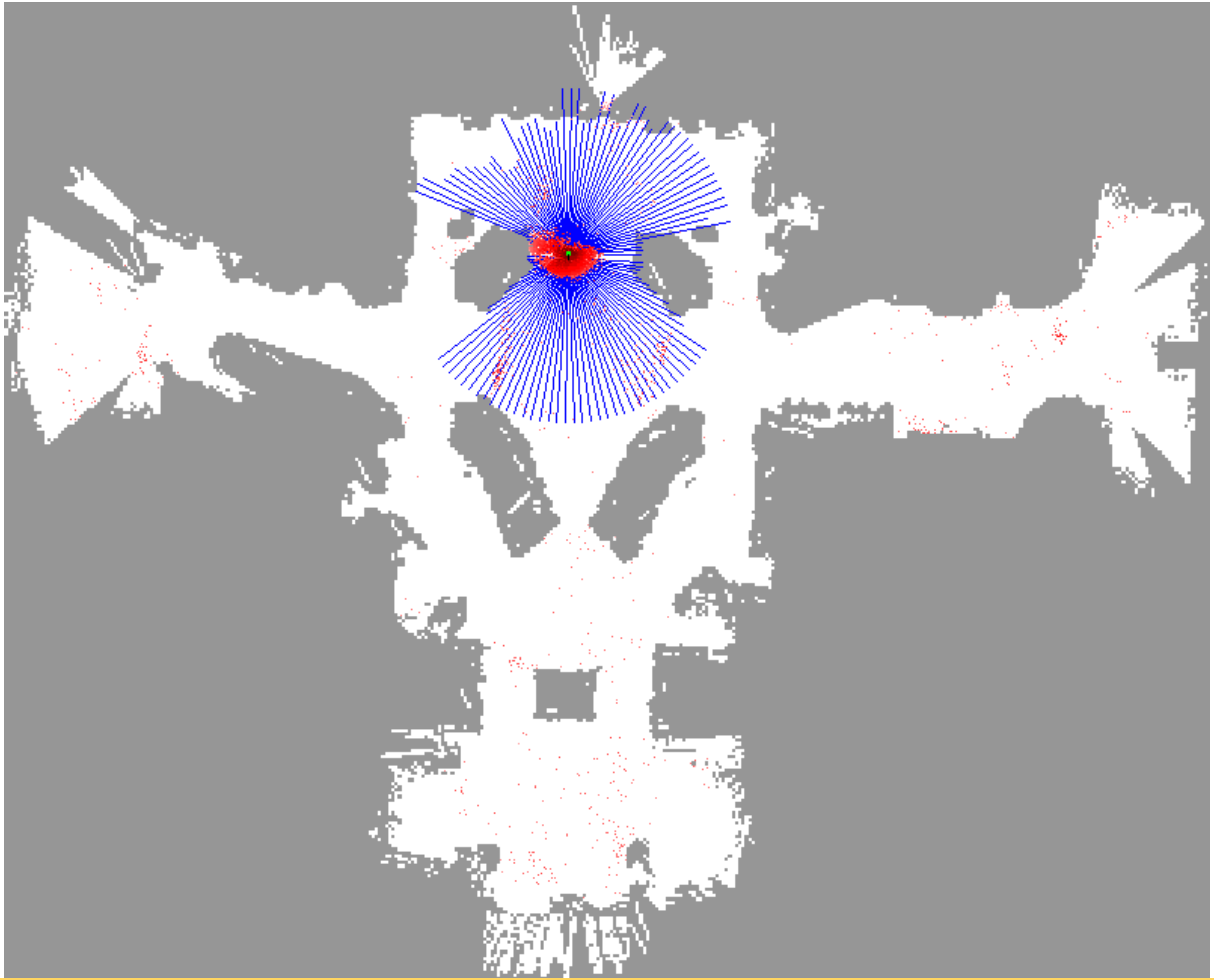
Motion Update

Observation Taken



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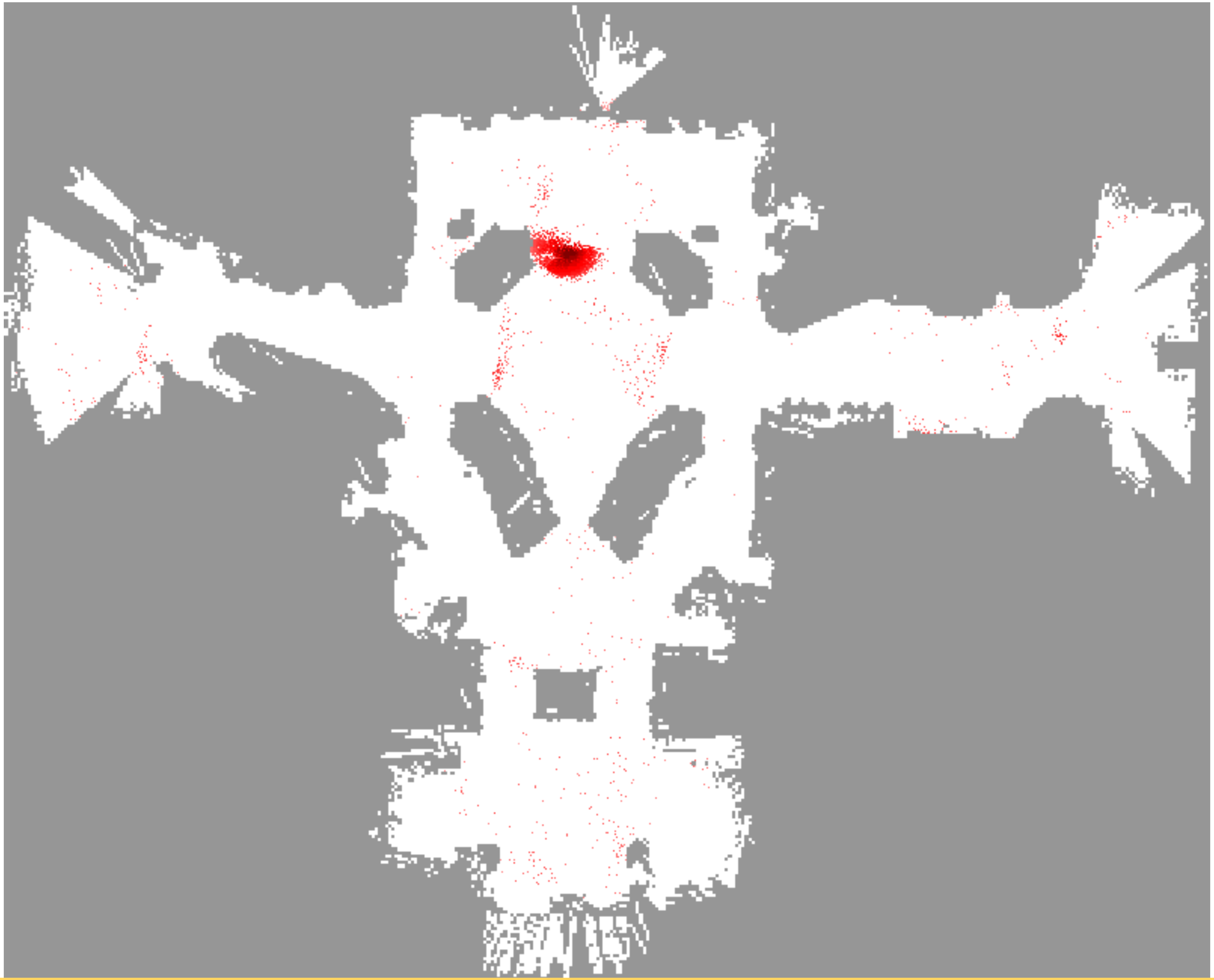

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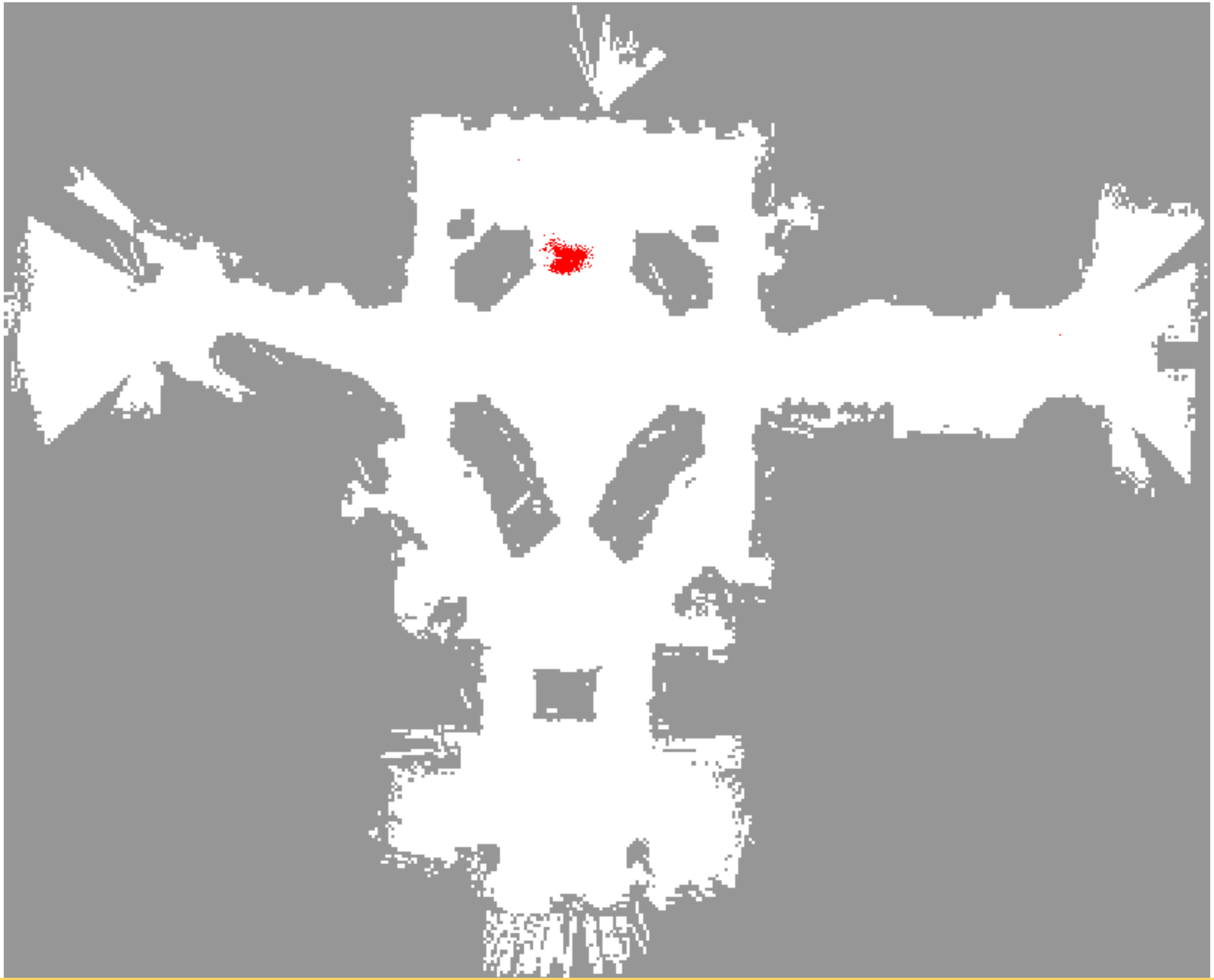
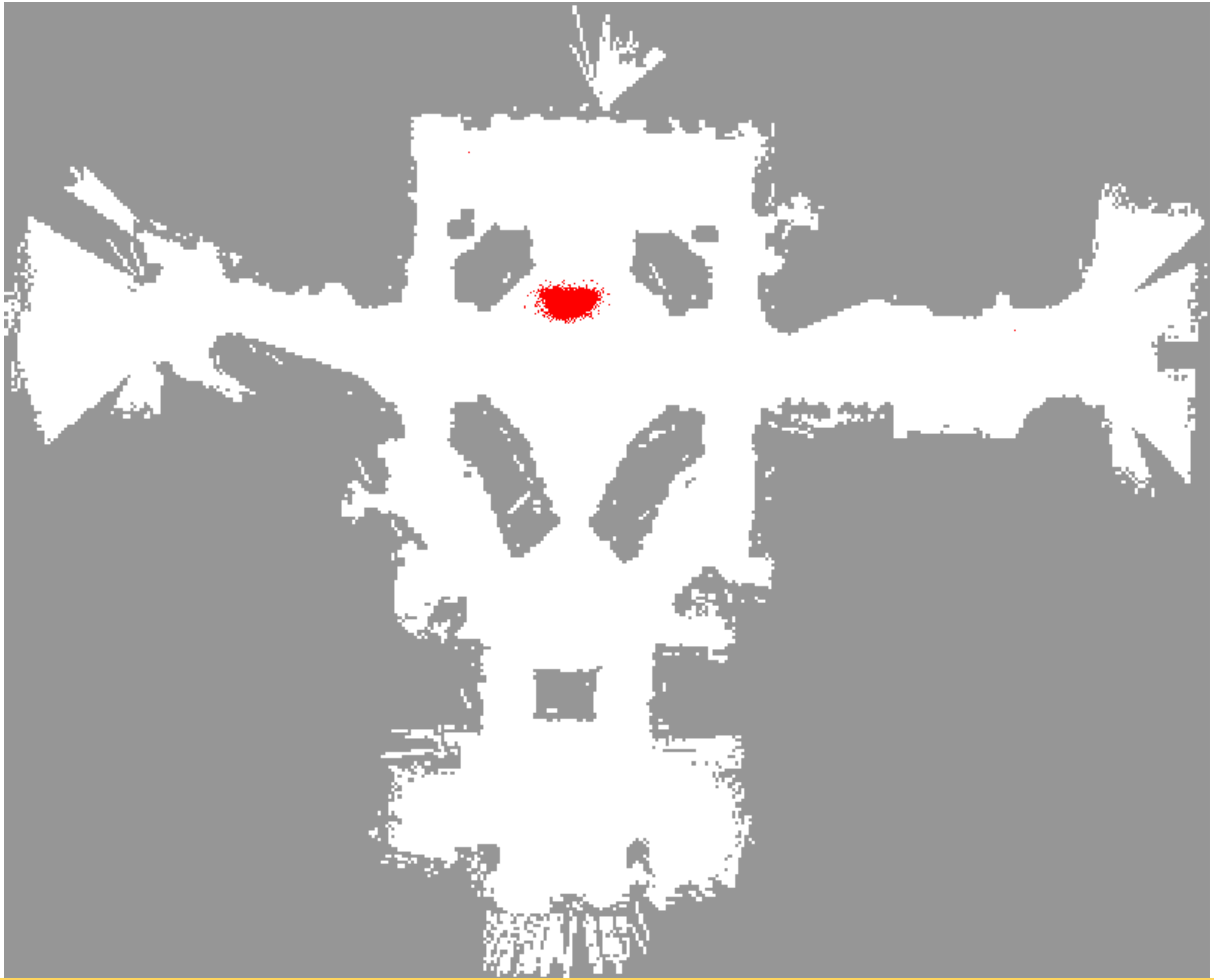


Motion Update



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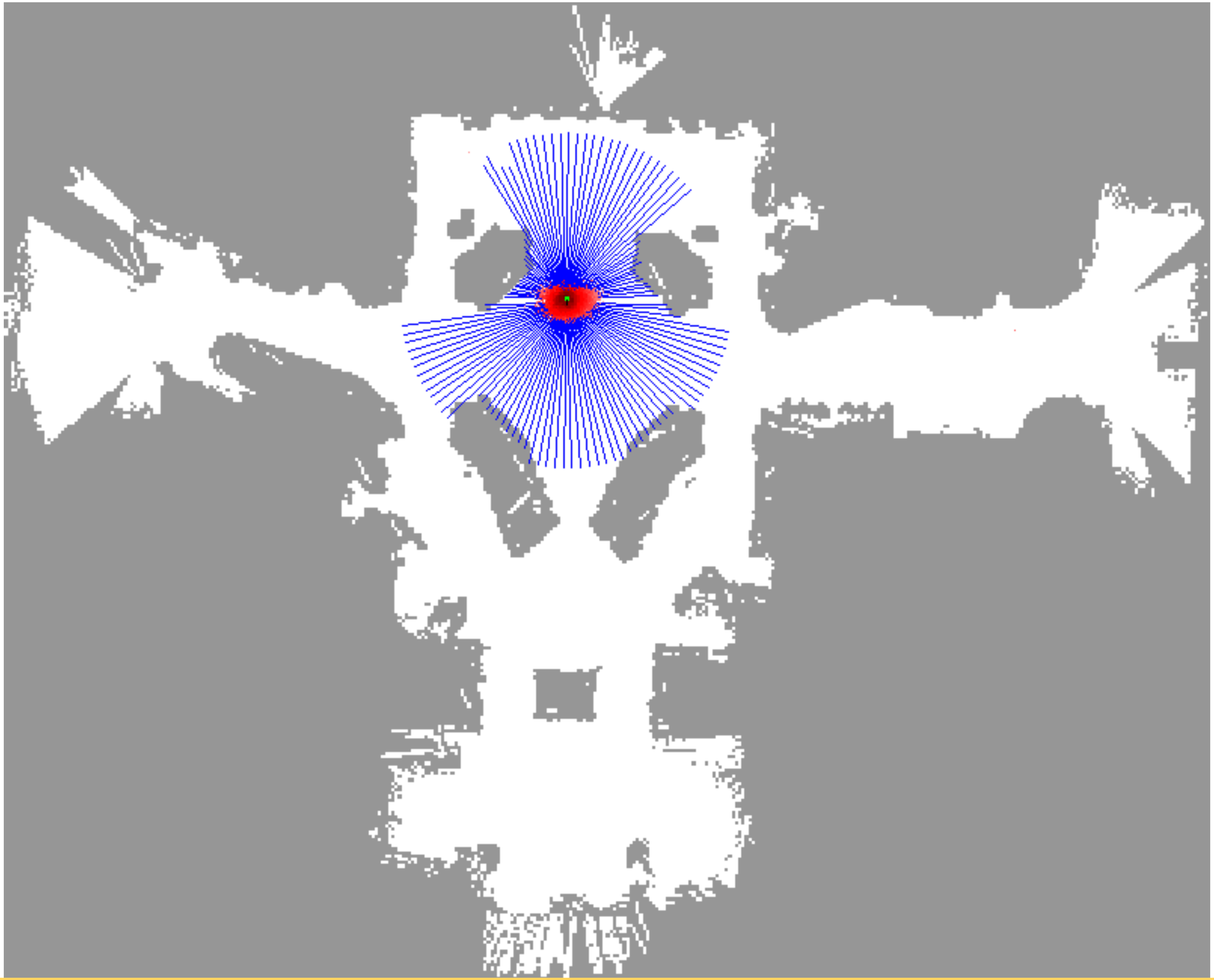

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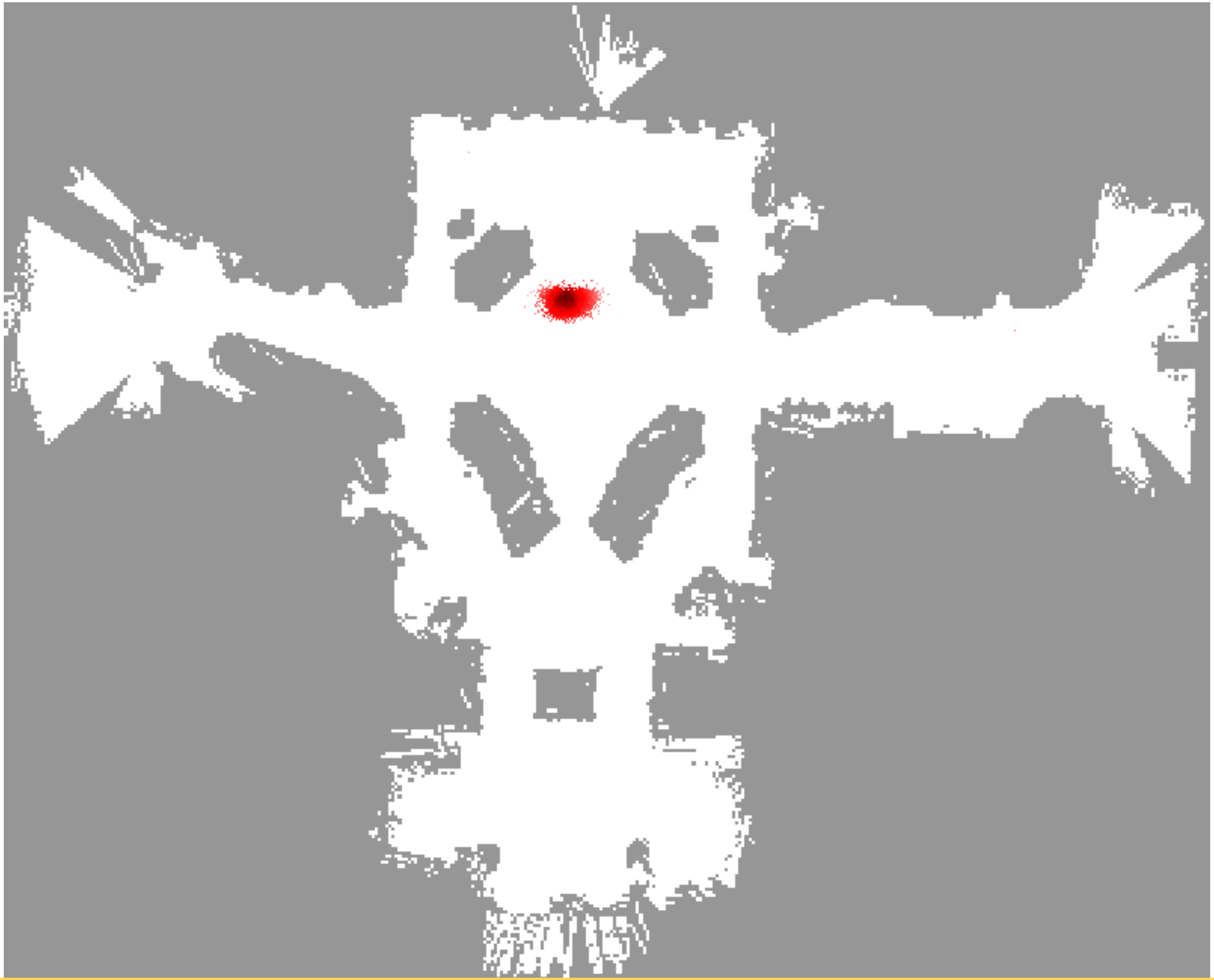

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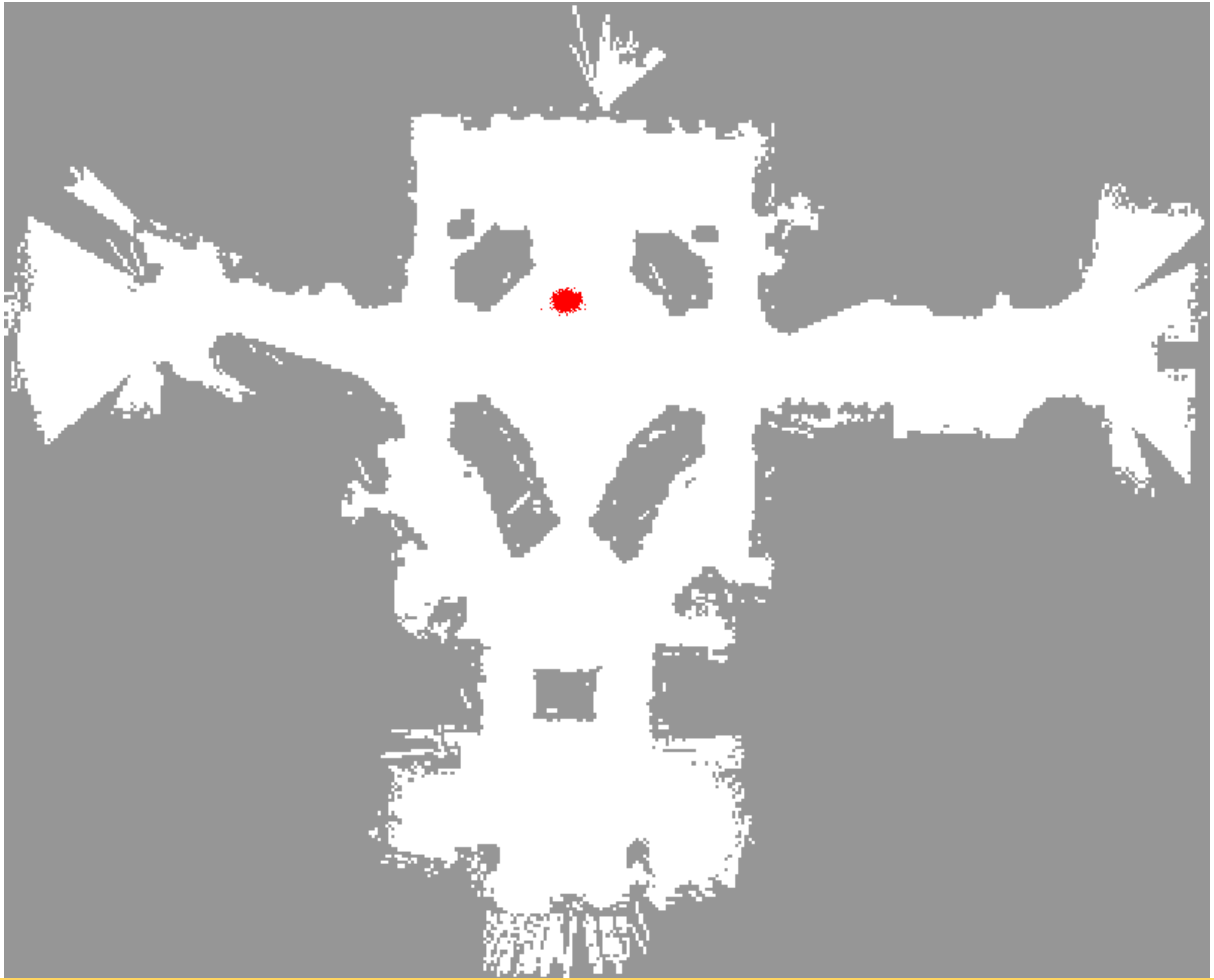
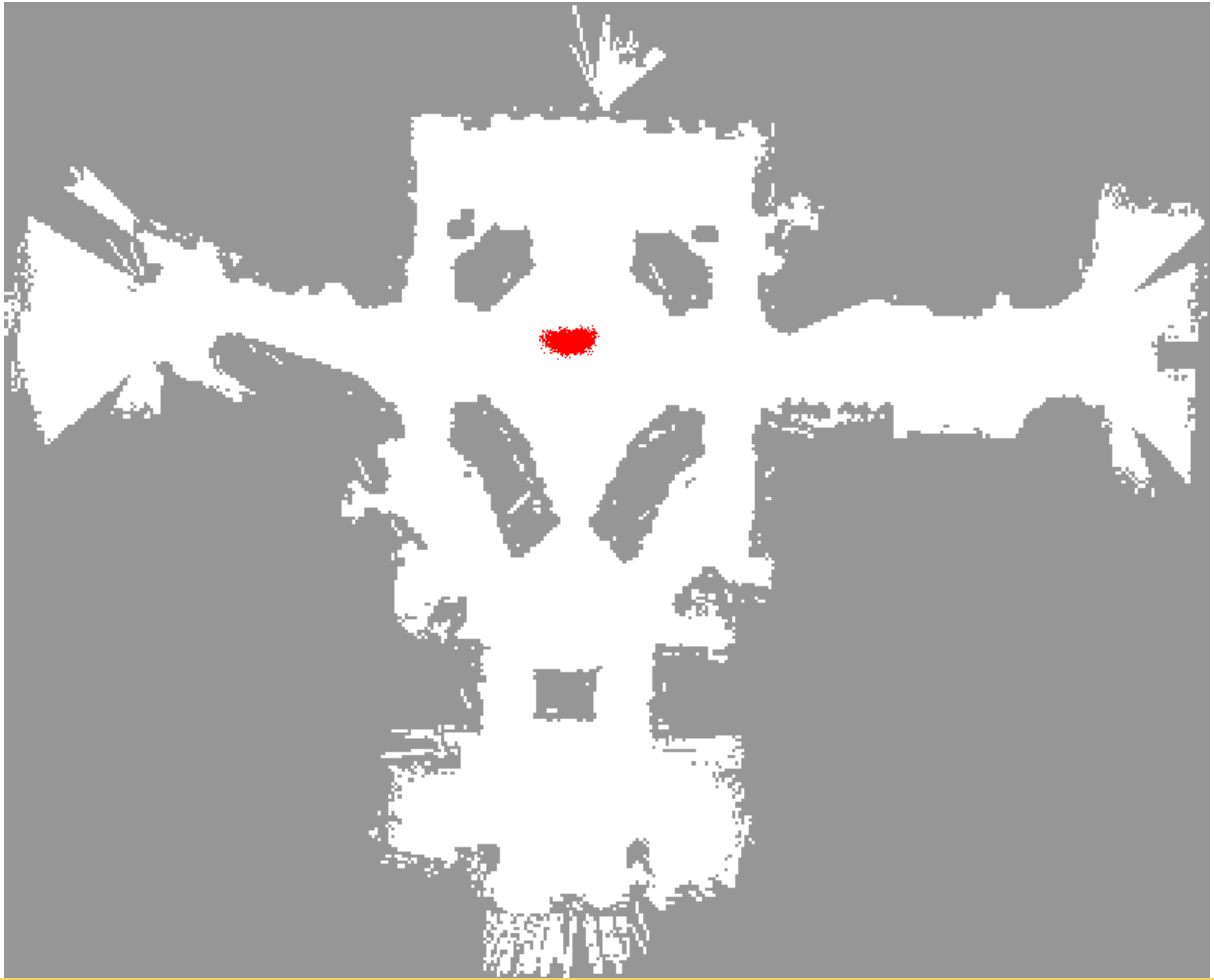


Motion Update



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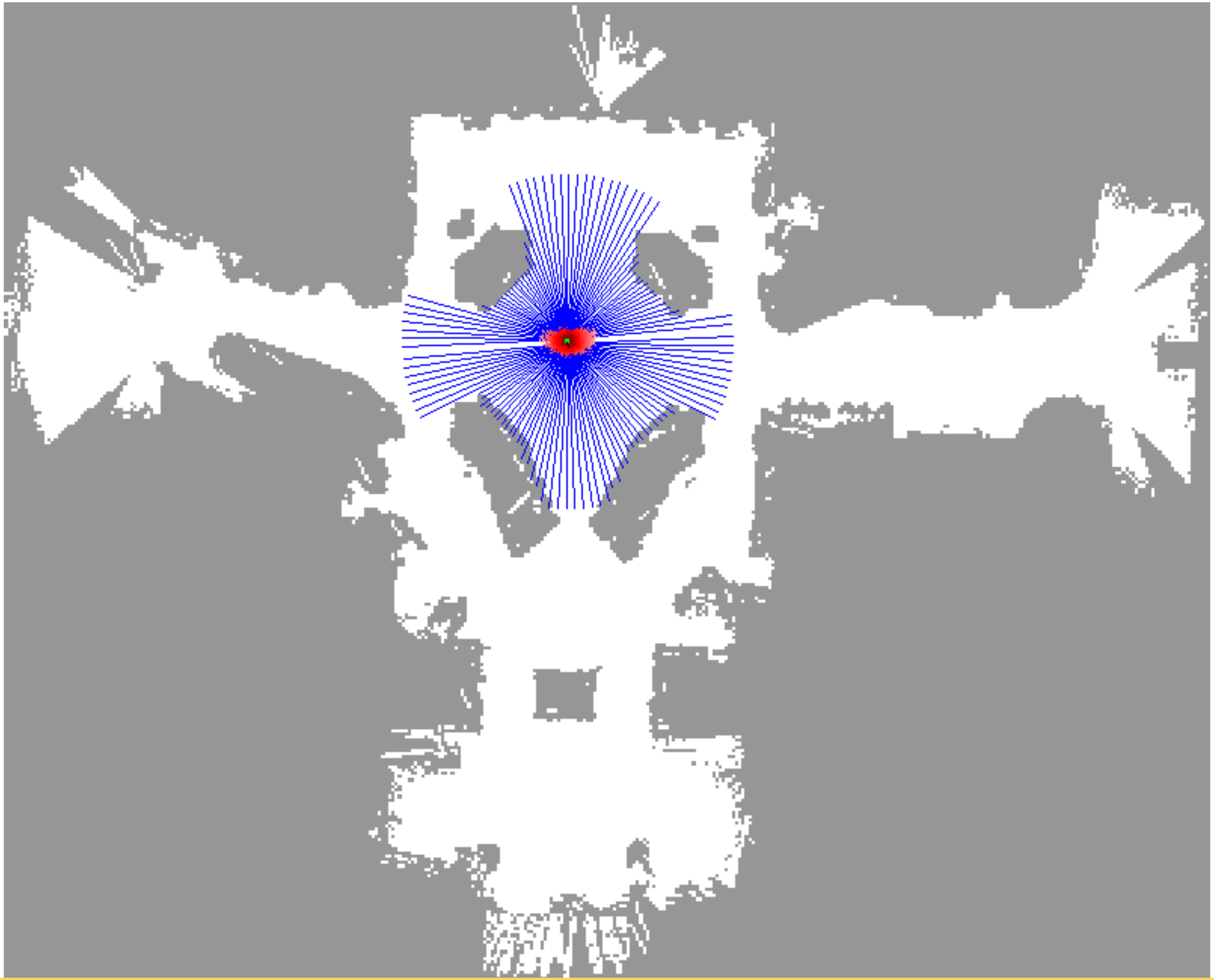
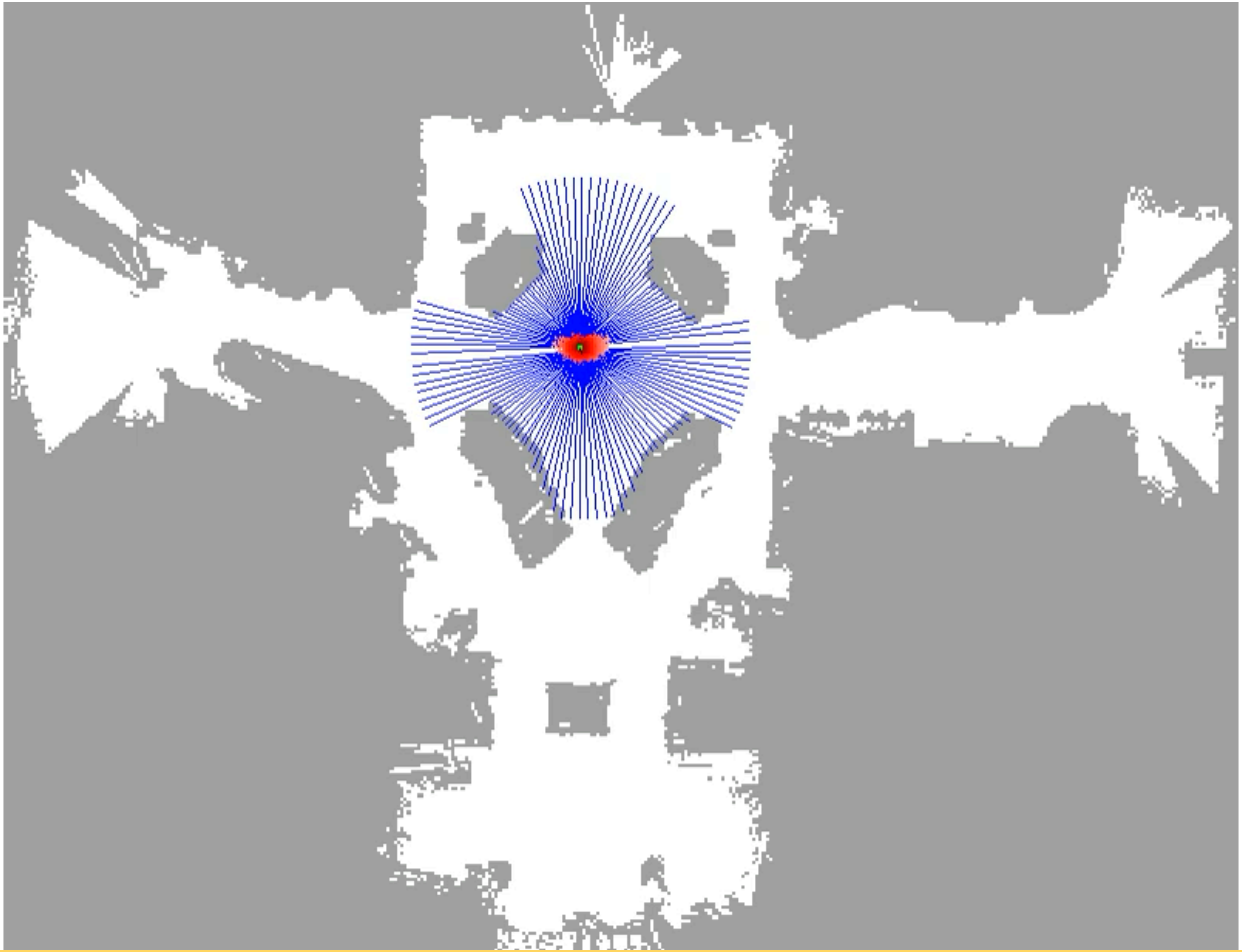


Particle Filter in Action



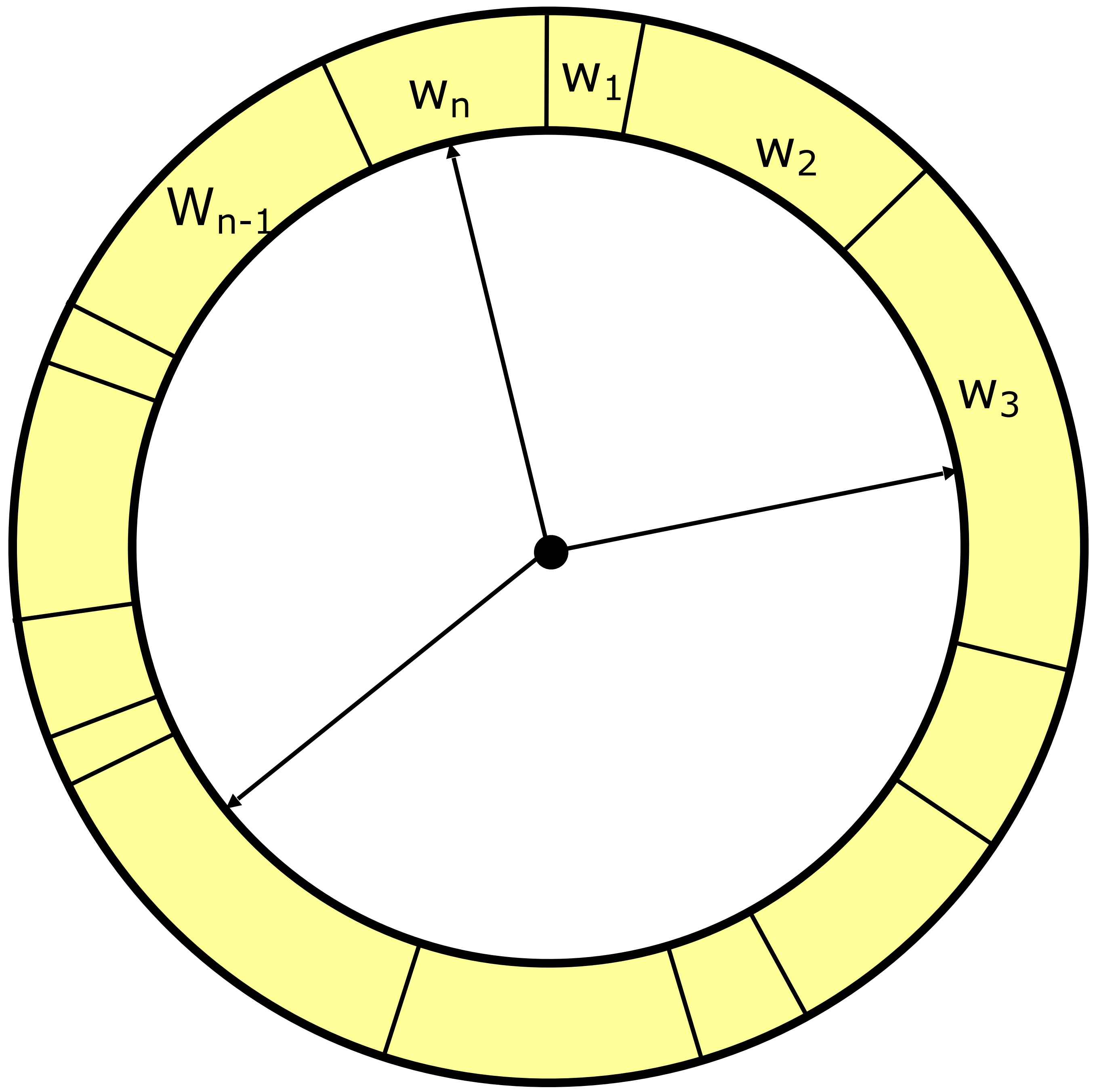
Resampling

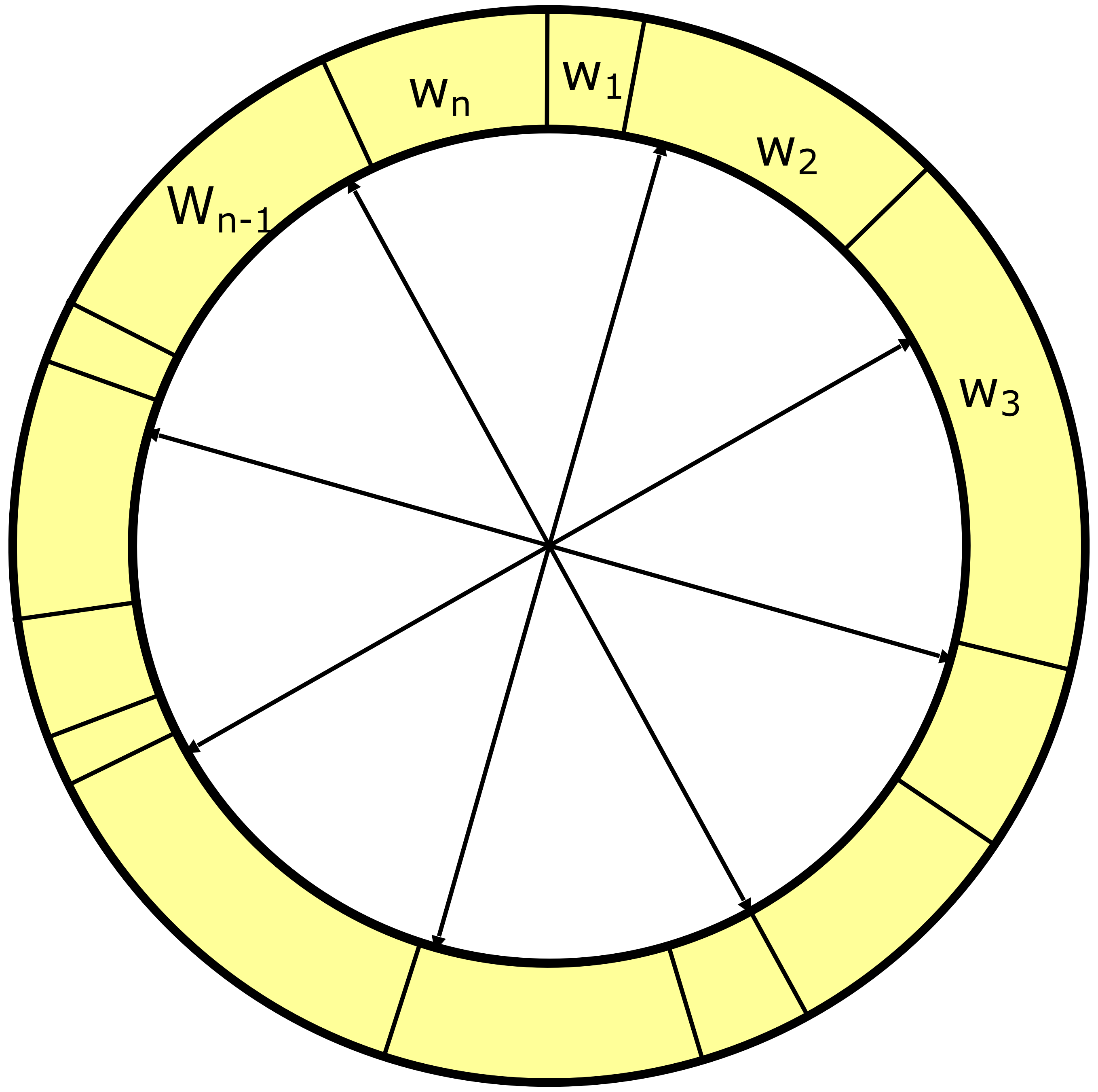
- **Given**: Set *S* of weighted samples.

- **Wanted** : Random sample, where the probability of drawing x_i is given by w_i.

- Typically done *n* times with replacement to generate new sample set *S′*.

Resampling





- Roulette wheel
- Binary search, n log n

- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance

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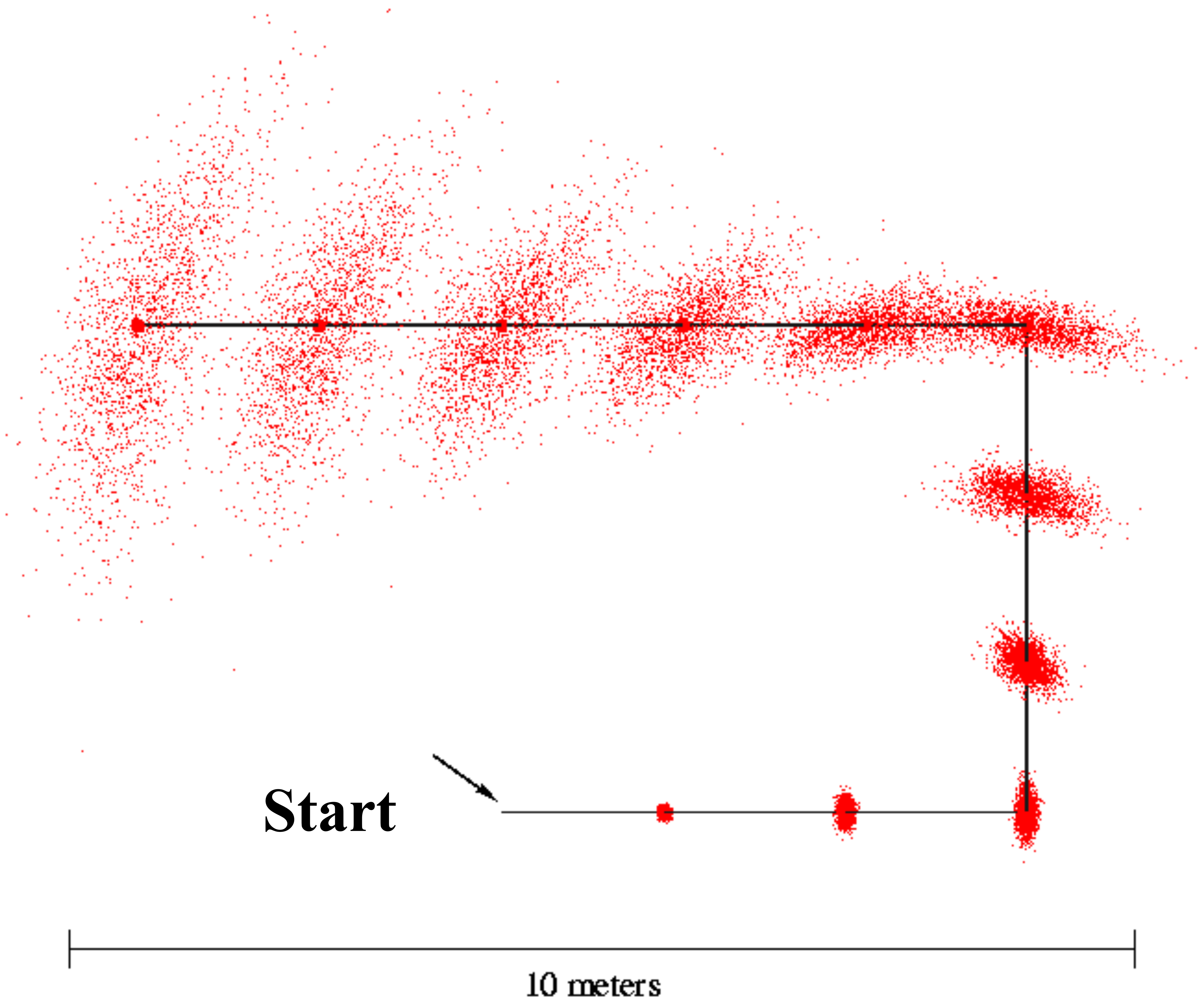
Motion Model Reminder



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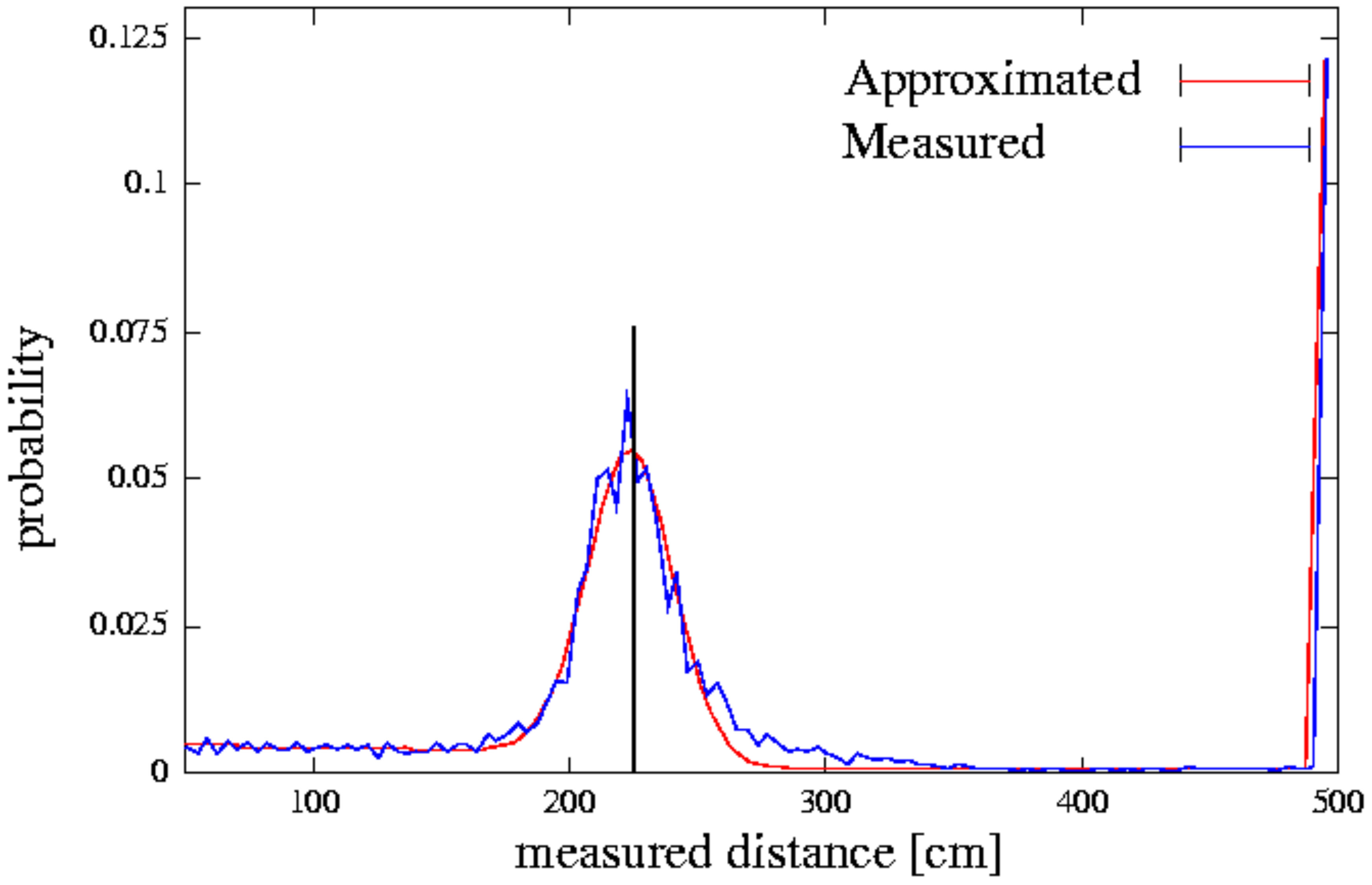
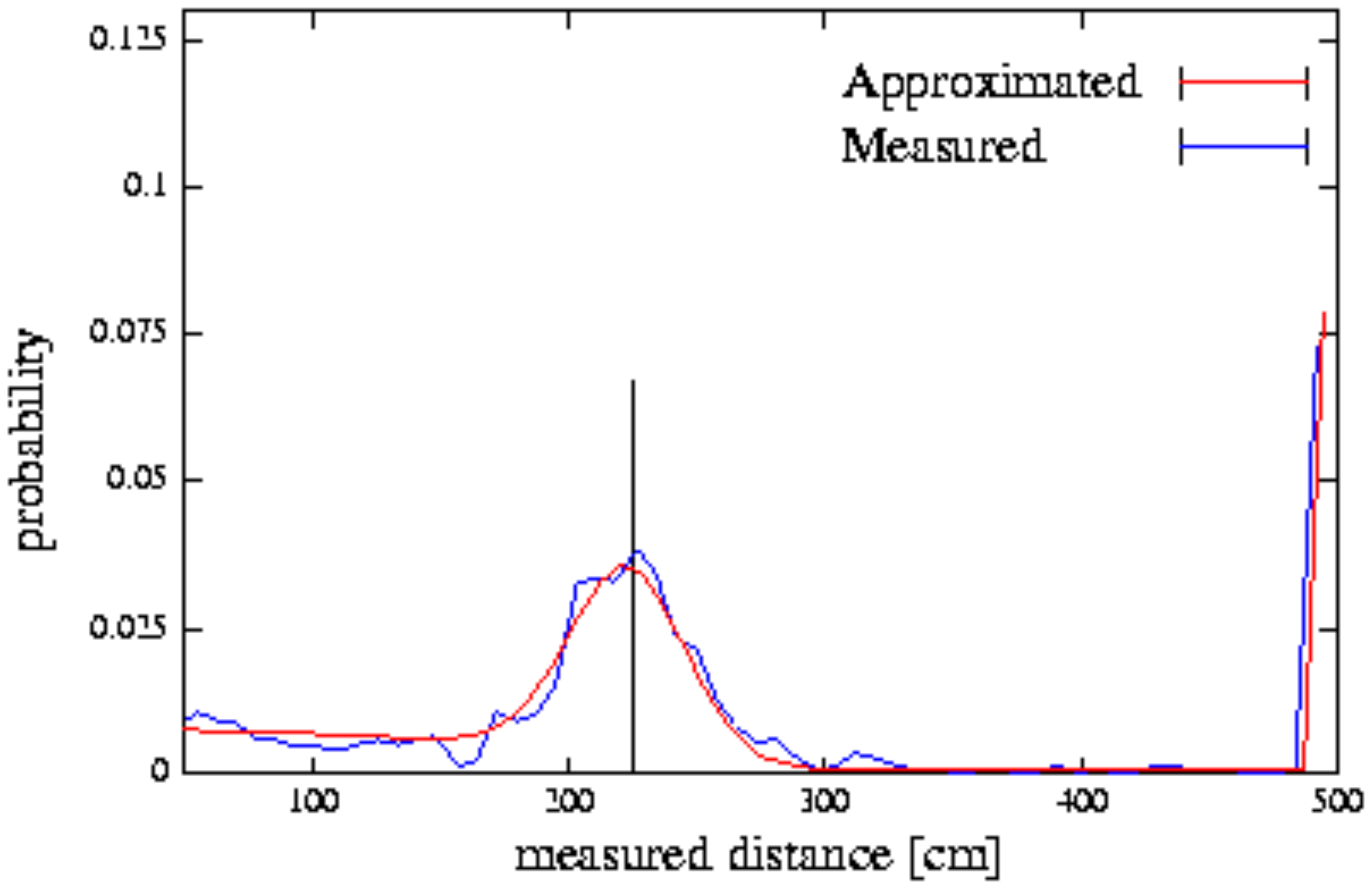
Proximity Sensor Model Reminder

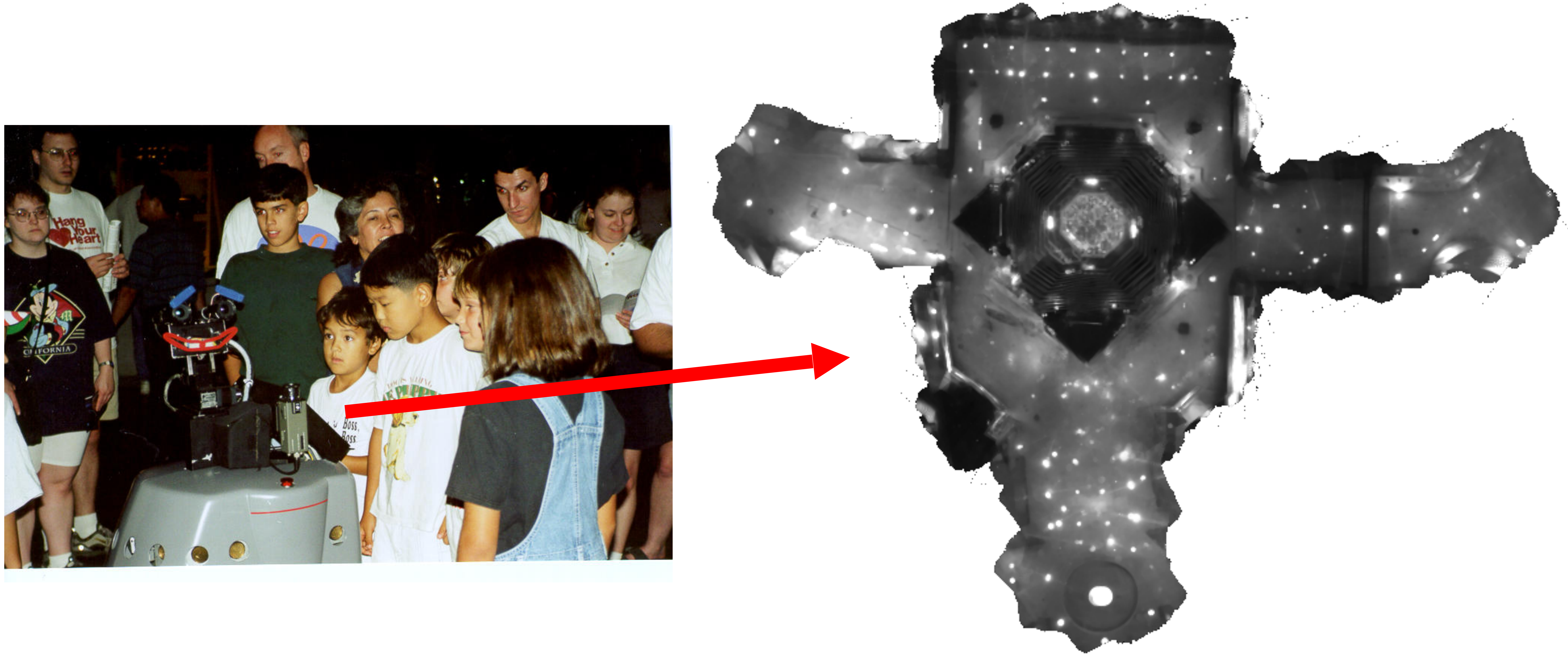


Laser sensor

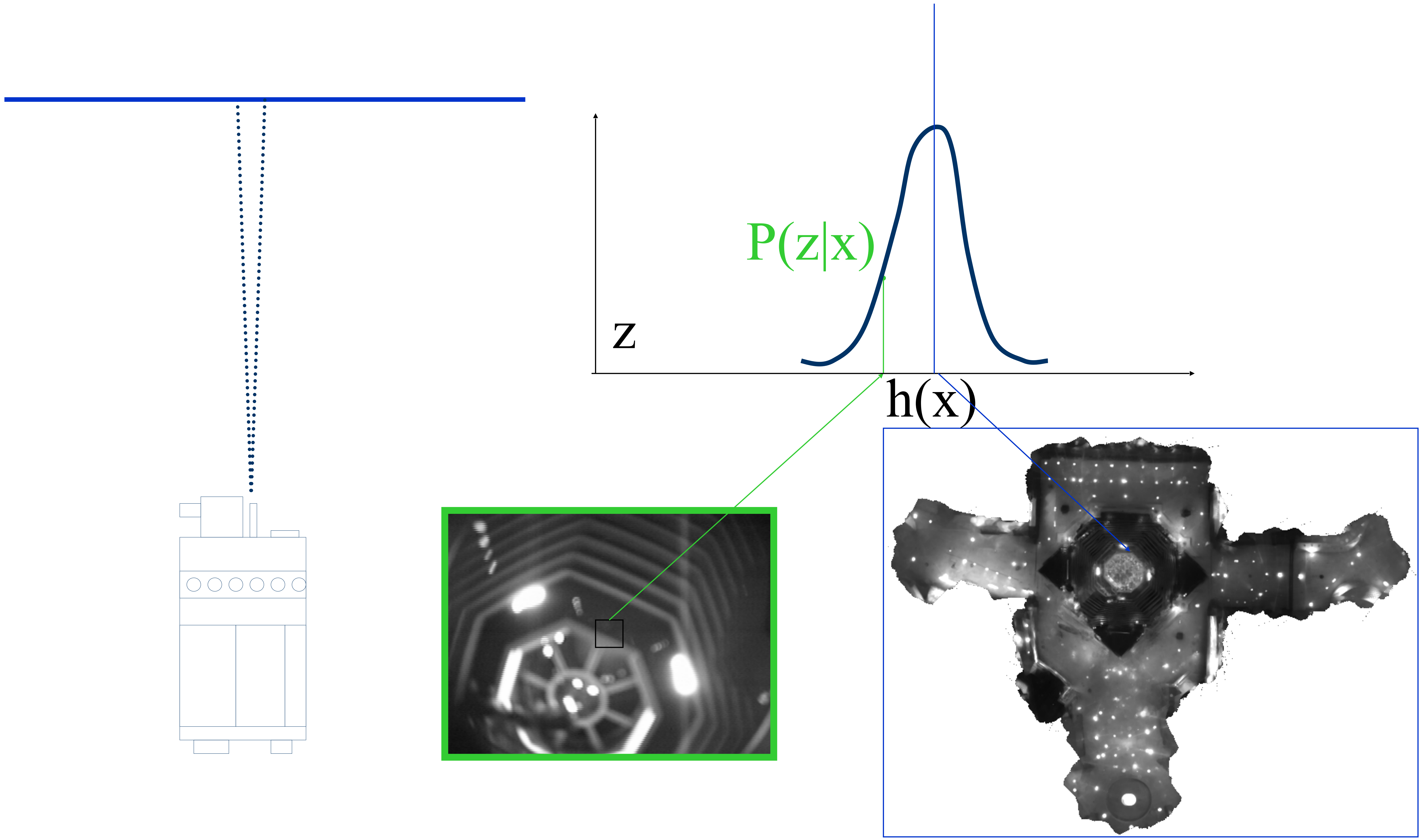


Sonar sensor

Using Ceiling Maps for Localization



Vision-based Localization



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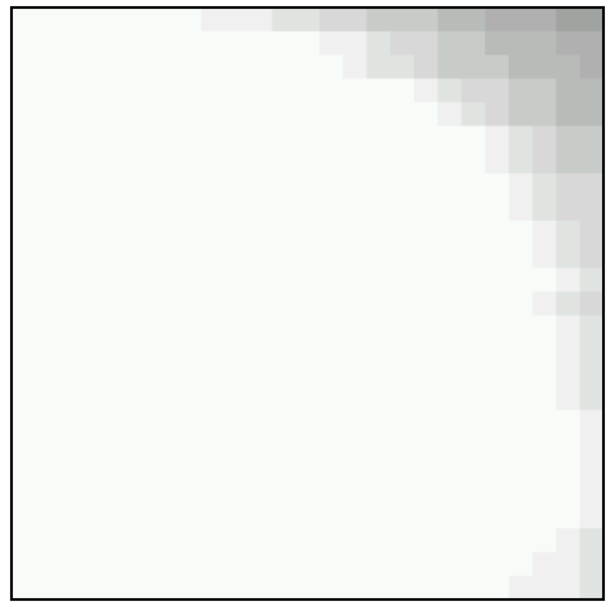
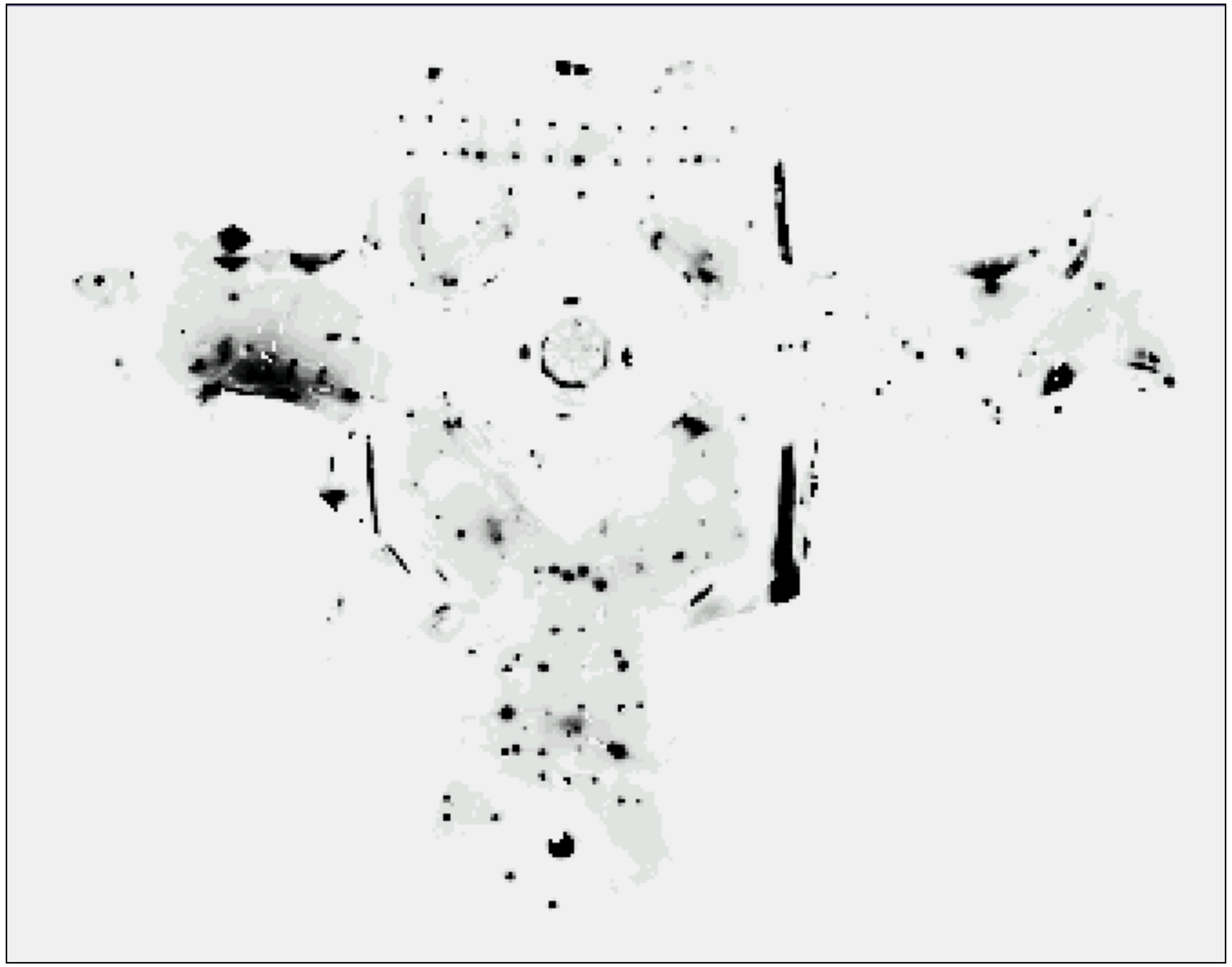
Under a Light

Measurement z:

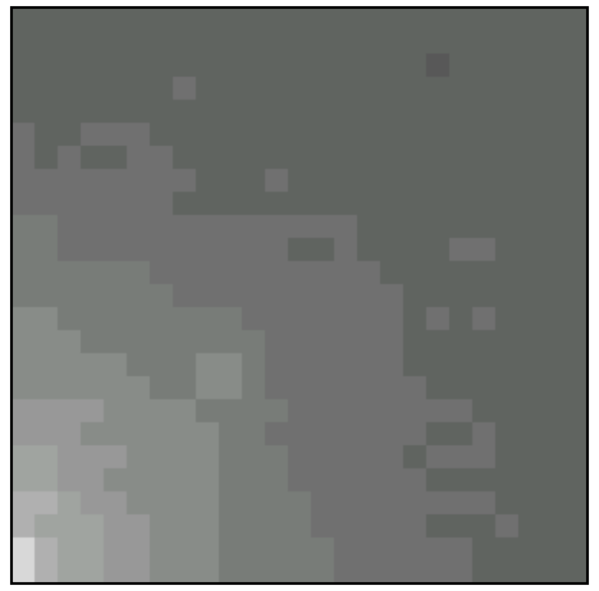


***P(z|x)*:**

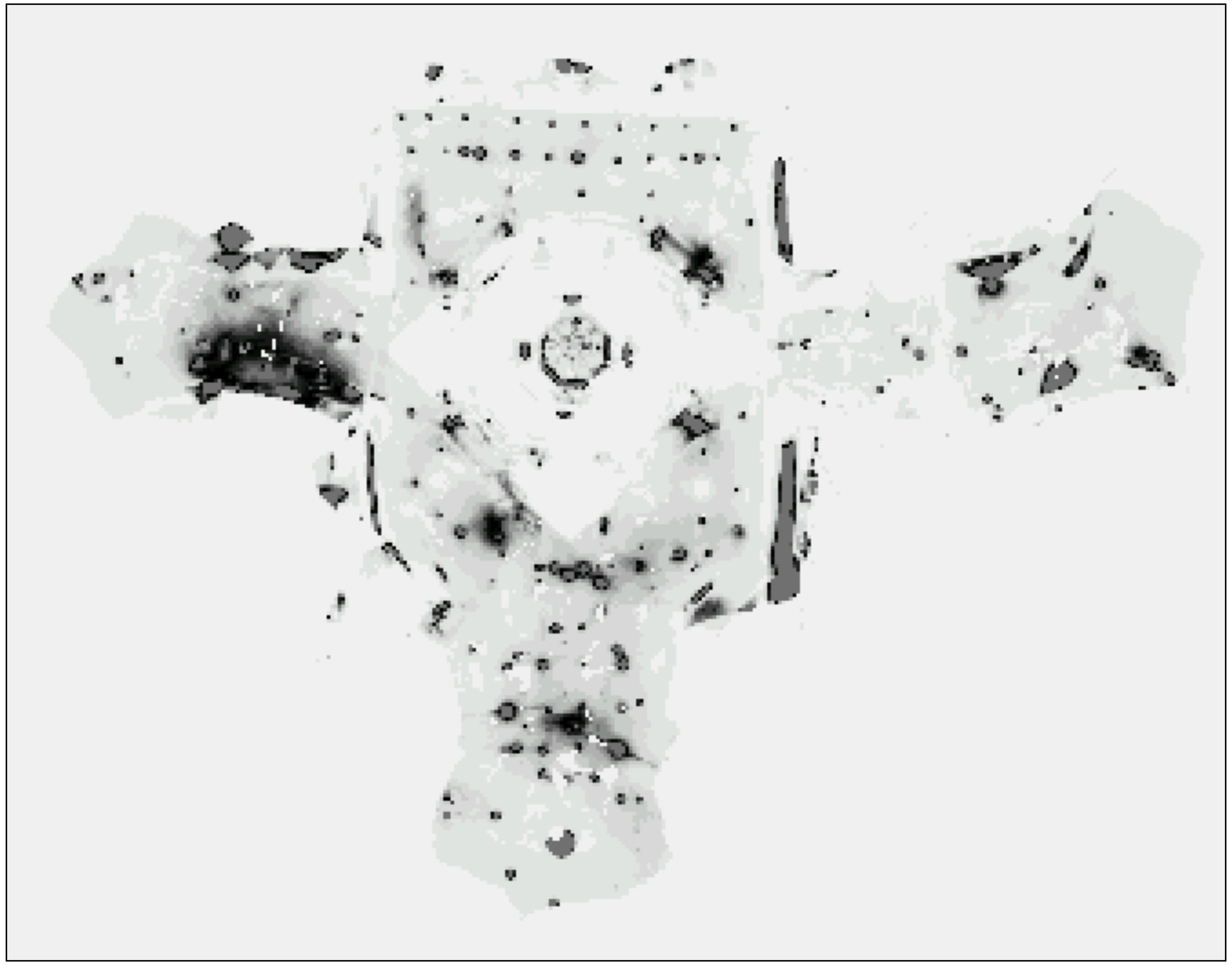


Next to a Light

Measurement z:



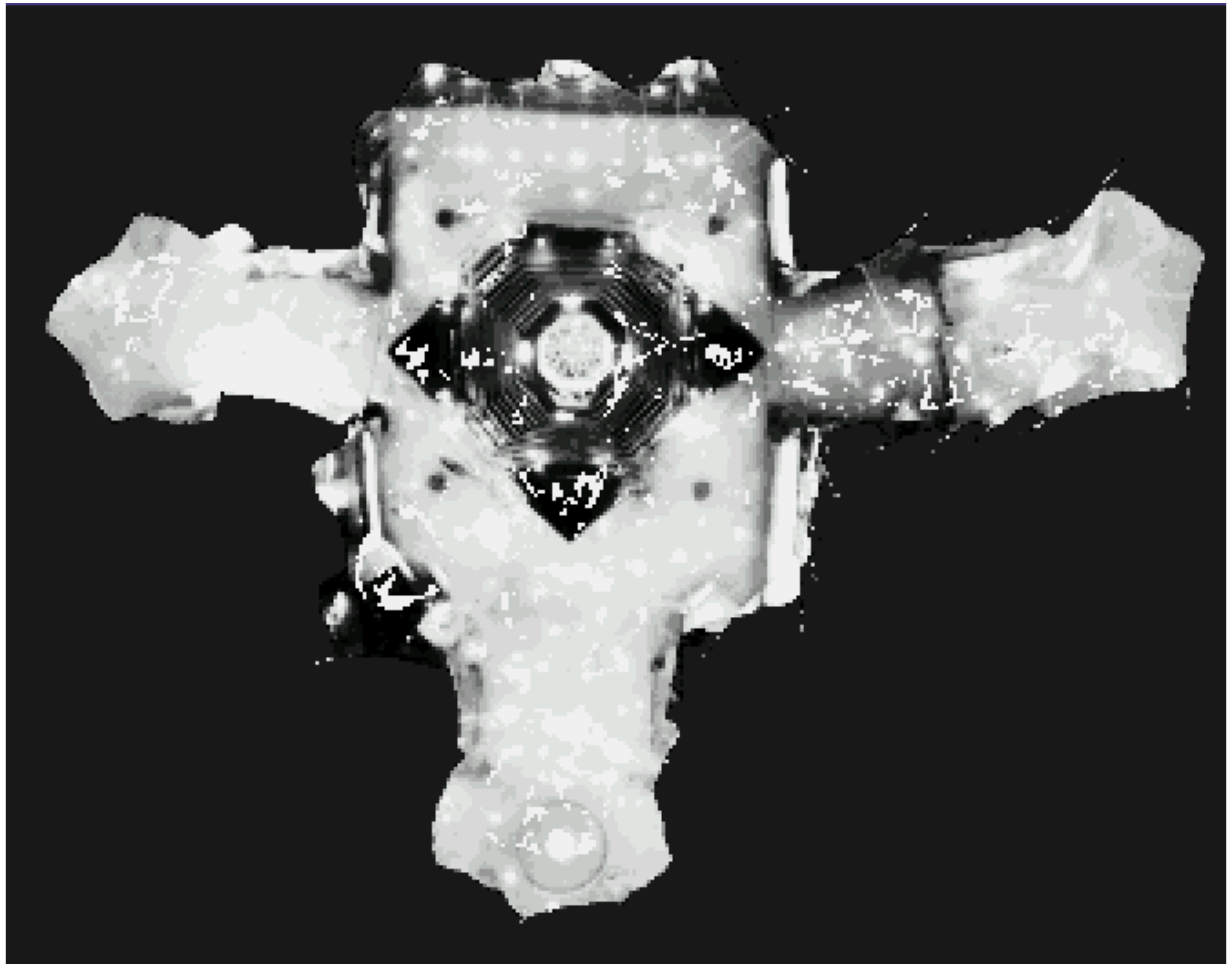
***P(z|x)*:**



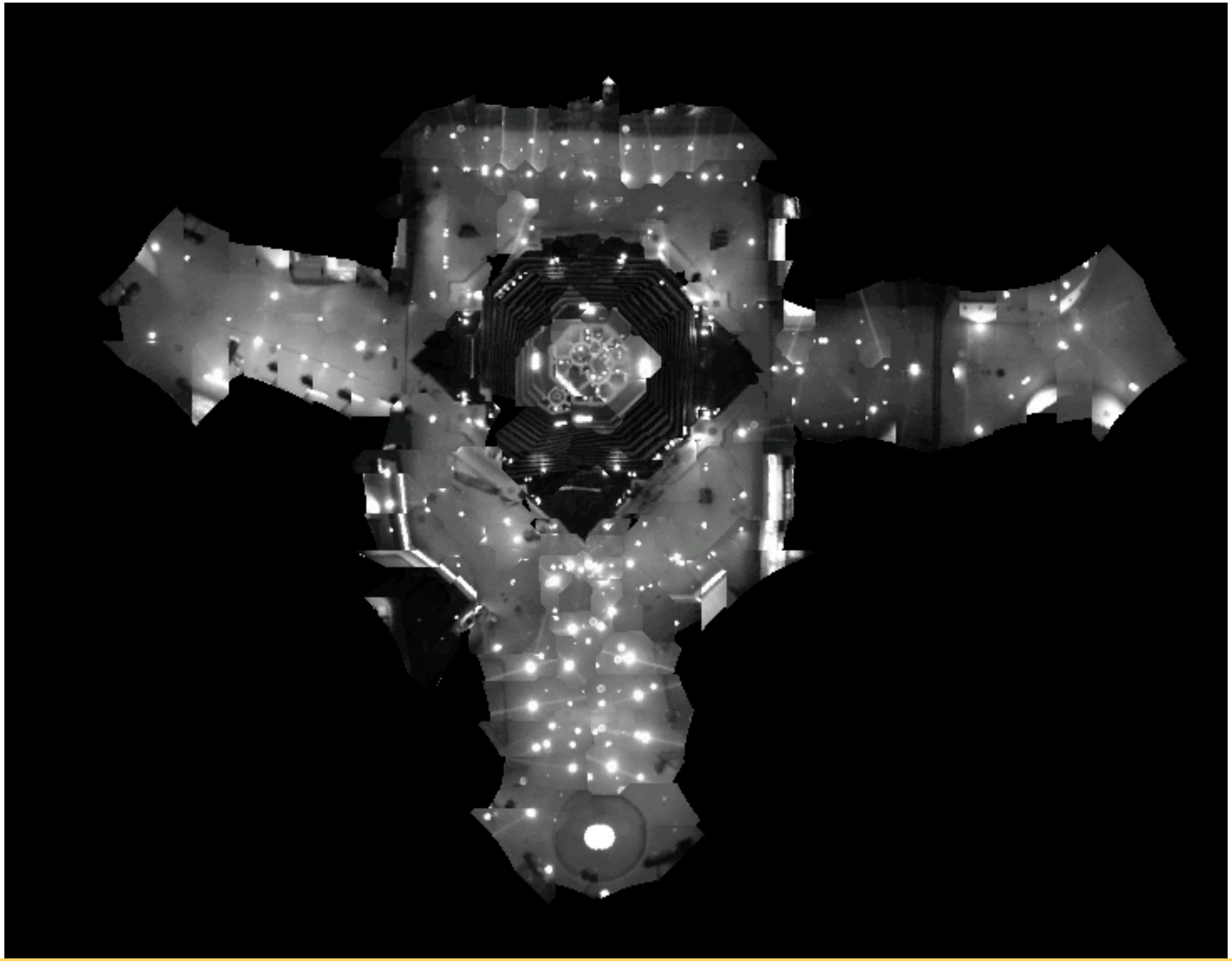
Elsewhere

Measurement z:

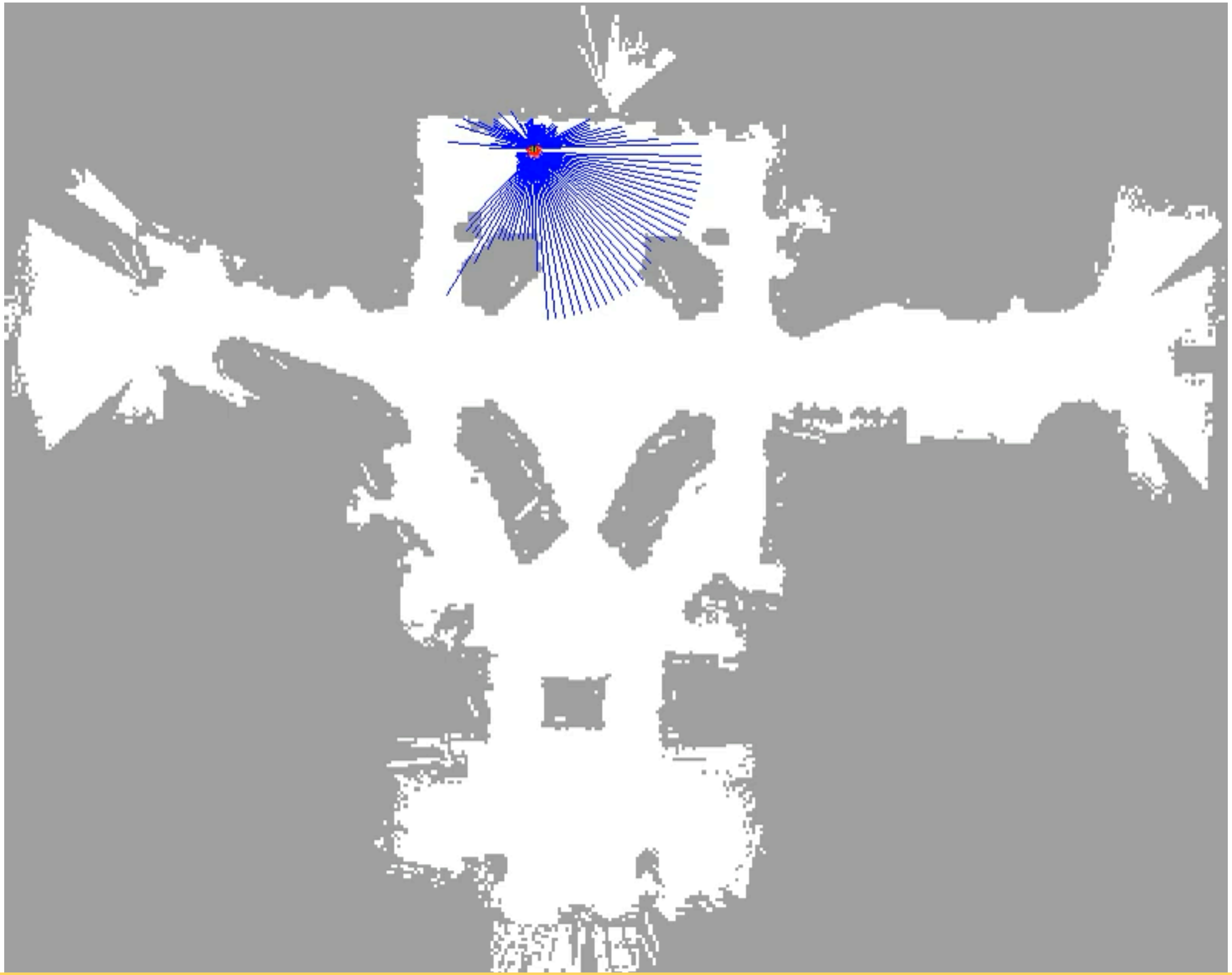
***P(z|x)*:**



Global Localization Using Vision



Recovery from Failure



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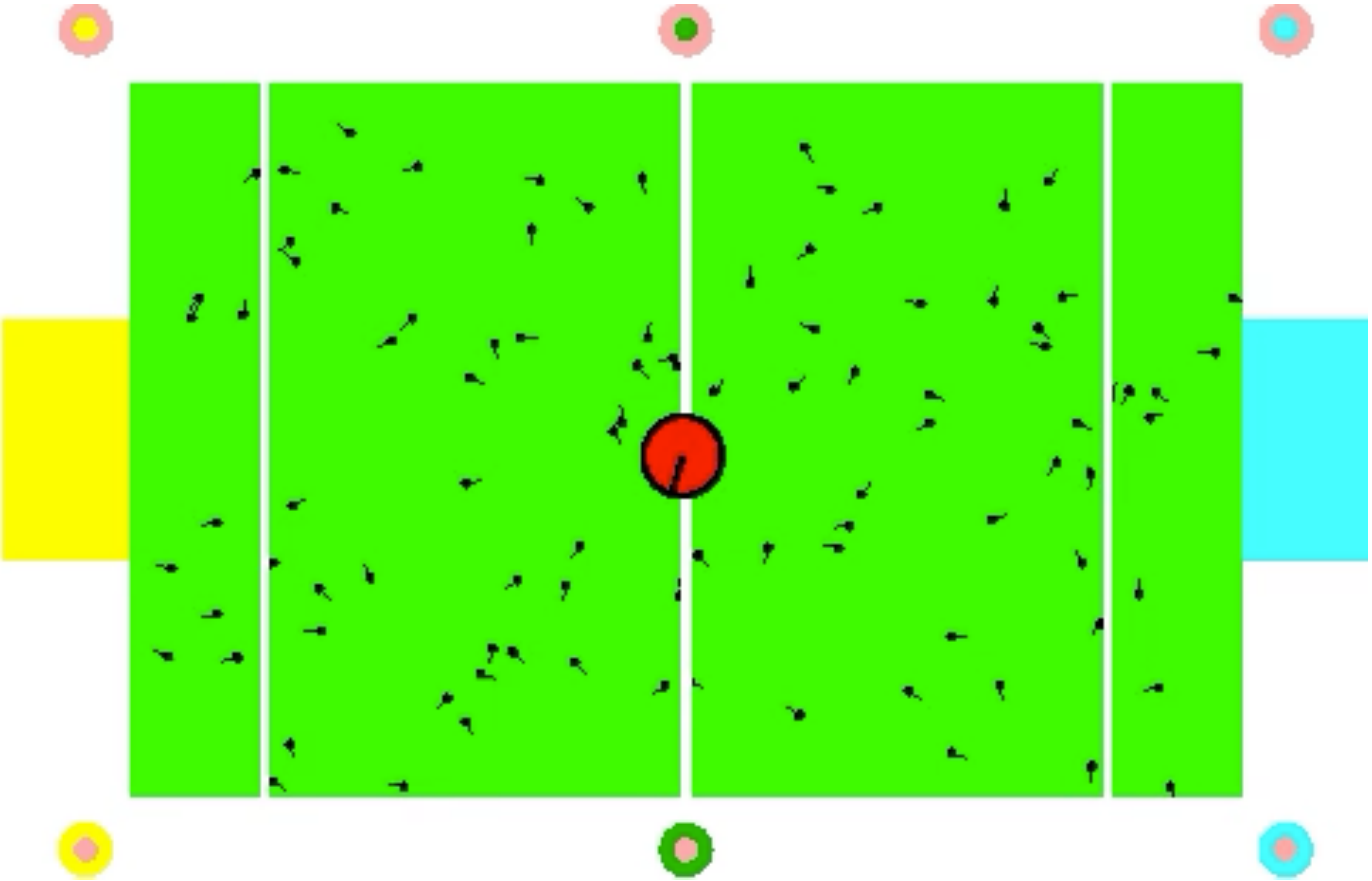
Localization for AIBO robots



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Next Lecture:
More PF and Mapping