

# Lecture 10

## Planning - II - Bugs

# Course Logistics

- Project 4 was posted on 02/19 and will be due on 03/05.
  - Start early!
- Quiz 5 will be posted tomorrow at noon and will be due at noon on Wed.



# Previously in Manipulation Lectures

## Robot Kinematics

**Goal:** Given the structure of a robot arm, compute

- Forward kinematics:** infer the pose of the end-effector, given the state of each joint.
- Inverse kinematics:** infer the joint states to reach a desired end-effector pose.

**Forward kinematics restated:** Given  $q$ , find  $T_n^w$ ;  
 $T_n^w$  transforms endeffector into workspace

$q = \{q_1, \dots, q_n\}$

**Inverse kinematics:** how to solve for  $q = \{\theta_1, \dots, \theta_n\}$  from  $T_n^w$ ?

## Inverse Kinematics: 2 possibilities

- Closed-form solution:** geometrically infer satisfying configuration
  - Speed:* solution often computed in constant time
  - Predictability:* solution is selected in a consistent manner
- Solve by optimization:** minimize error of endeffector to desired pose
  - often some form of Gradient Descent (a la Jacobian Transpose)
  - Generality:* same solver can be used for many different robots

## Robot arm and its Jacobian

*Lets focus on i-1th frame*

*i-1th frame maps to i-th column in*

### The Jacobian

A  $6 \times N$  matrix

$$J = [J_1 J_2 \dots J_n]$$

consisting of two  $3 \times N$  matrices

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

Figure 5.1: Motion of the end-effector due to link  $i$ .

$J_i$  for a prismatic joint       $J_i$  for a rotational joint

$$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} \quad J_i = \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix}$$

## How to use this Jacobian for IK as optimization?

**IK Procedure restated:**

- compute endpoint error  $\Delta x_n = x_d - x_n$
- compute step direction  $\Delta q_n = J(q_n)^{-1} \Delta x_n$  (repeat)
- perform step direction  $q_{n+1} = q_n + \gamma \Delta q_n$

Check point:  
How will you get  $x_{n+1}$  given  $q_{n+1}$ ?

## Matlab 5-link arm example: Jacobian transpose

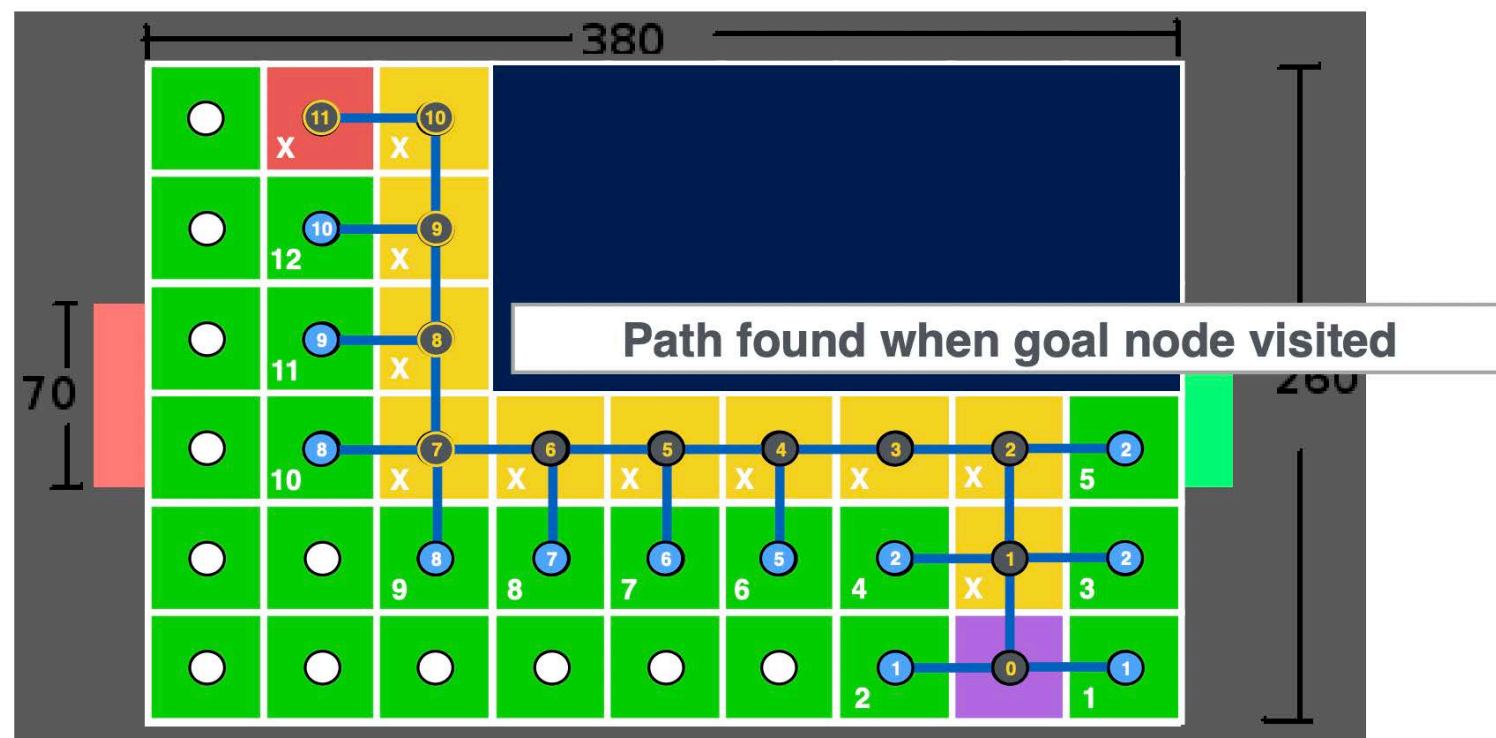
## Definition of Manipulation

Mason, Matthew T. "Toward robotic manipulation." *Annual Review of Control, Robotics, and Autonomous Systems* 1 (2018): 1-28.

This lecture uses the structure and material from this review paper!

# Previously in Planning, Decision Making, Control

## Depth-first search

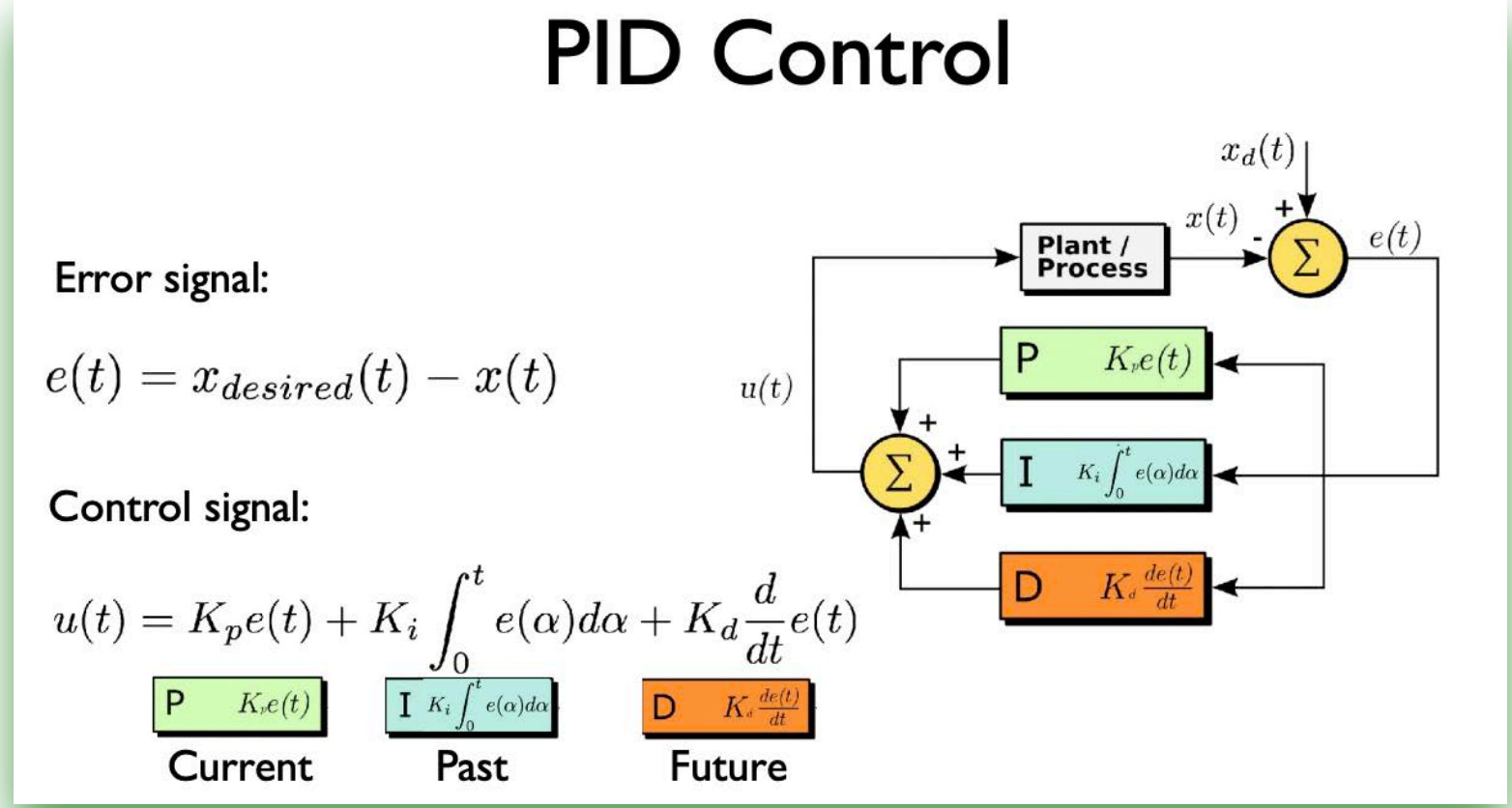
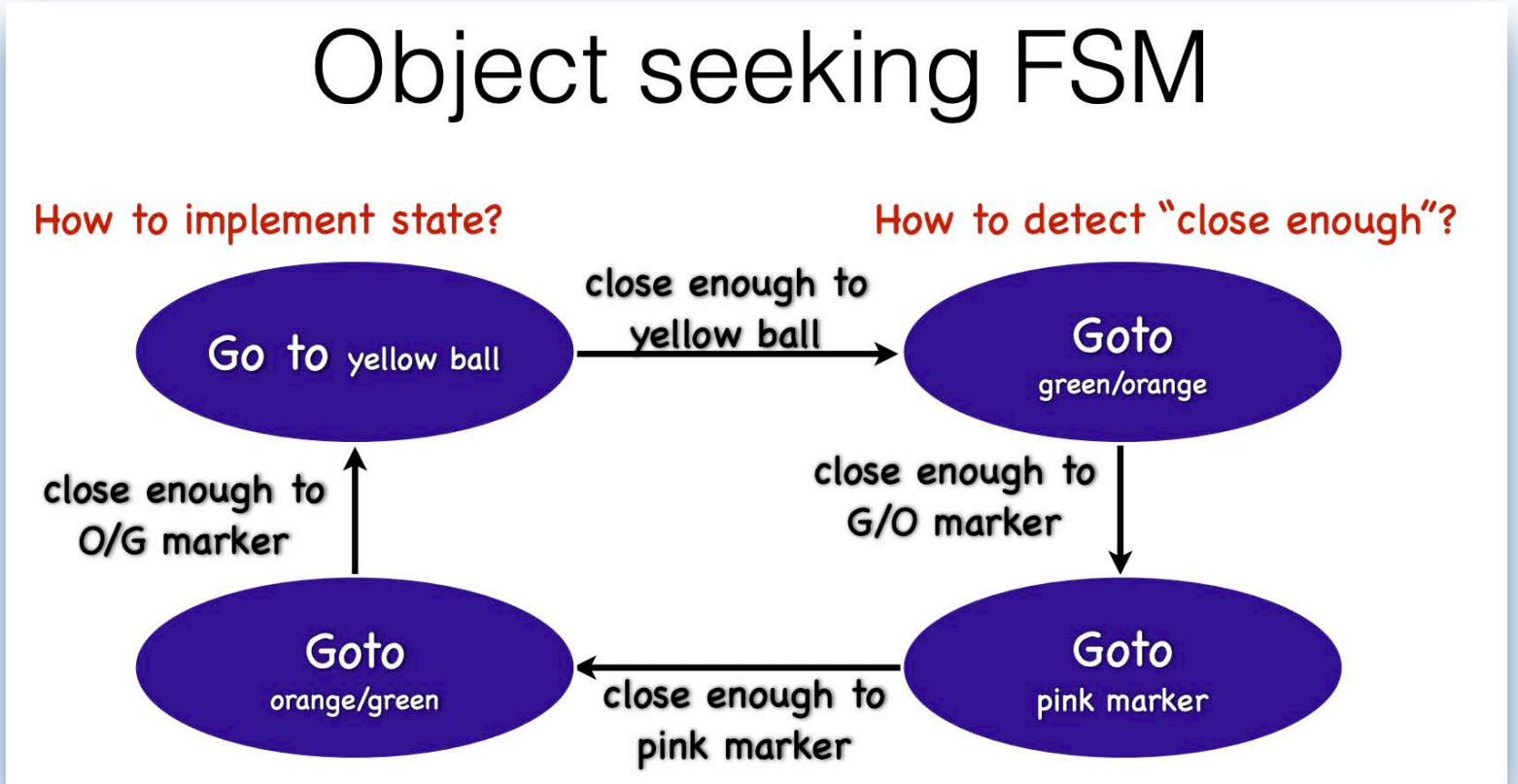
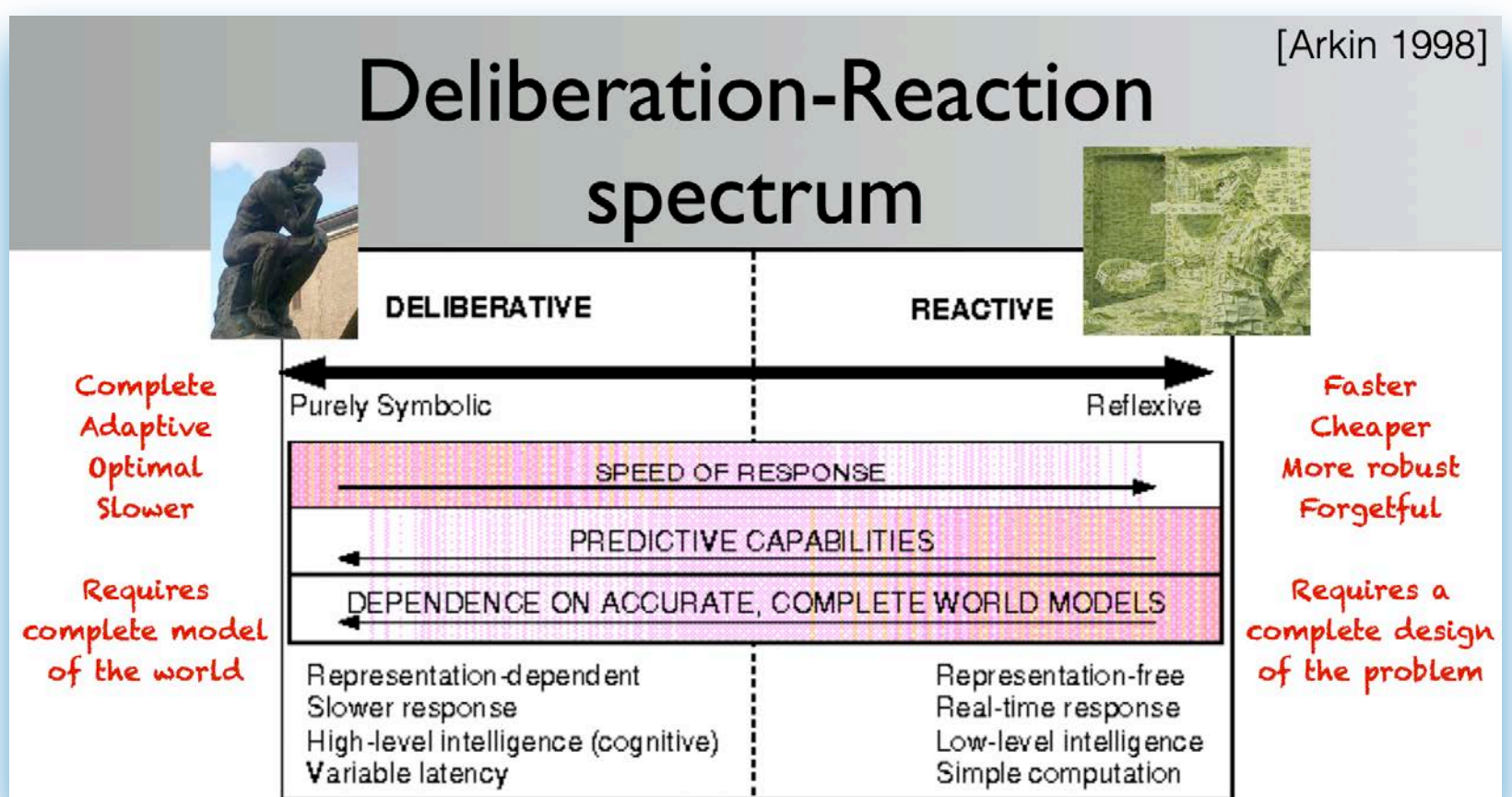
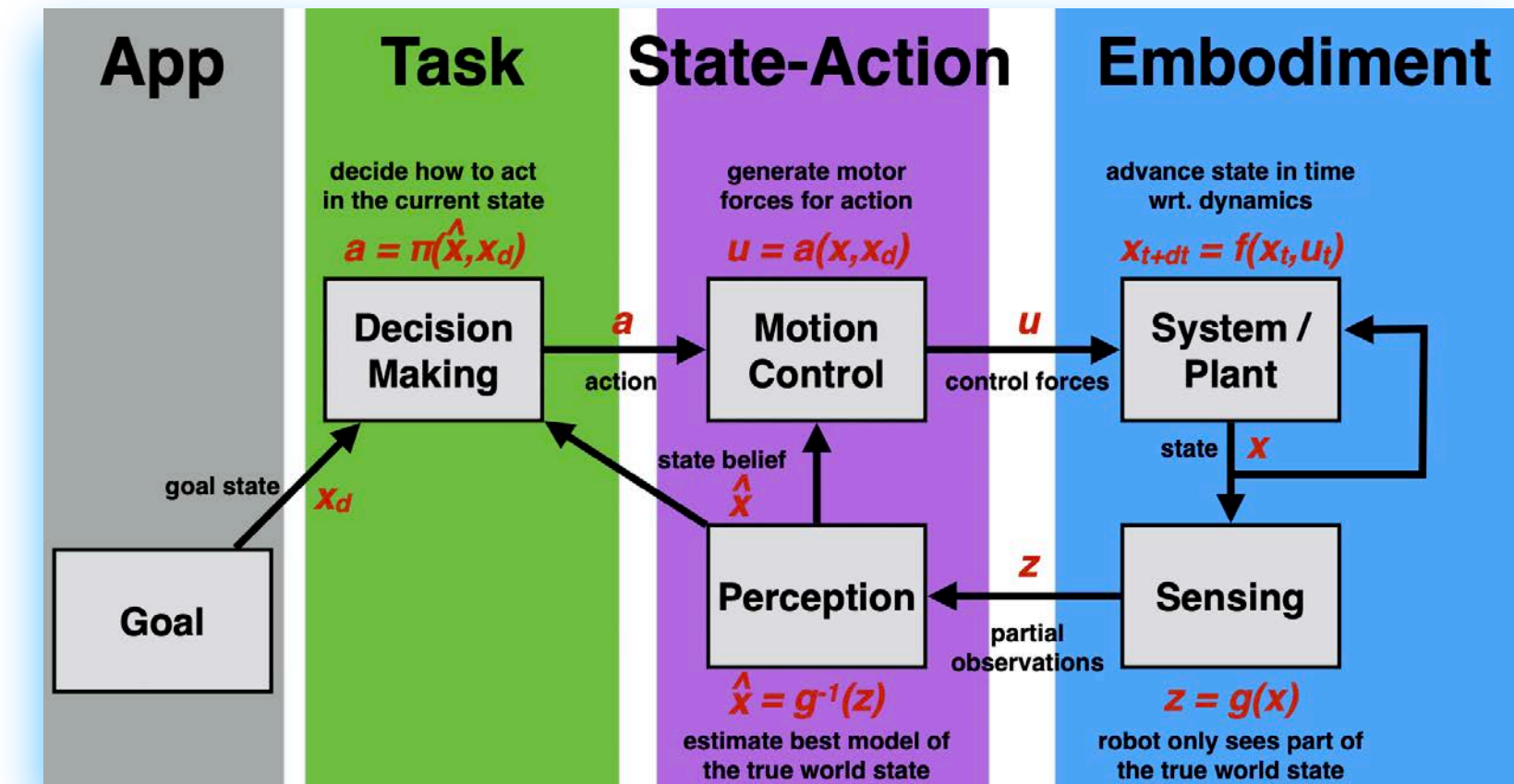


### Search algorithm template

```

all nodes ← {dist_start ← infinity, parent_start ← none, visited_start ← false}
start_node ← {dist_start ← 0, parent_start ← none, visited_start ← true}
visit_list ← start_node

while visit_list != empty && current_node != goal
  cur_node ← highestPriority(visit_list)
  visited_cur_node ← true
  for each nbr in not_visited(adjacent(cur_node))
    add(nbr to visit_list)
    if dist_nbr > dist_cur_node + distStraightLine(nbr, cur_node)
      parent_nbr ← current_node
      dist_nbr ← dist_cur_node + distStraightLine(nbr, cur_node)
    end if
  end for loop
end while loop
output ← parent, distance
  
```

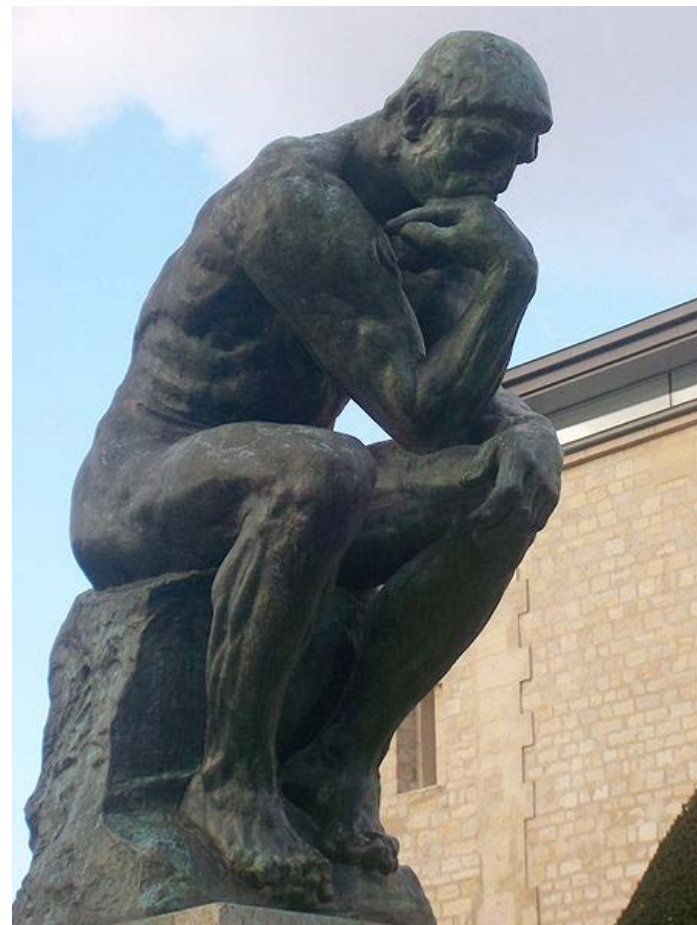
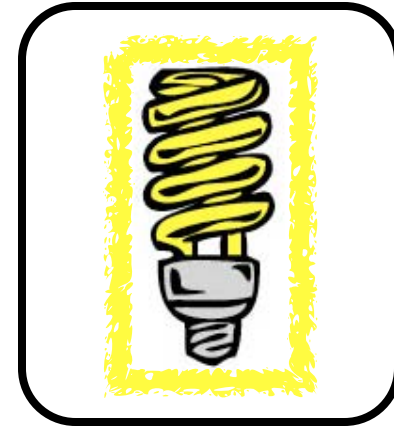


# Approaches to motion planning

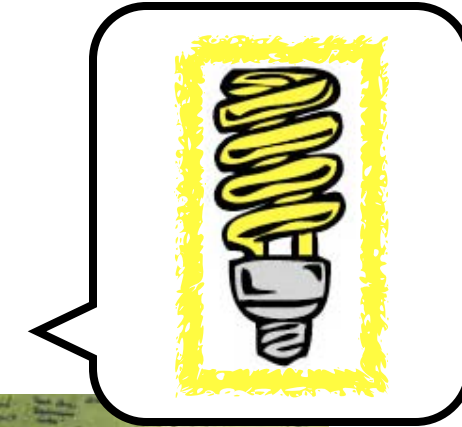
- **Bug algorithms: Bug[0-2], Tangent Bug**
- Graph Search (fixed graph)
  - Depth-first, Breadth-first, Dijkstra, A-star
- Sampling-based Search (build graph):
  - Probabilistic Road Maps, Rapidly-exploring Random Trees
- Optimization (local search):
  - Gradient descent, potential fields, Wavefront



# Should your robot's decision making



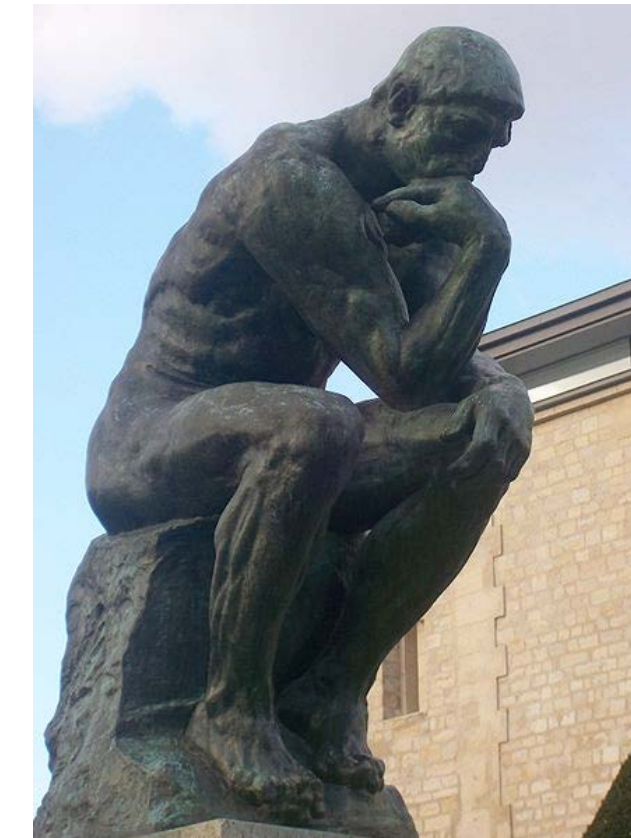
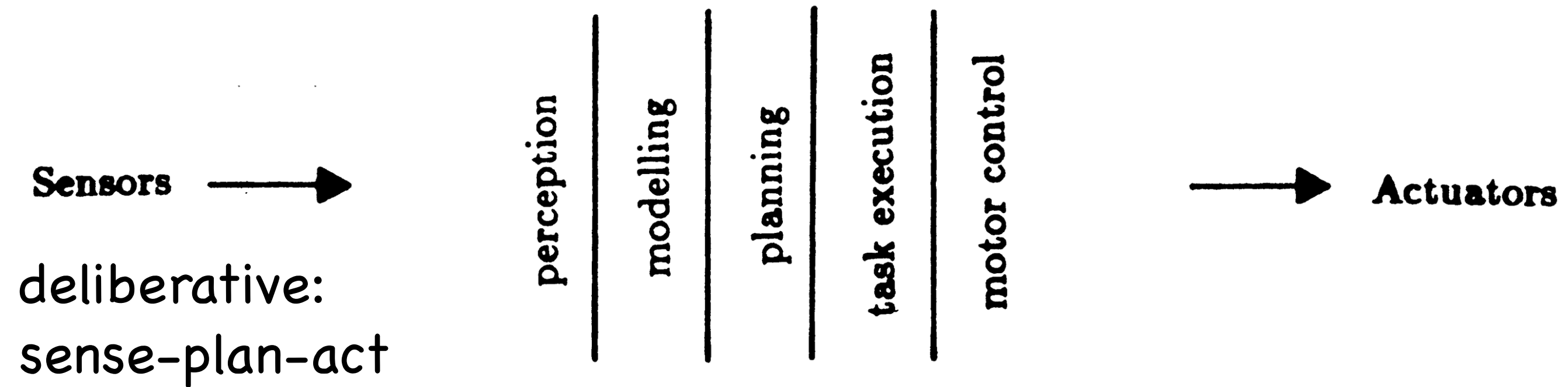
OR



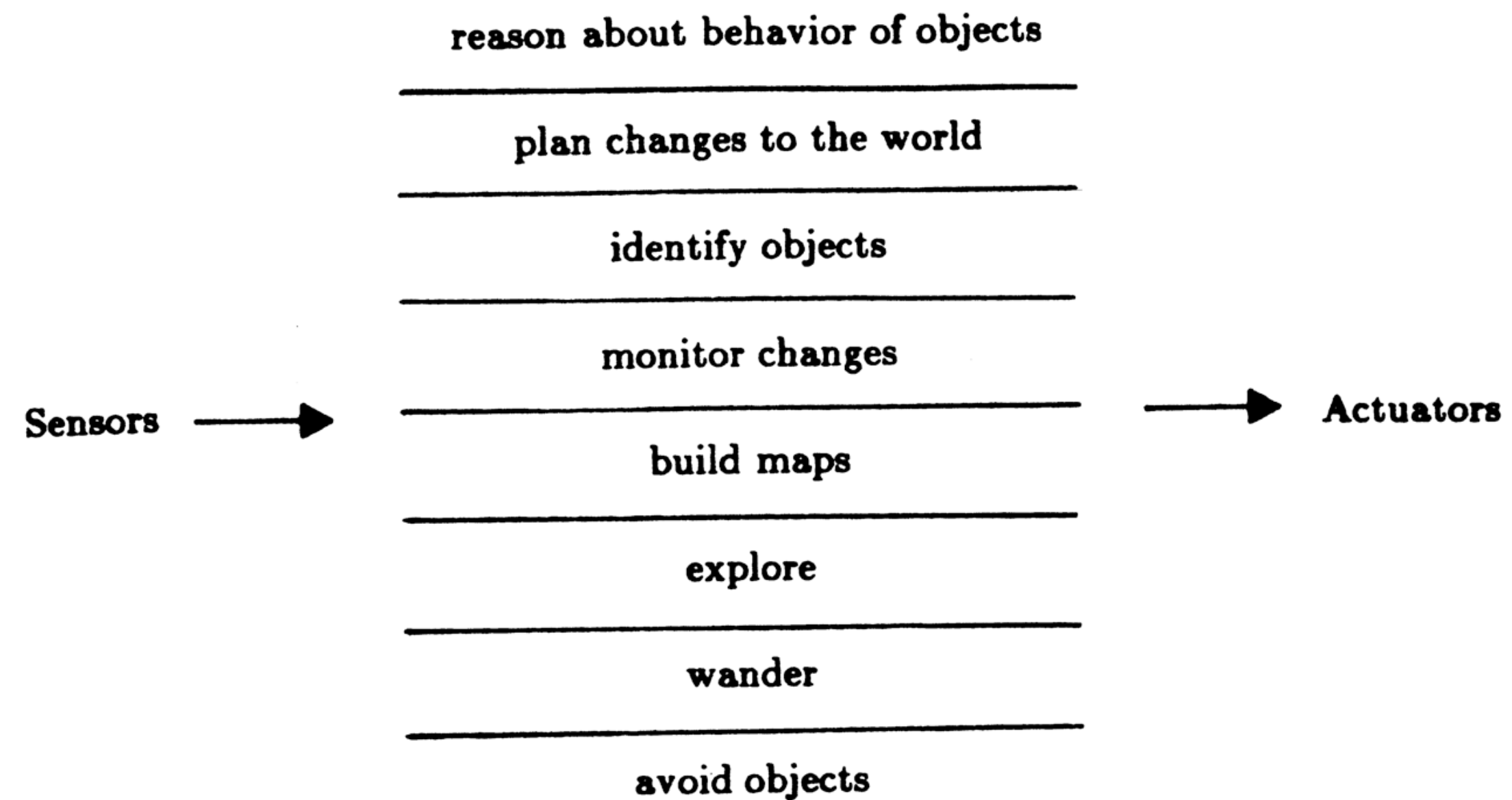
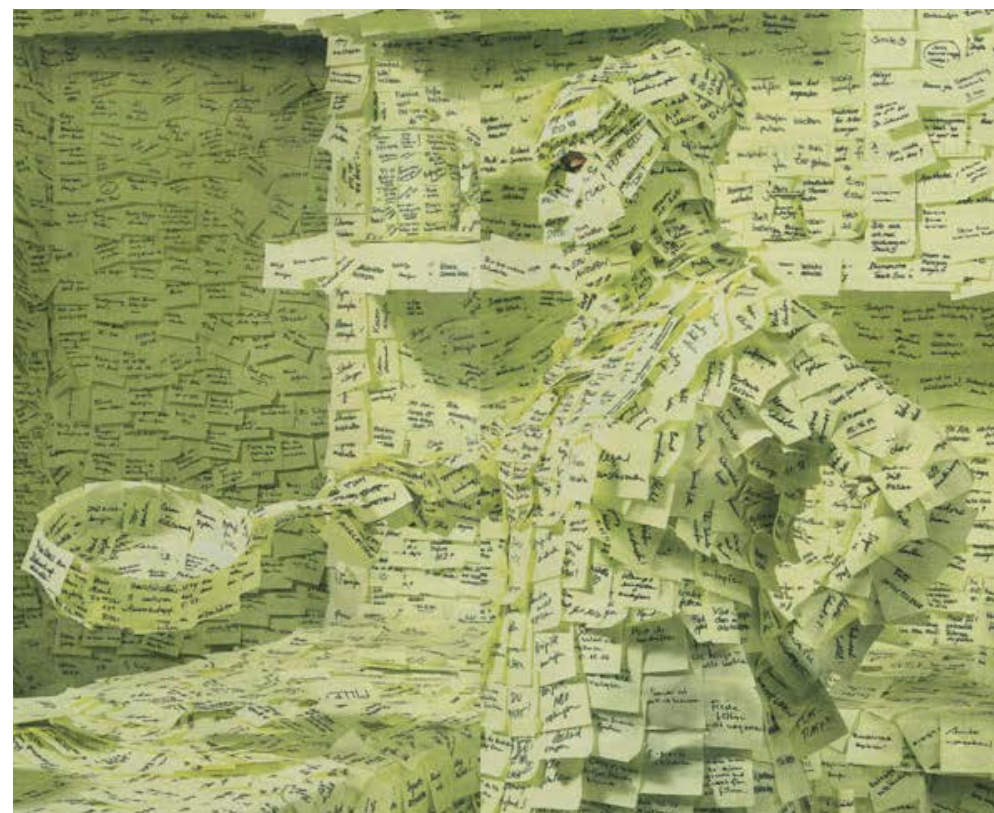
fully think through  
solving a problem?

react quickly to  
changes in its world?

# Deliberation v. Reaction



reaction: subsumption,  
Finite State Machine  
controllers act in parallel

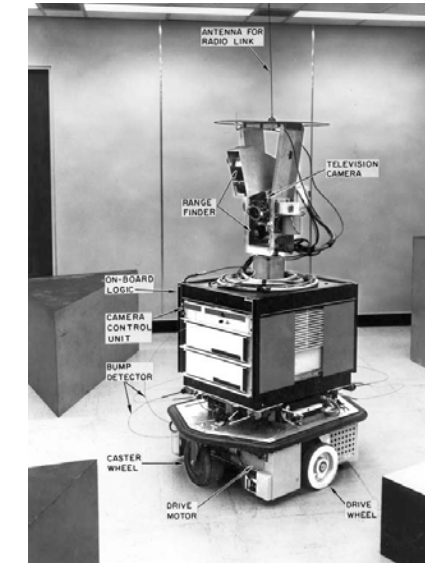


# Deliberation

## “Sense-Plan-Act” paradigm

- sense: build most complete model of world
- GPS, SLAM, 3D reconstruction, affordances
- plan: search over all possible outcomes
- BFS, DFS, Dijkstra, A\*, RRT
- act: execute plan through motor forces

Sensors →



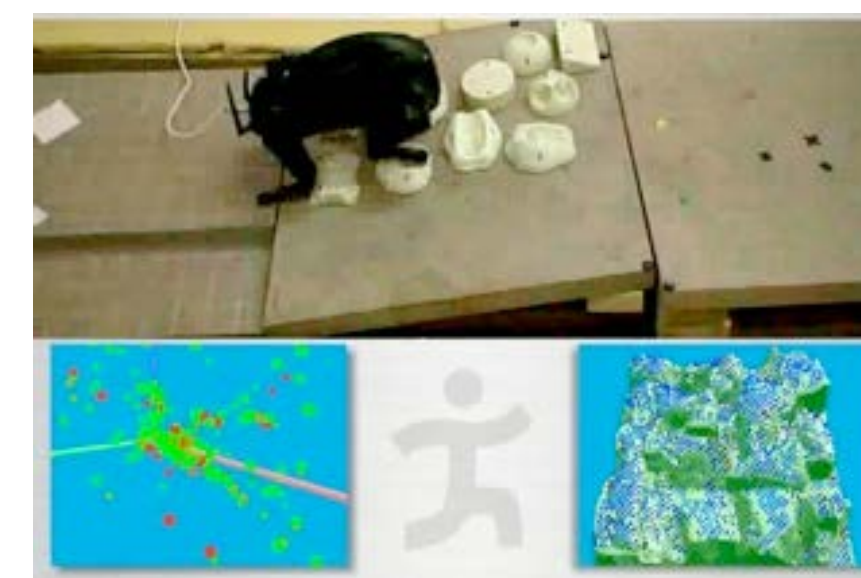
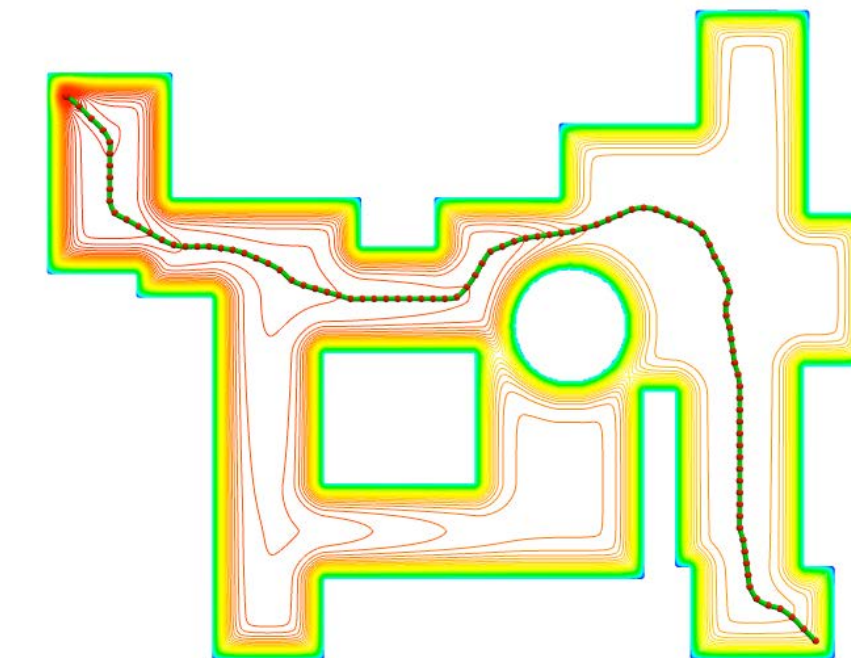
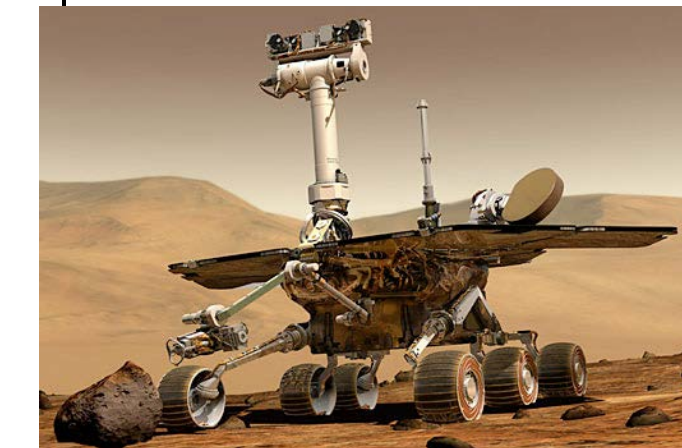
Sense

Model

Plan

Act

→ Actuators





# Reaction

Sensors →

Explore

Wander Around

Avoid Obstacles

Avoid Collision

→ Actuators

- No representation of state
- Typically, fast hardcoded rules
- Embodied intelligence
  - behavior := control + embodiment
  - ant analogy, stigmergy
- Subsumption architecture
  - prioritized reactive policies
- Ghengis hexpod video

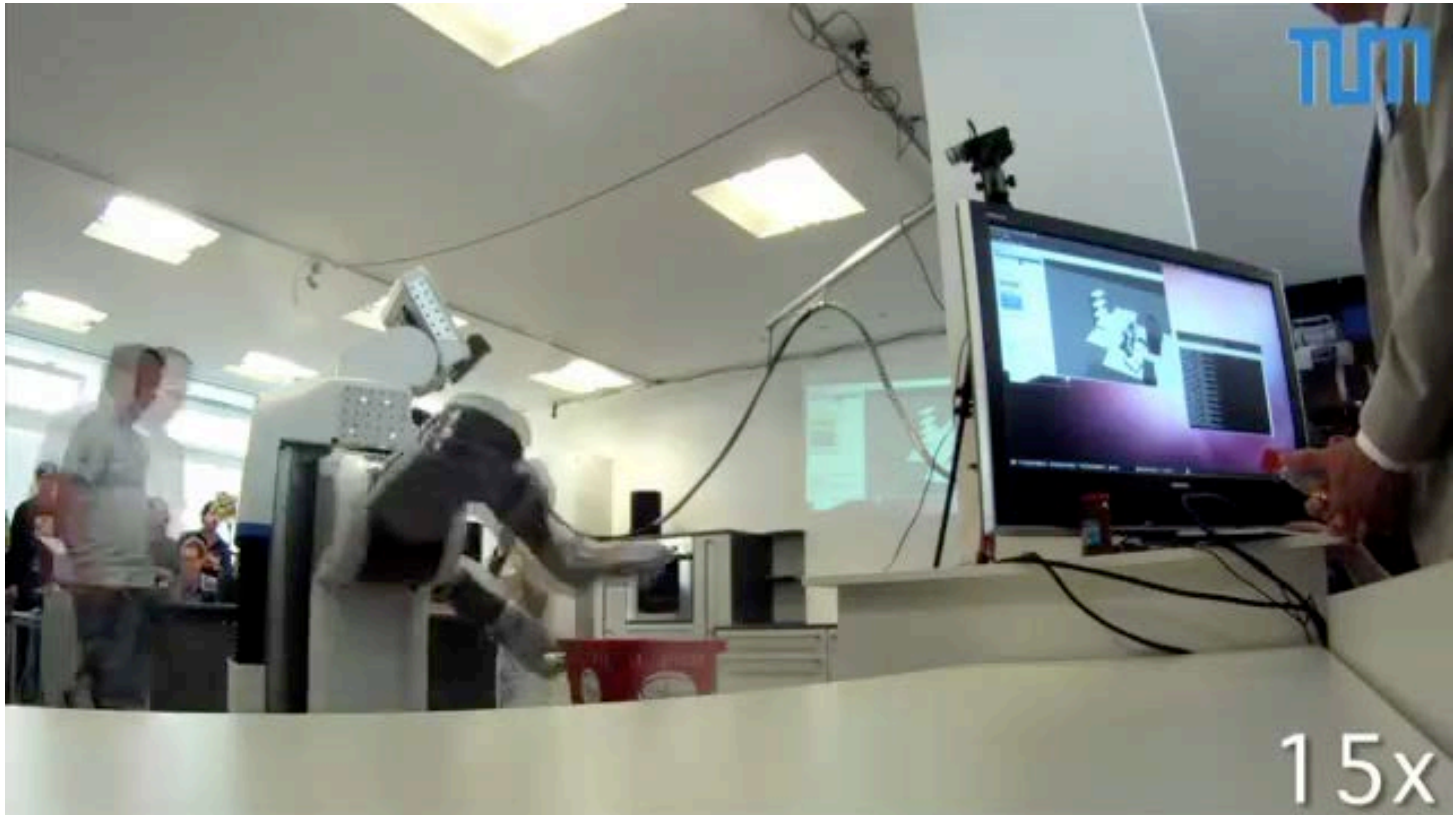




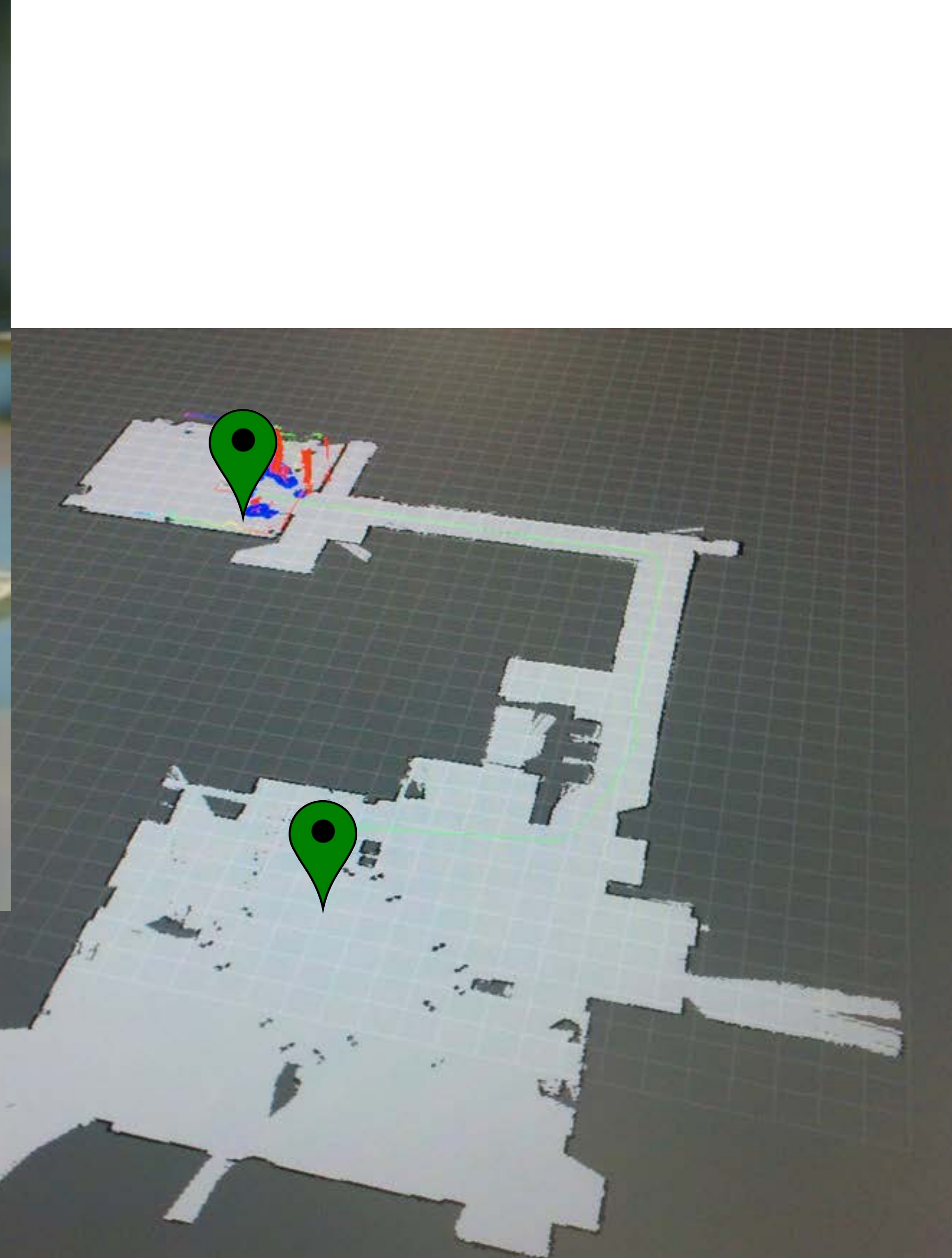
MIT Genghis

<https://www.youtube.com/watch?v=1j6CliOwRng>





Robots have to make lots of decisions



Remember your path planner?

How to get from Location A to  
Location B?



# Base Navigation

- How get from point A to point B
- **What is the simplest policy to perform navigation?**
  - Remember: simplest reactive policy?



# Random Walk: Goal Seeking

- Move in a random direction until you hit something
- Then go in a new direction
- Stop when you get to the goal, assuming it can be recognized



Lisa Miller, <http://www.youtube.com/watch?v=VBzXDrz8rMI>

goal: exit here



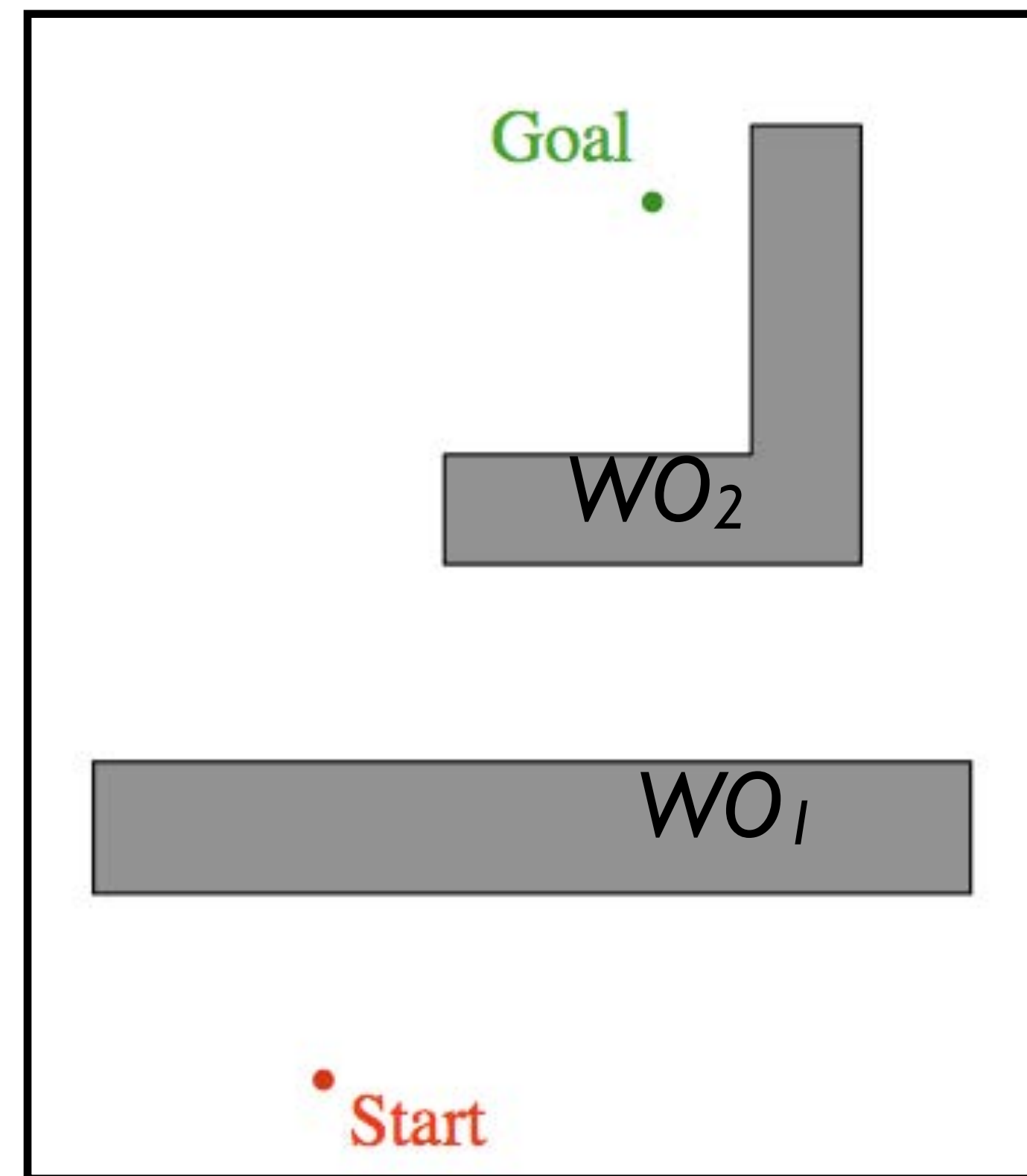
# Base Navigation

- How get from point A to point B
- What is the simplest policy to perform navigation?
  - random walk
  - reactive: embodied intelligence
- **What is a “simple” deliberative policy?**



# Bug Algorithms

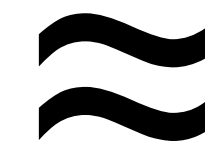
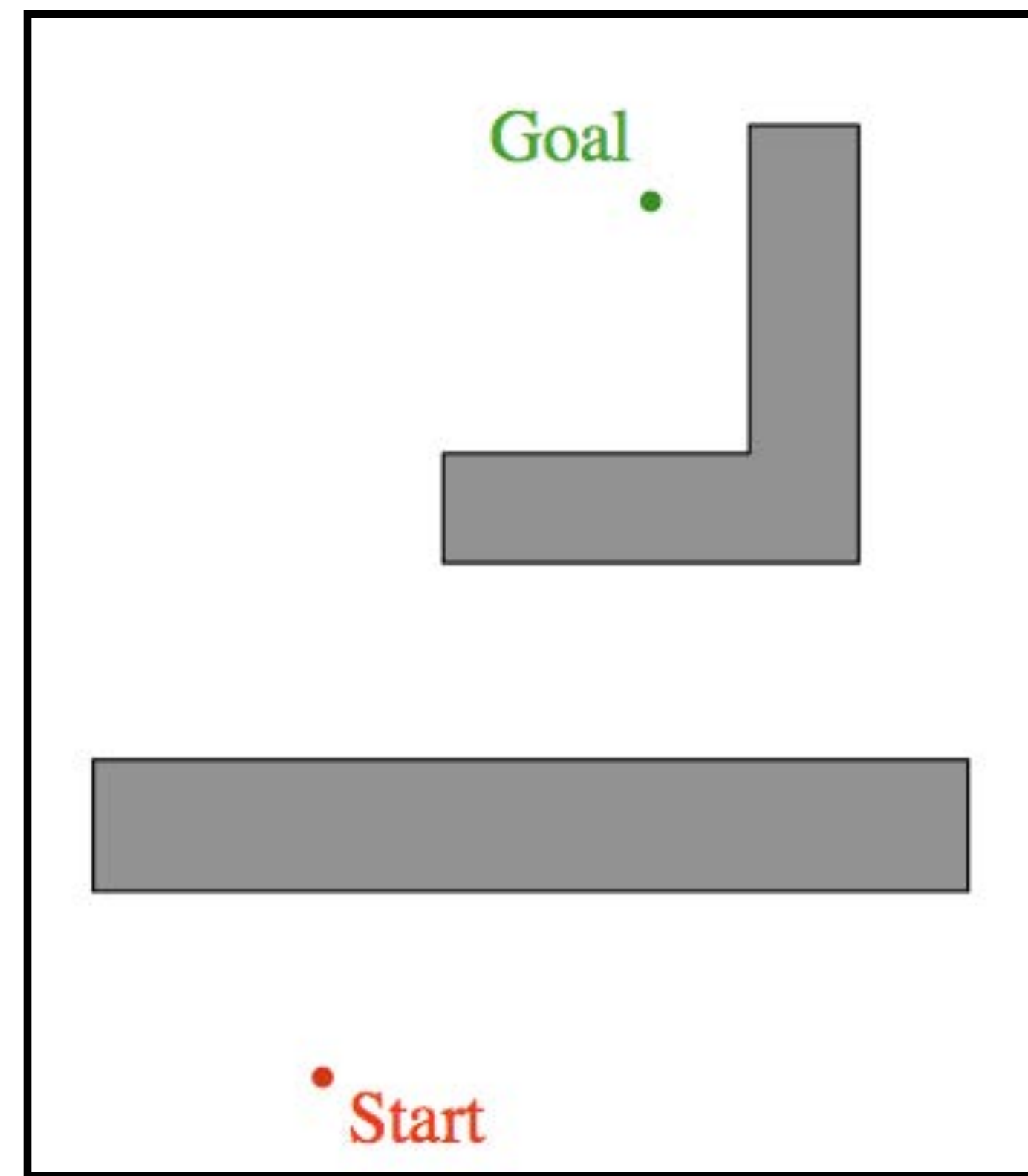
- Assume bounded world  $W$
- Known: global goal
  - measurable distance  $d(x,y)$
- Unknown: obstacles  $WO_i$
- Local sensing
  - tactile
  - distance traveled





# Bug Algorithms

- Assume bounded world  $W$
- Known: global goal
  - measurable distance  $d(x,y)$
- Unknown: obstacles  $WO_i$
- Local sensing
  - **bump sensor**
  - distance traveled

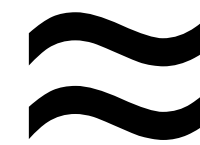
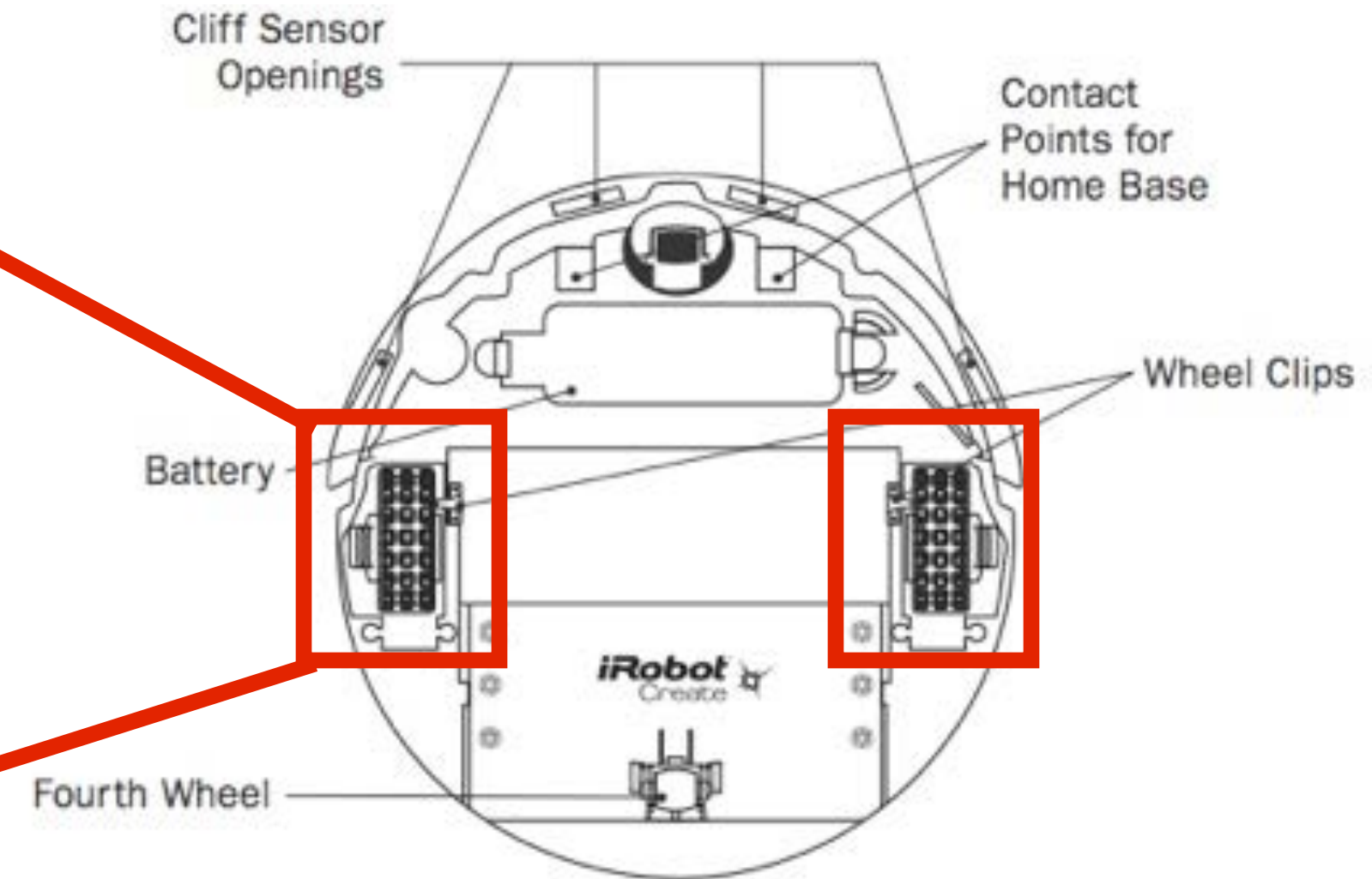
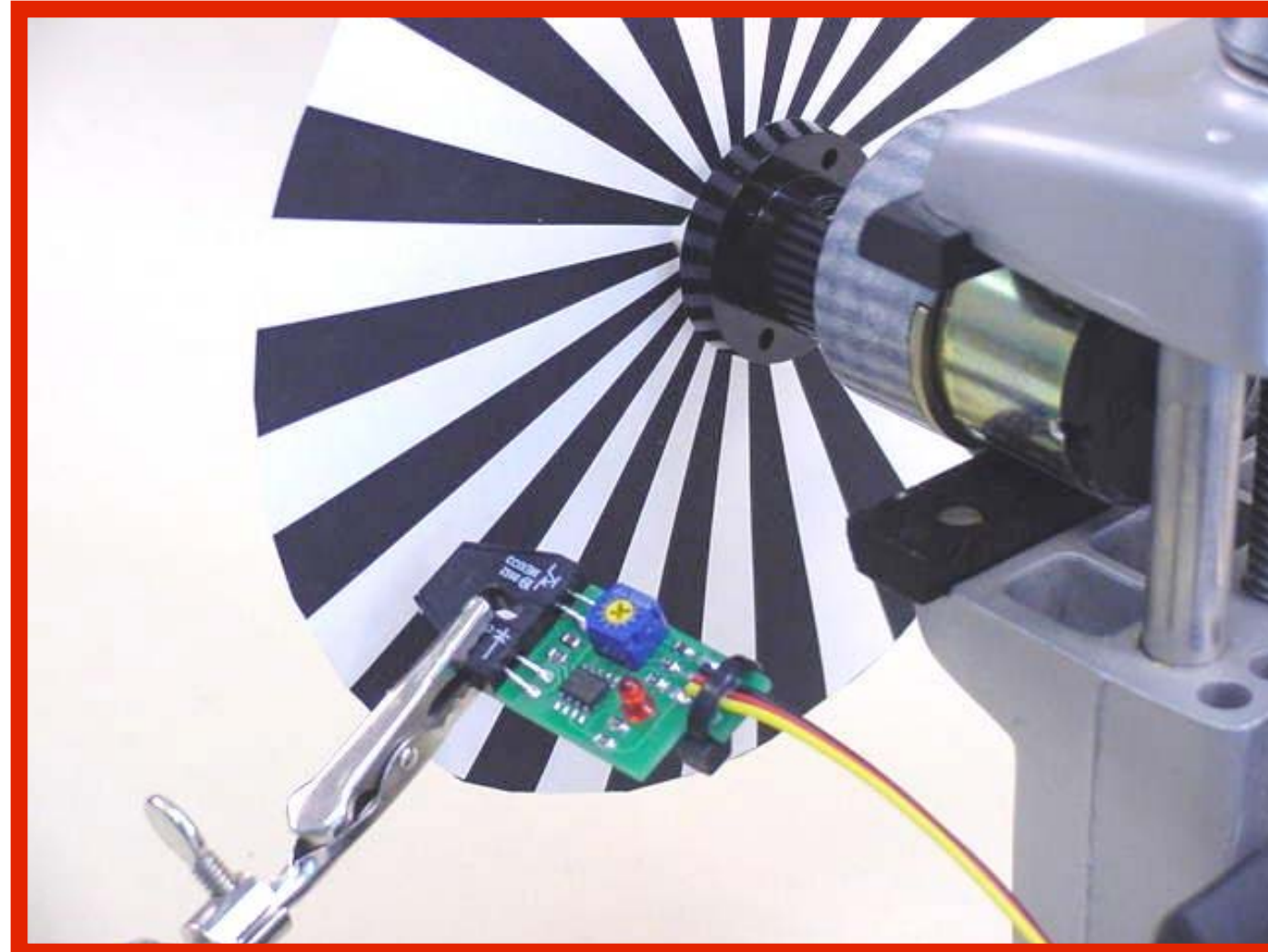


bumper is essentially an on/off button

# Bug

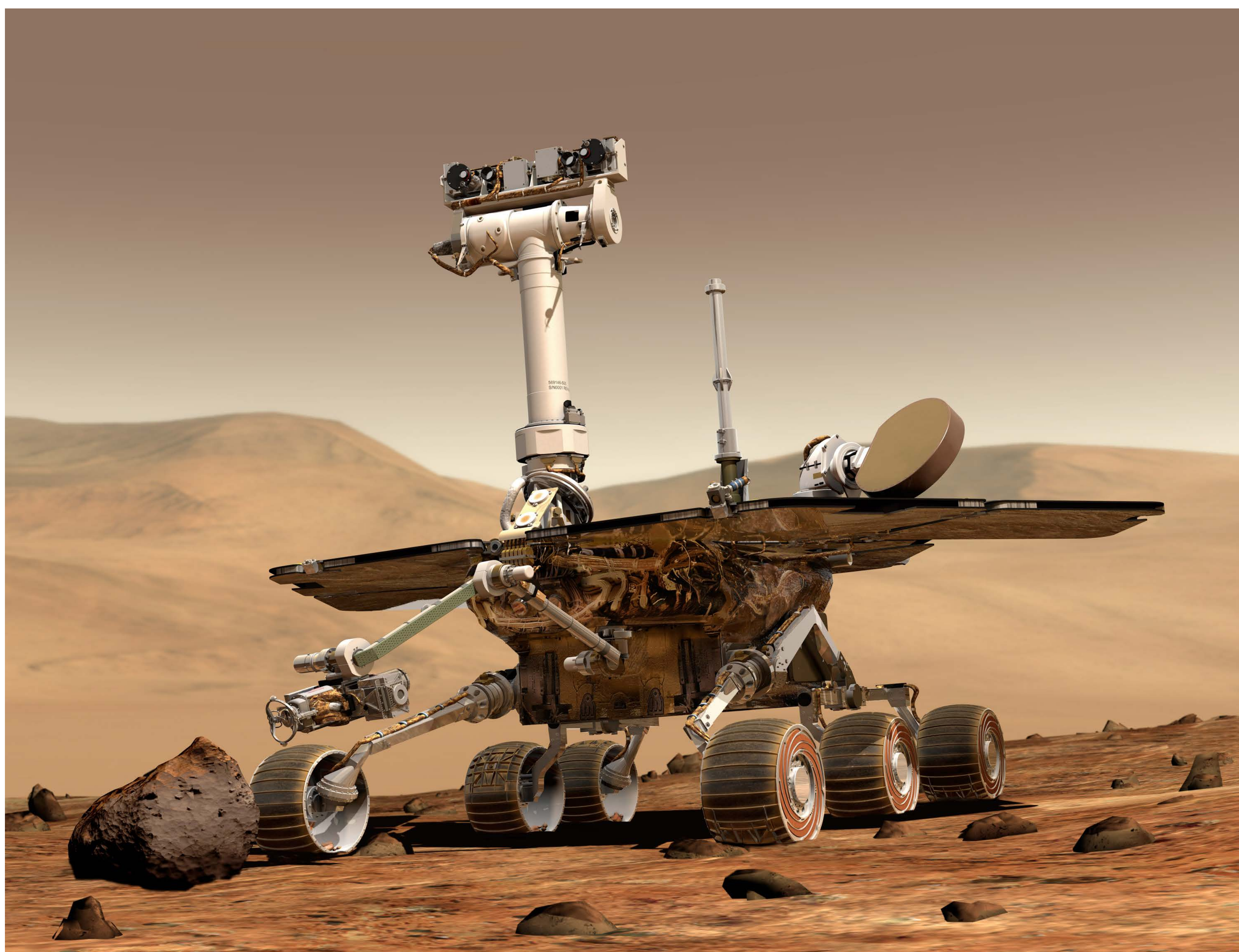
## Optical encoders

- Assume
- Known:
  - meas
- Unknow
- Local sensing
  - bump
  - odometry

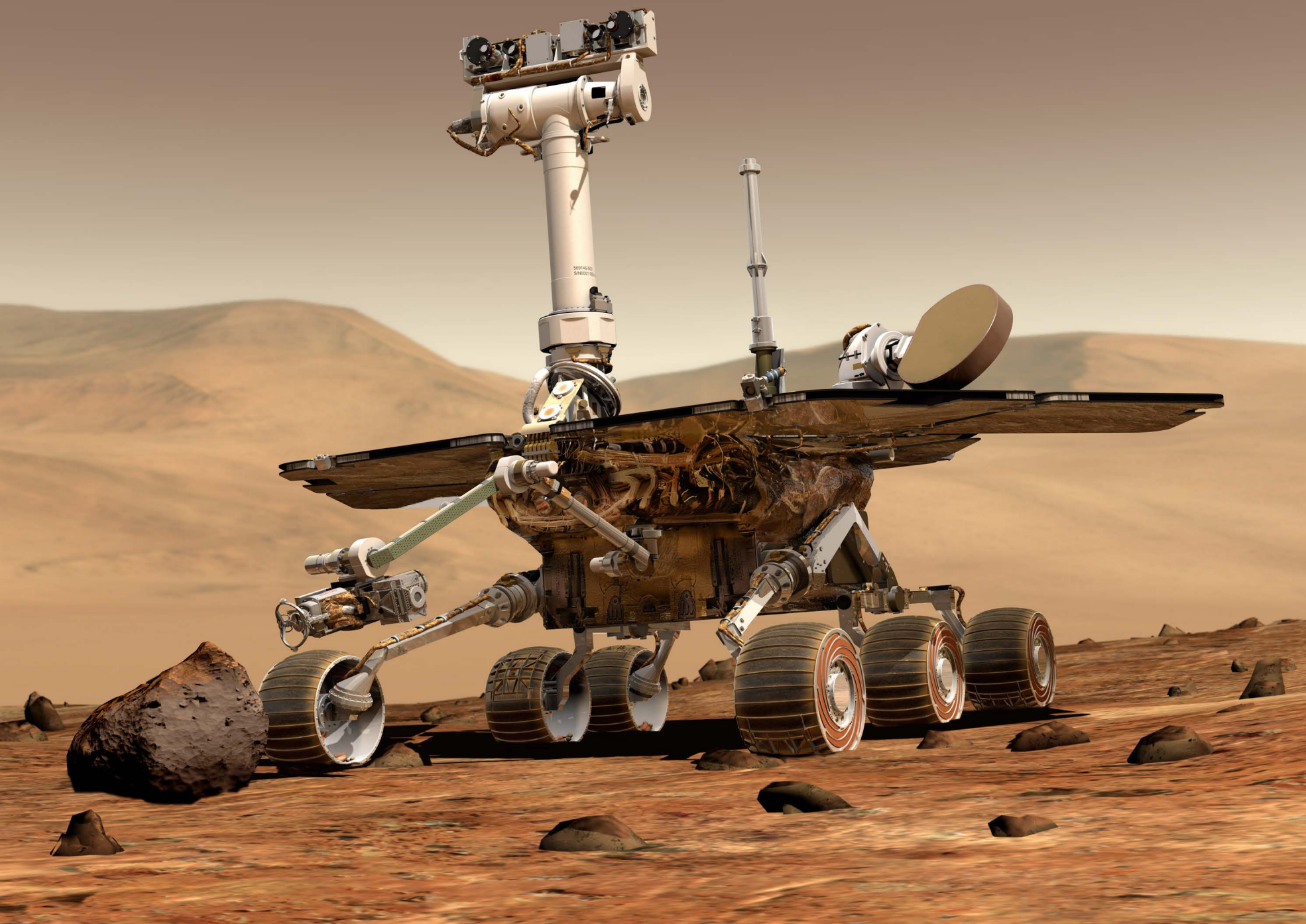


# Interesting application of Bug algorithms ?



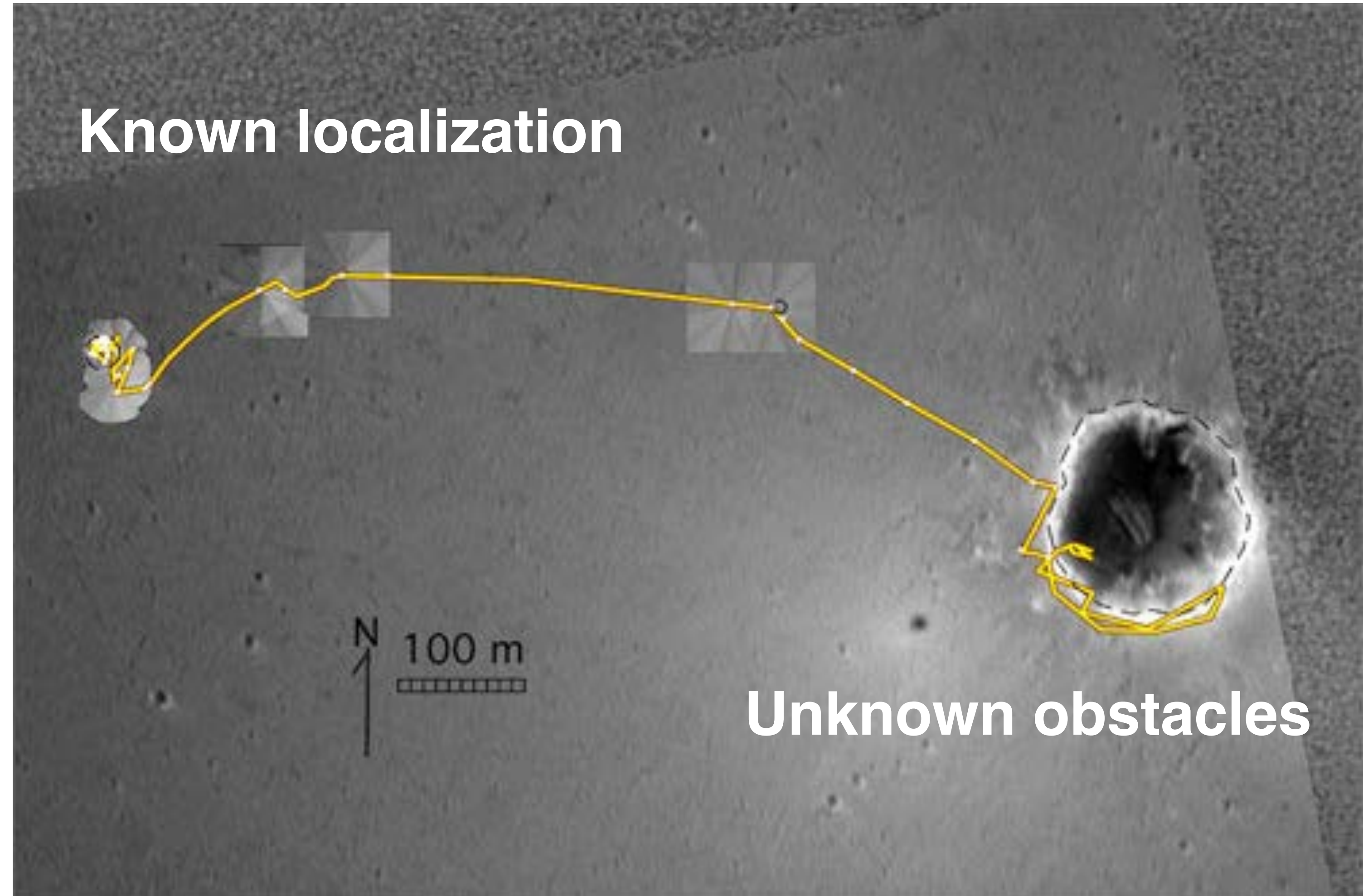


# Mars Exploration Rover



<http://mars.nasa.gov/mer/gallery/press/opportunity/20040921a.html>

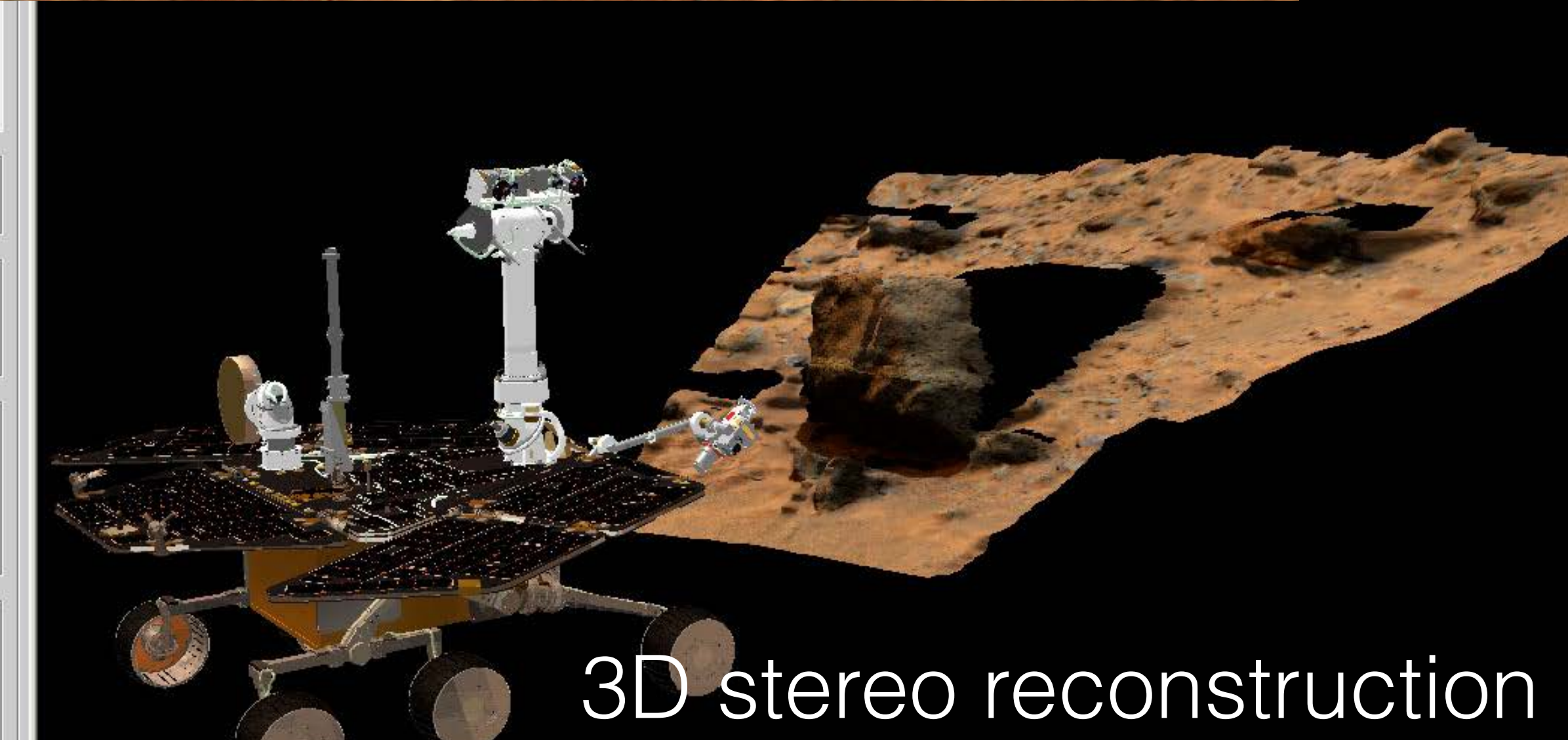
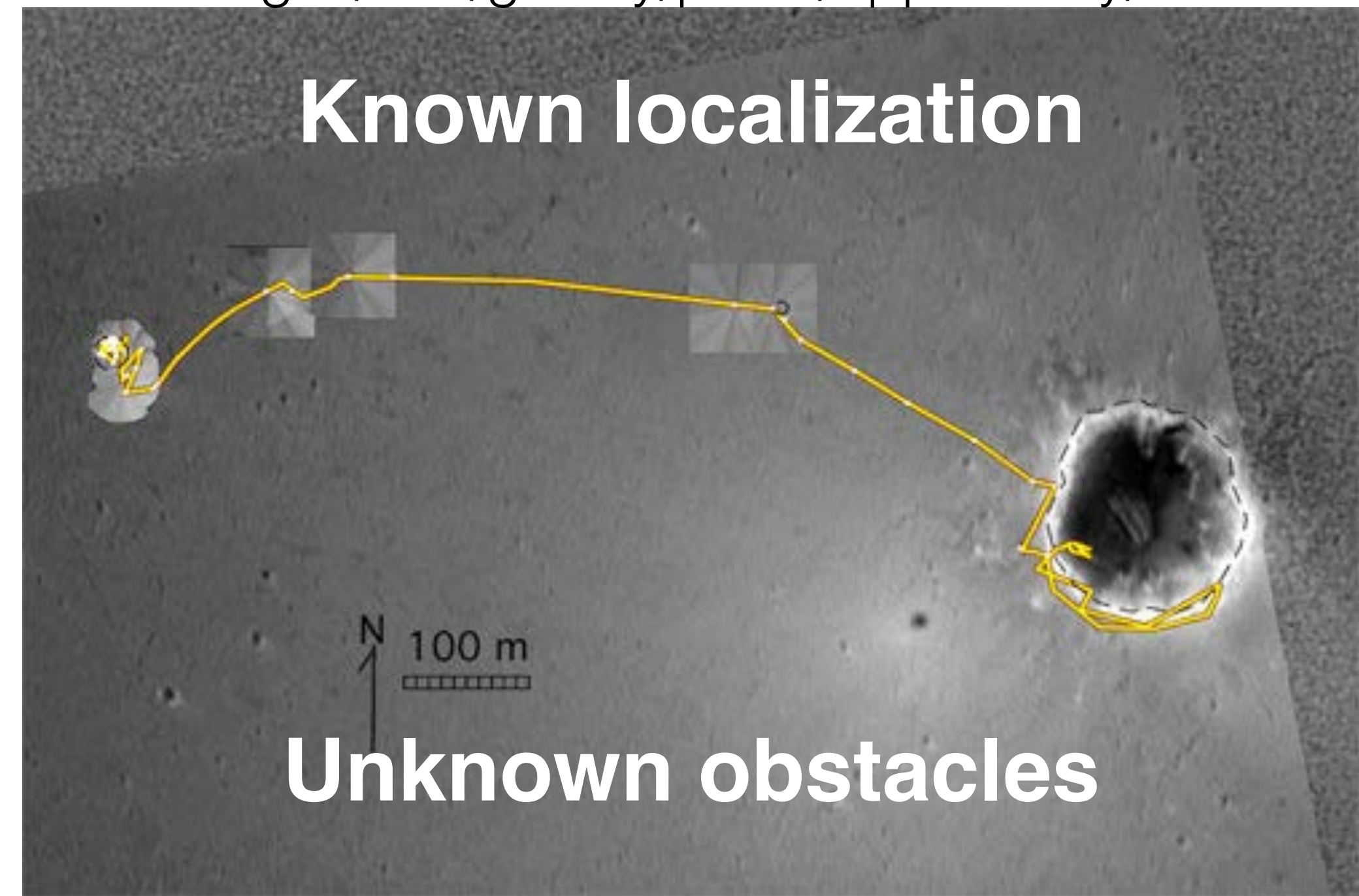
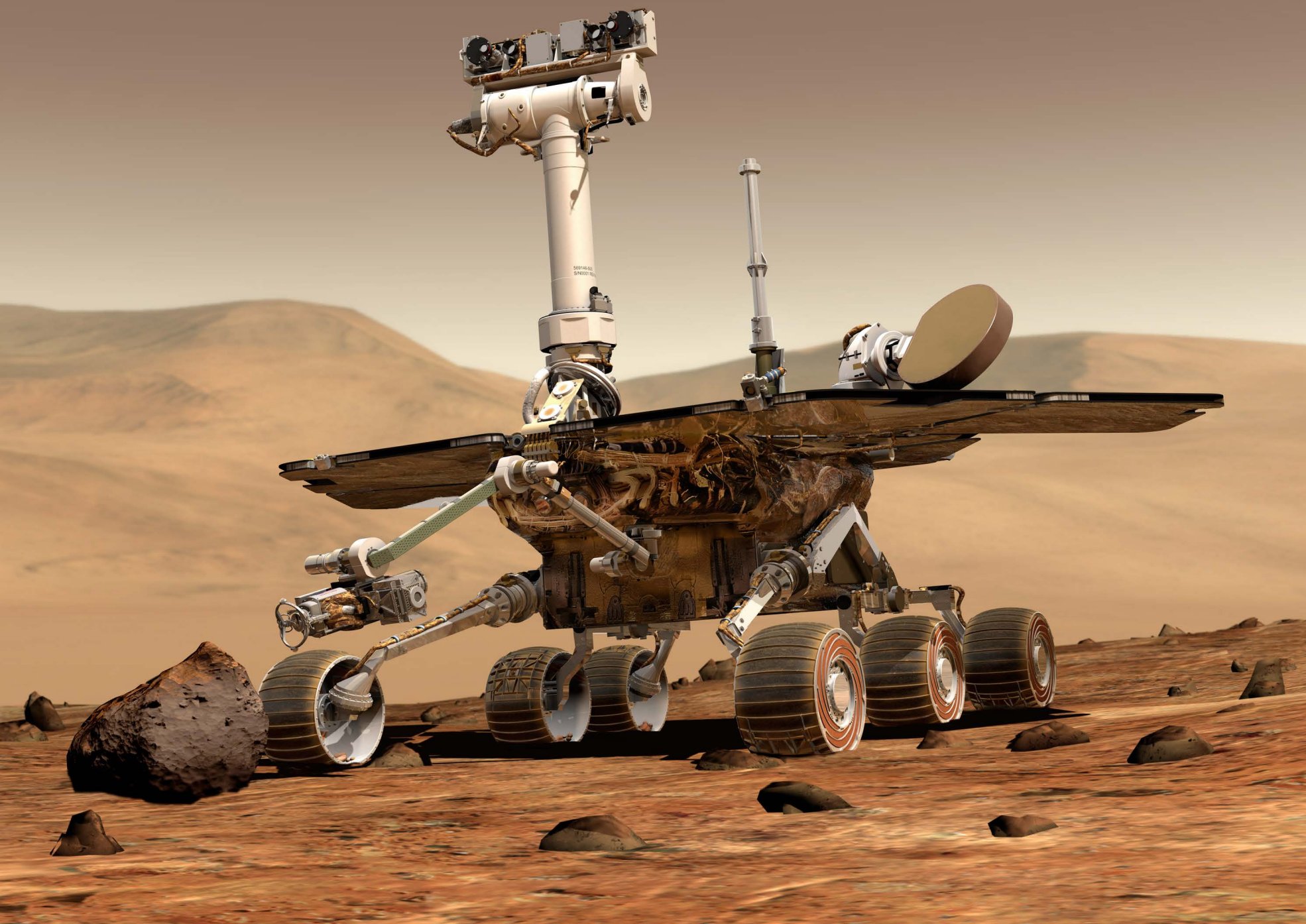
Known localization



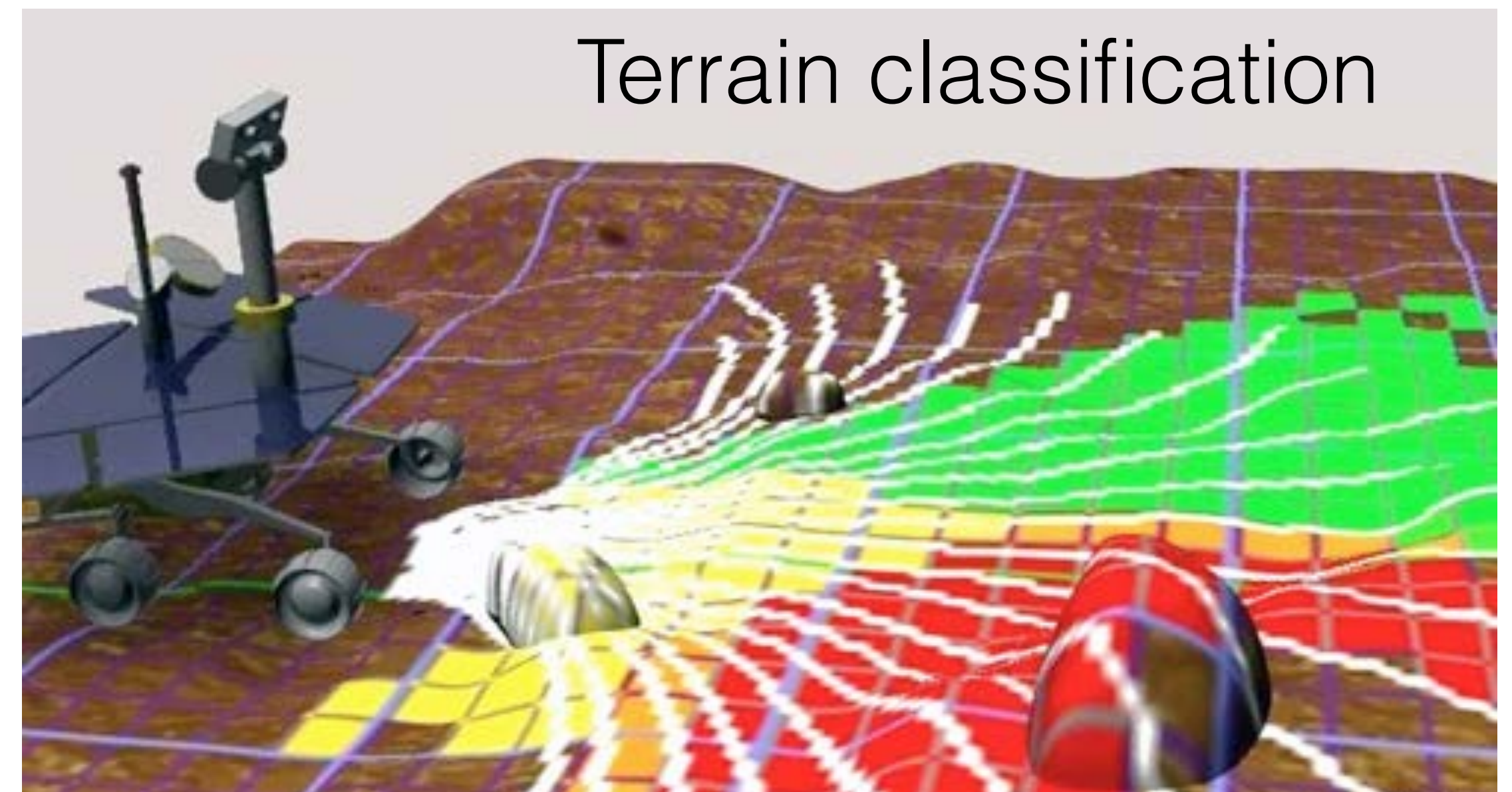
Unknown obstacles

# Mars Exploration Rover

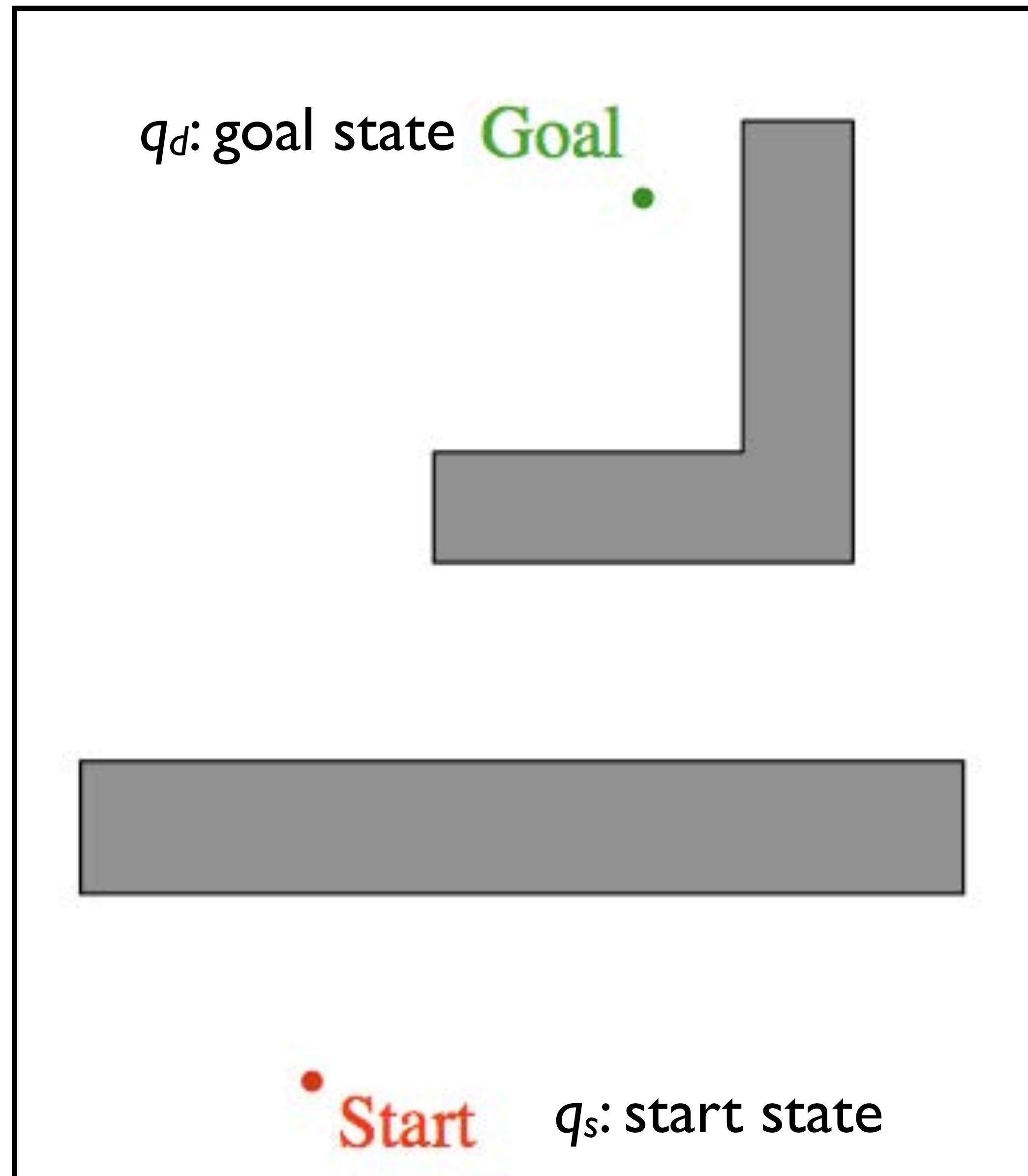
<http://mars.nasa.gov/mer/gallery/press/opportunity/20040921a.html>



3D stereo reconstruction



# Bug Navigation

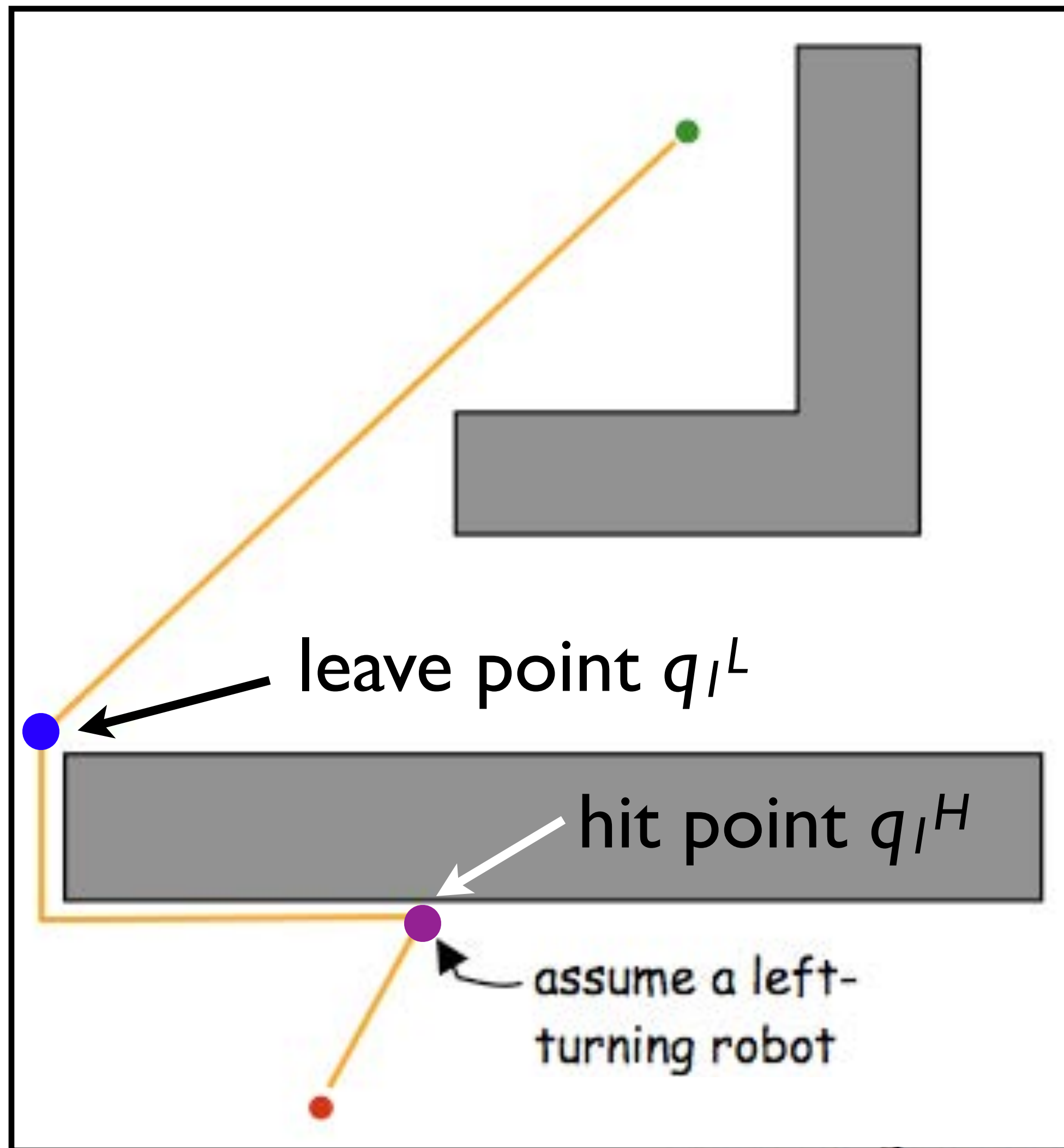


Plan navigation path from start  $q_s$   
to goal  $q_d$

as a sequence of hit/leave point  
pairs on obstacles

Hit point:  $q_i^H$   
Leave point:  $q_i^L$

# Bug 0



- 1) Head towards goal
- 2) When hit point set, **follow wall**, until you can move towards goal again (leave point)
- 3) continue from (1)



# Wall following

follow wall



One approach:

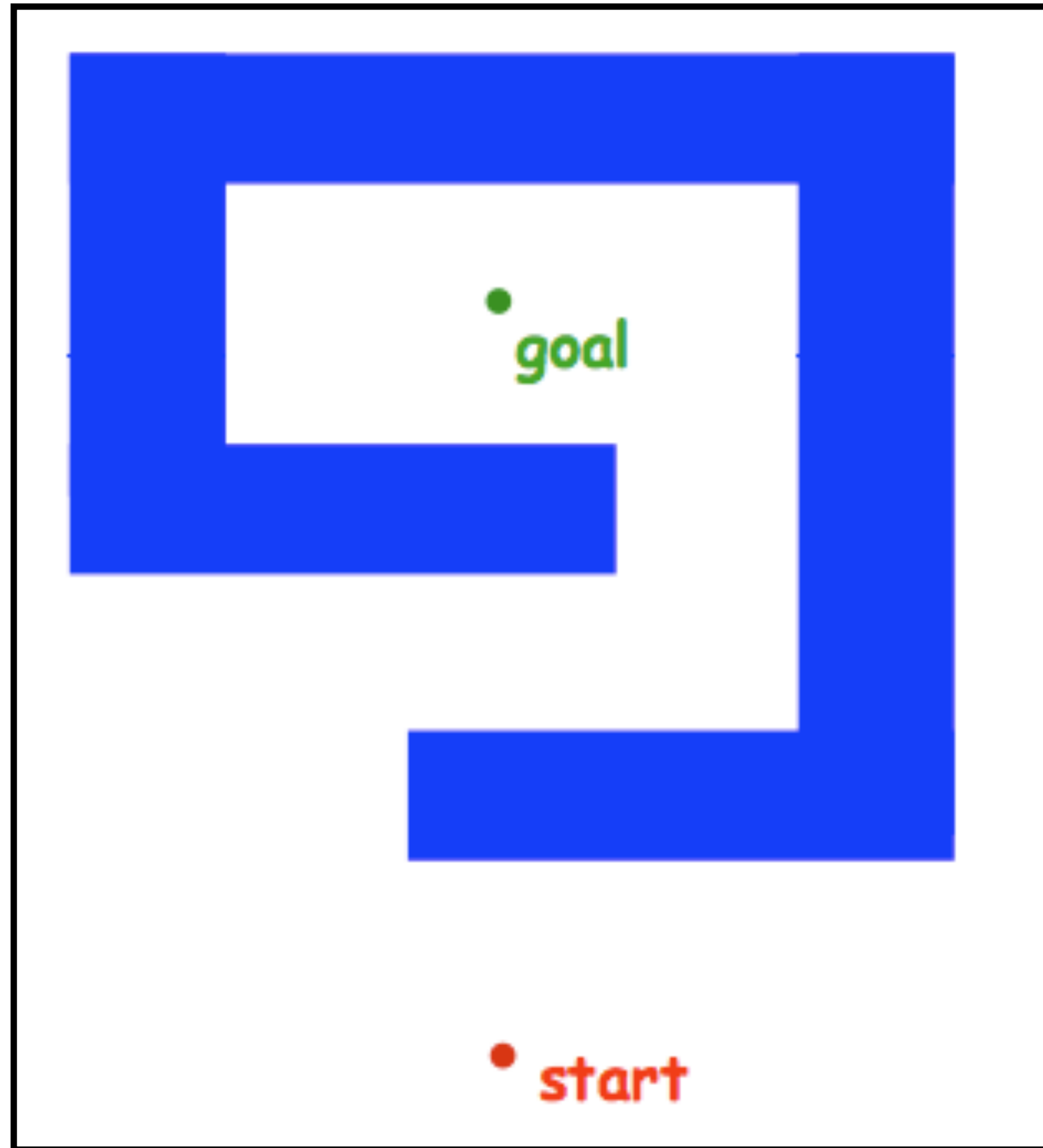
a) move forward with slight turn

b) when bumped, turn opposite direction

c) goto (a)



# Bug 0

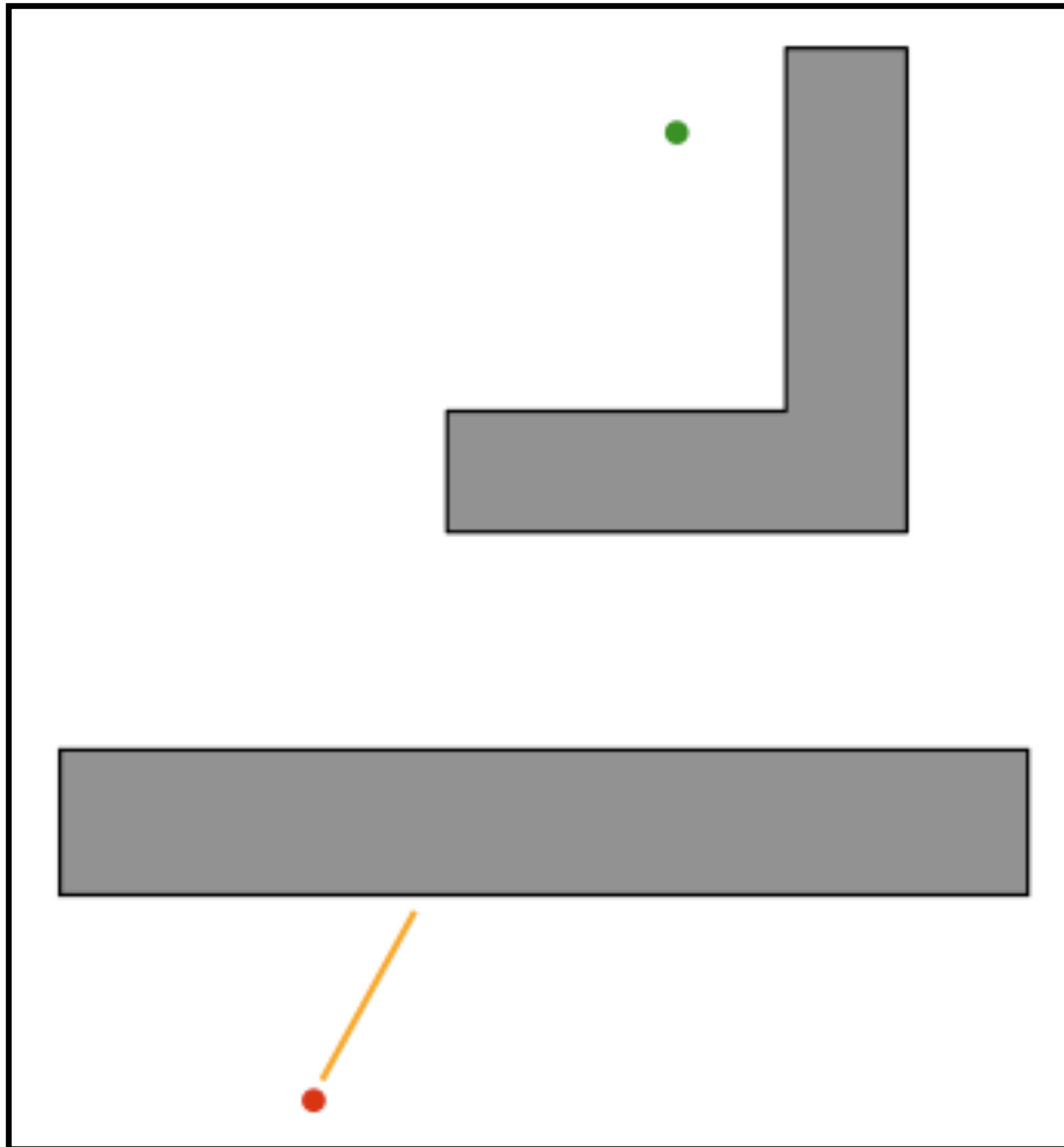


- 1) Head towards goal
- 2) When hit point set, follow wall, until you can move towards goal again (leave point)
- 3) continue from (1)

Can you trace the Bug 0 path?

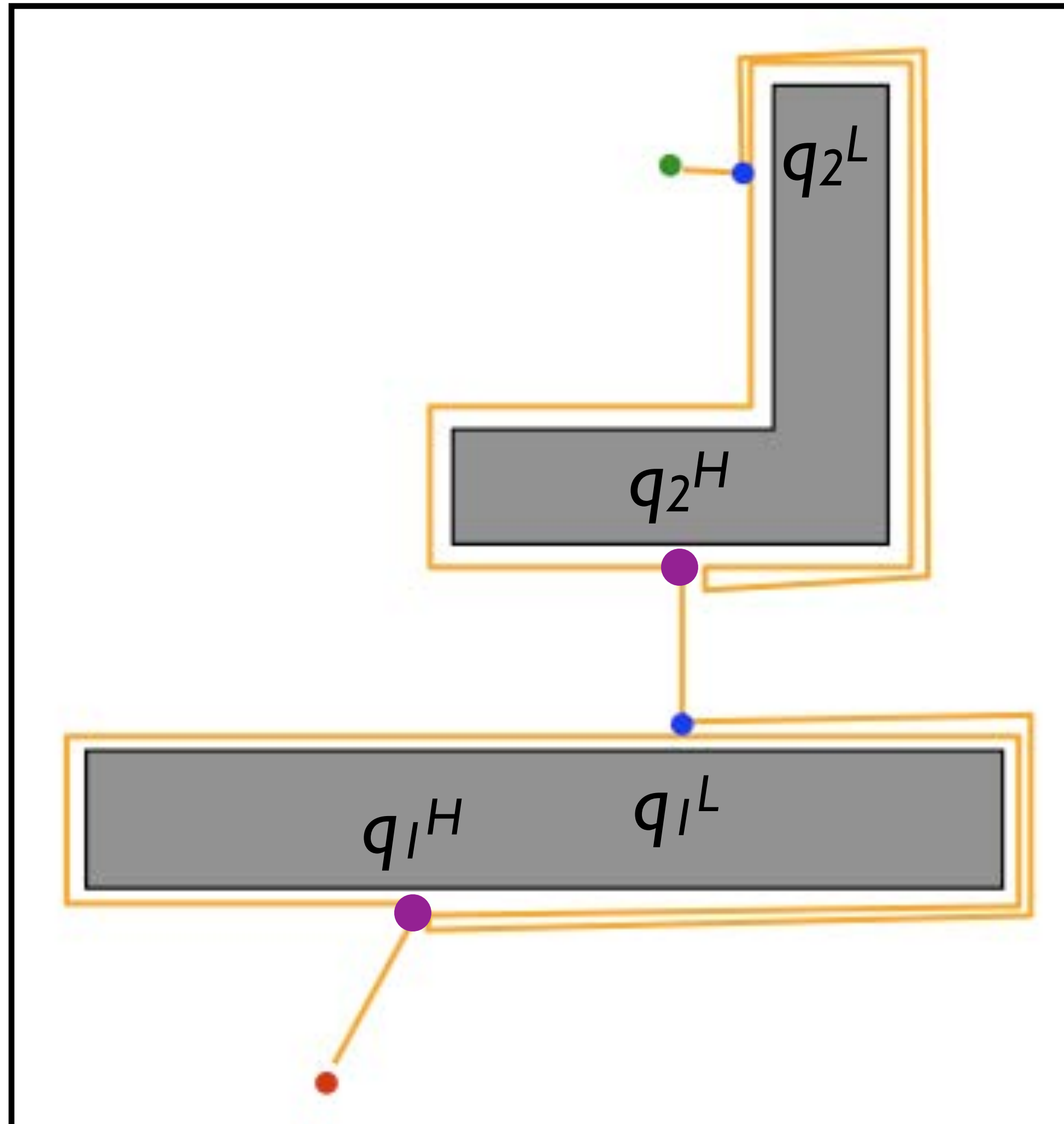


# Bug 1



- 1) Head towards goal
- 2) When hit point set, circumnavigate obstacle, setting leave point as closest to goal
- 3) return to leave point
- 4) continue from (1)

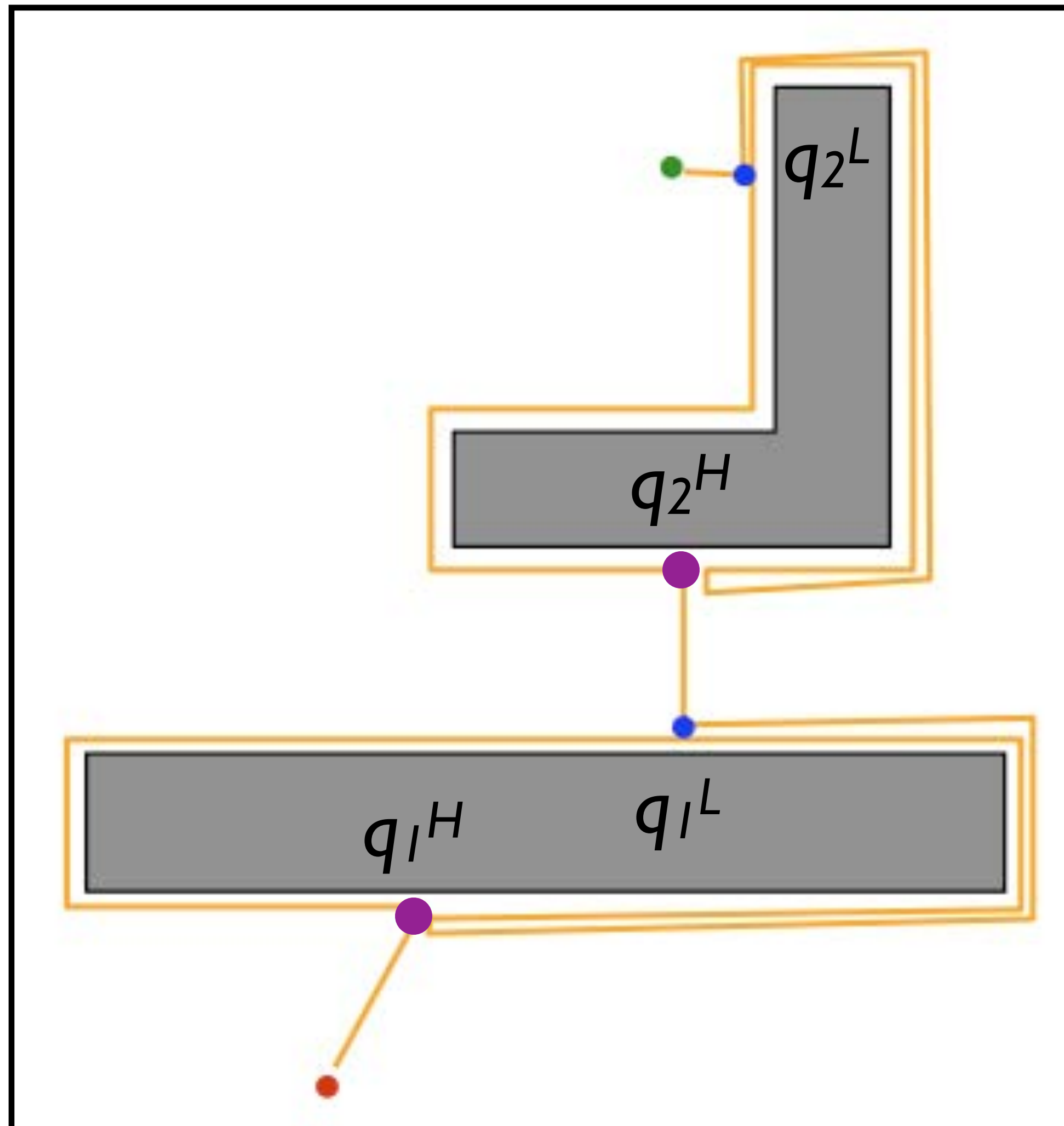
# Bug 1



- 1) Head towards goal
- 2) When hit point set, circumnavigate obstacle, setting leave point as closest to goal
- 3) return to leave point
- 4) continue from (1)

What map would foil Bug 1?

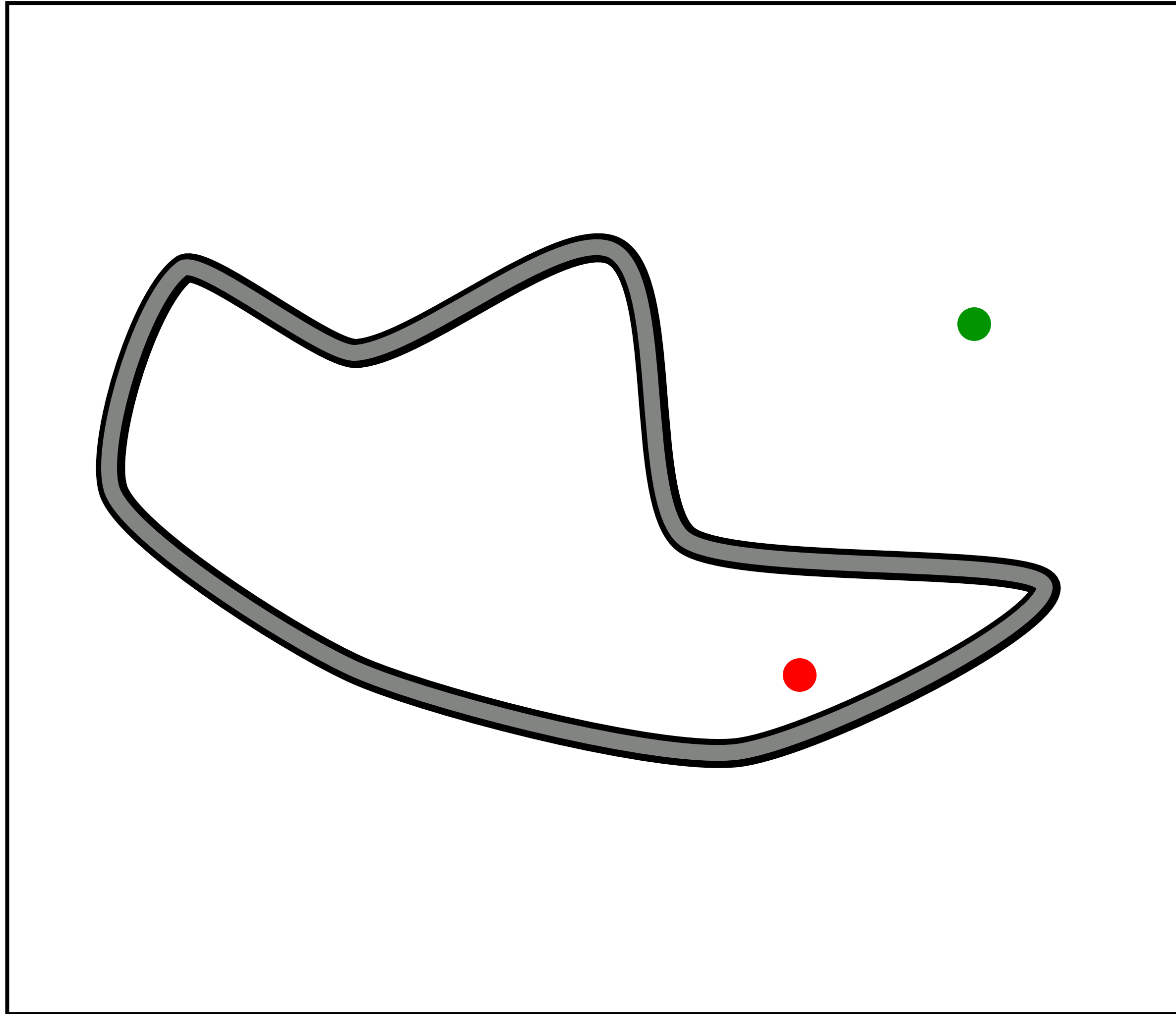
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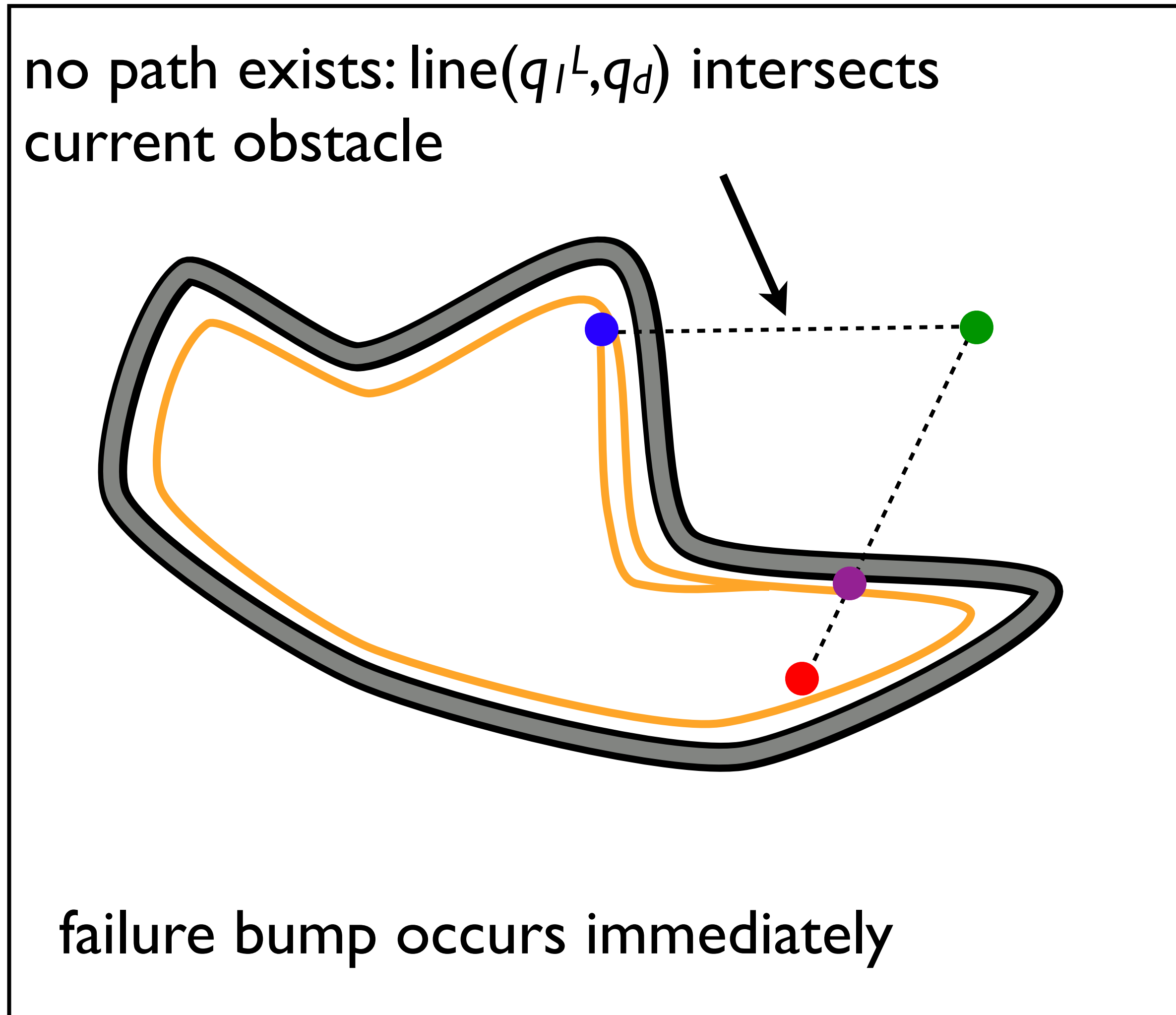
# Bug 1



- 1) Head towards goal
- 2) When hit point set, circumnavigate obstacle, setting leave point as closest to goal
- 3) return to leave point
- 4) if bump current obstacle, return fail; else, continue from (1)



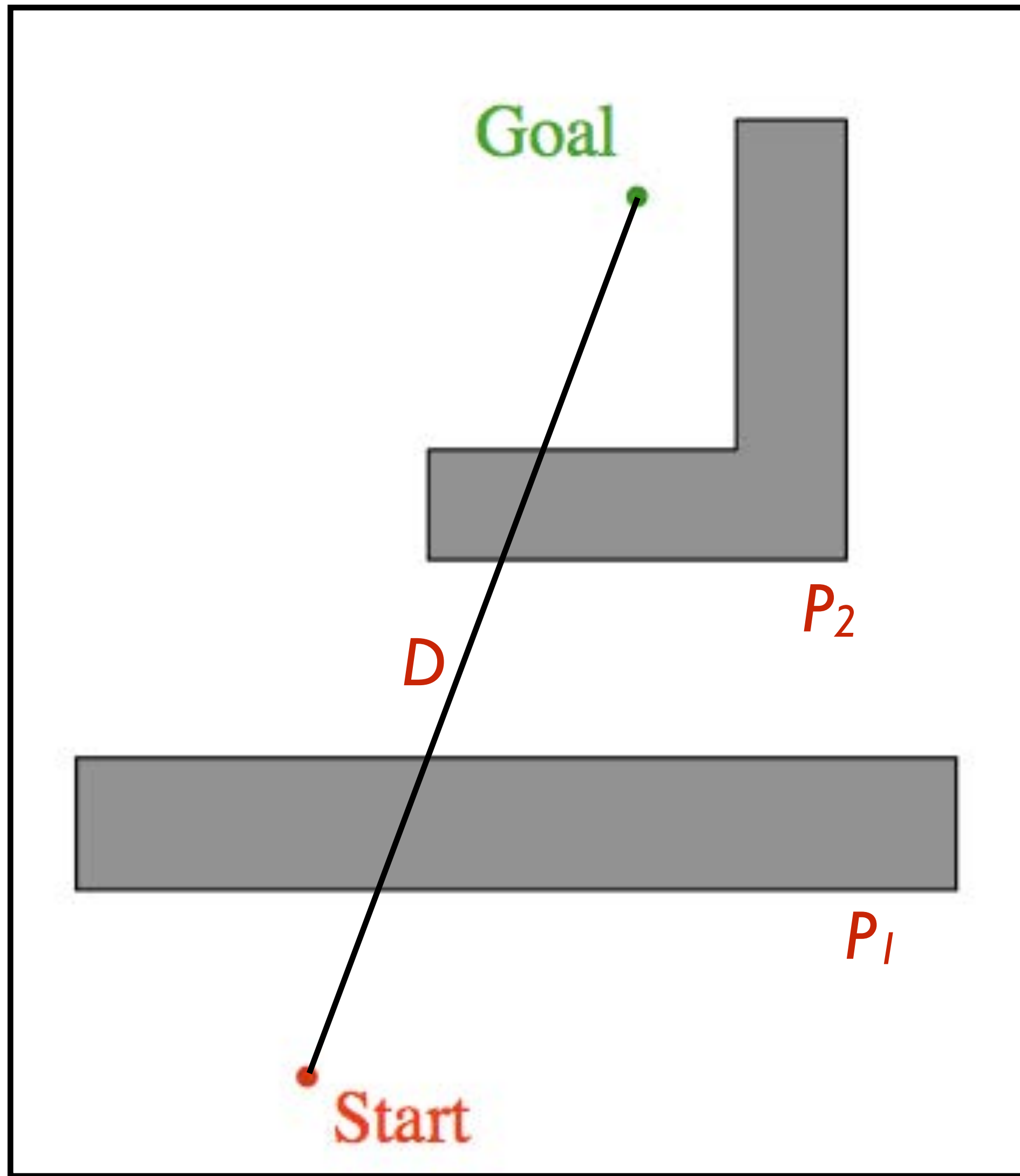
What map would foil Bug 1?



# Bug 1: Detecting Failure

- 1) Head towards goal
- 2) When hit point set, circumnavigate obstacle, setting leave point as closest to goal
- 3) return to leave point
- 4) if bump current obstacle, return **fail**;  
else, continue from (1)

# Bug 1: Search Bounds



Bounds on path distance, assuming

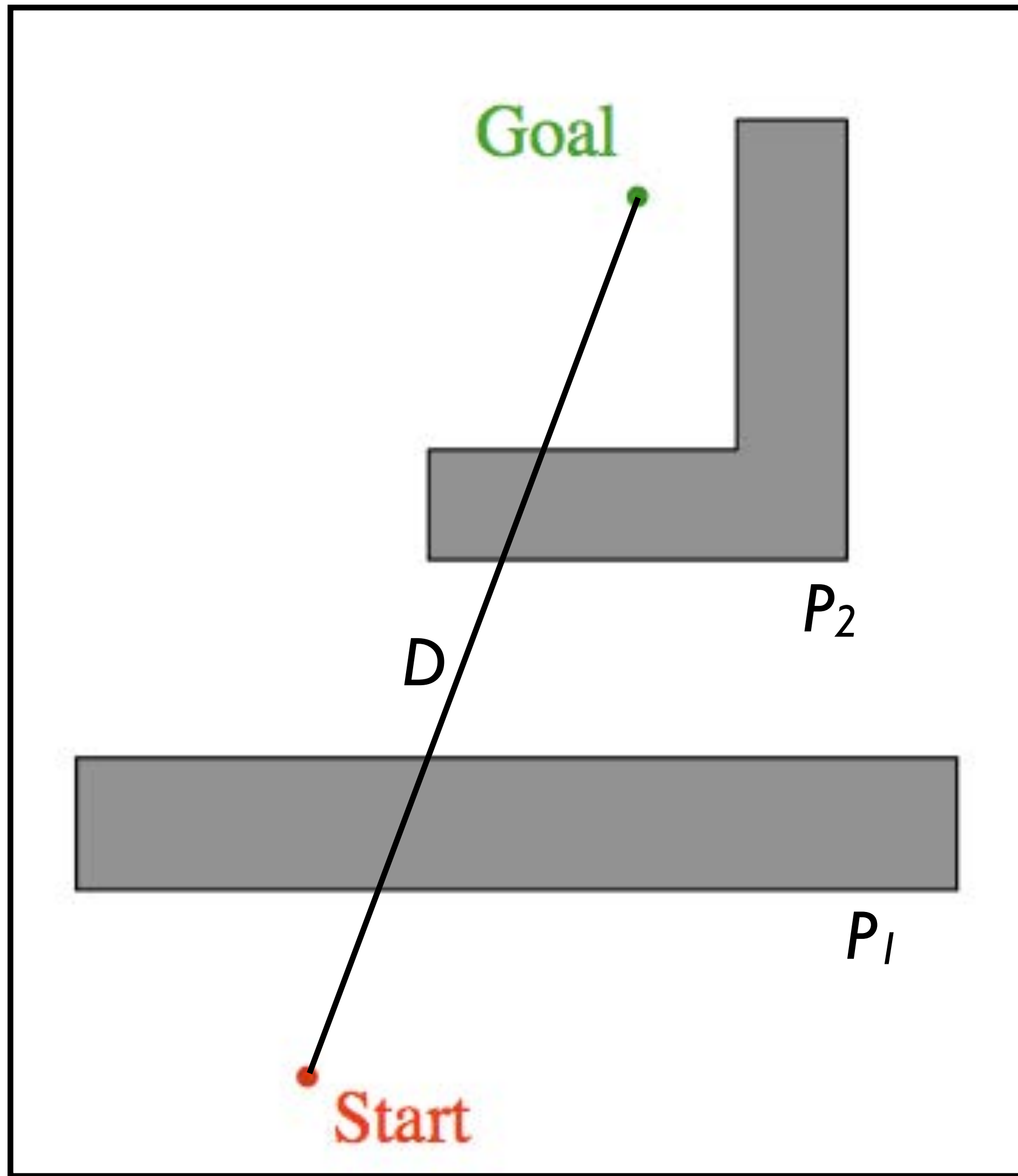
$D$ : distance start-to-goal

$P_i$ : obstacle perimeter

Best case: 

Worst case: 

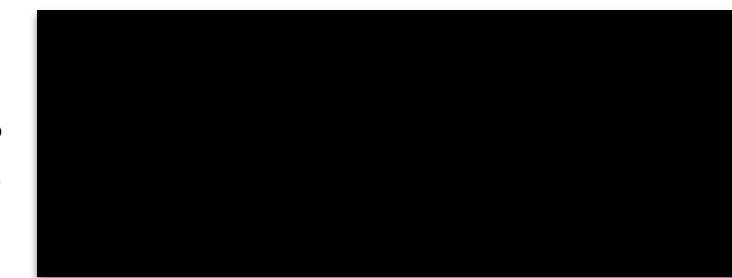
# Bug 1: Search Bounds



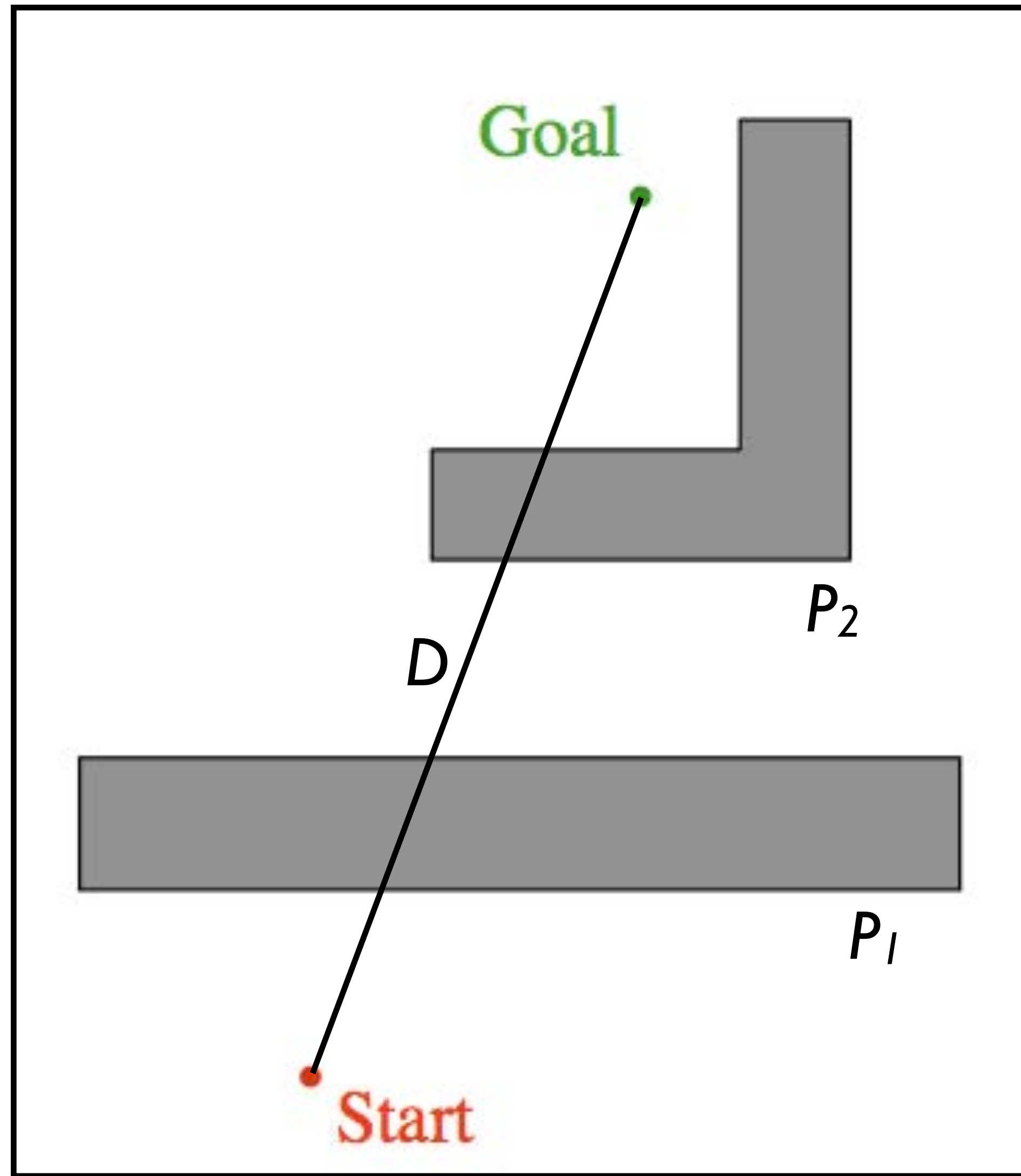
Bounds on path distance, assuming  
 $D$ : distance start-to-goal  
 $P_i$ : obstacle perimeter

Best case:  $D$

Worst case:



# Bug 1: Search Bounds



Bounds on path distance, assuming  
 $D$ : distance start-to-goal  
 $P_i$ : obstacle perimeter

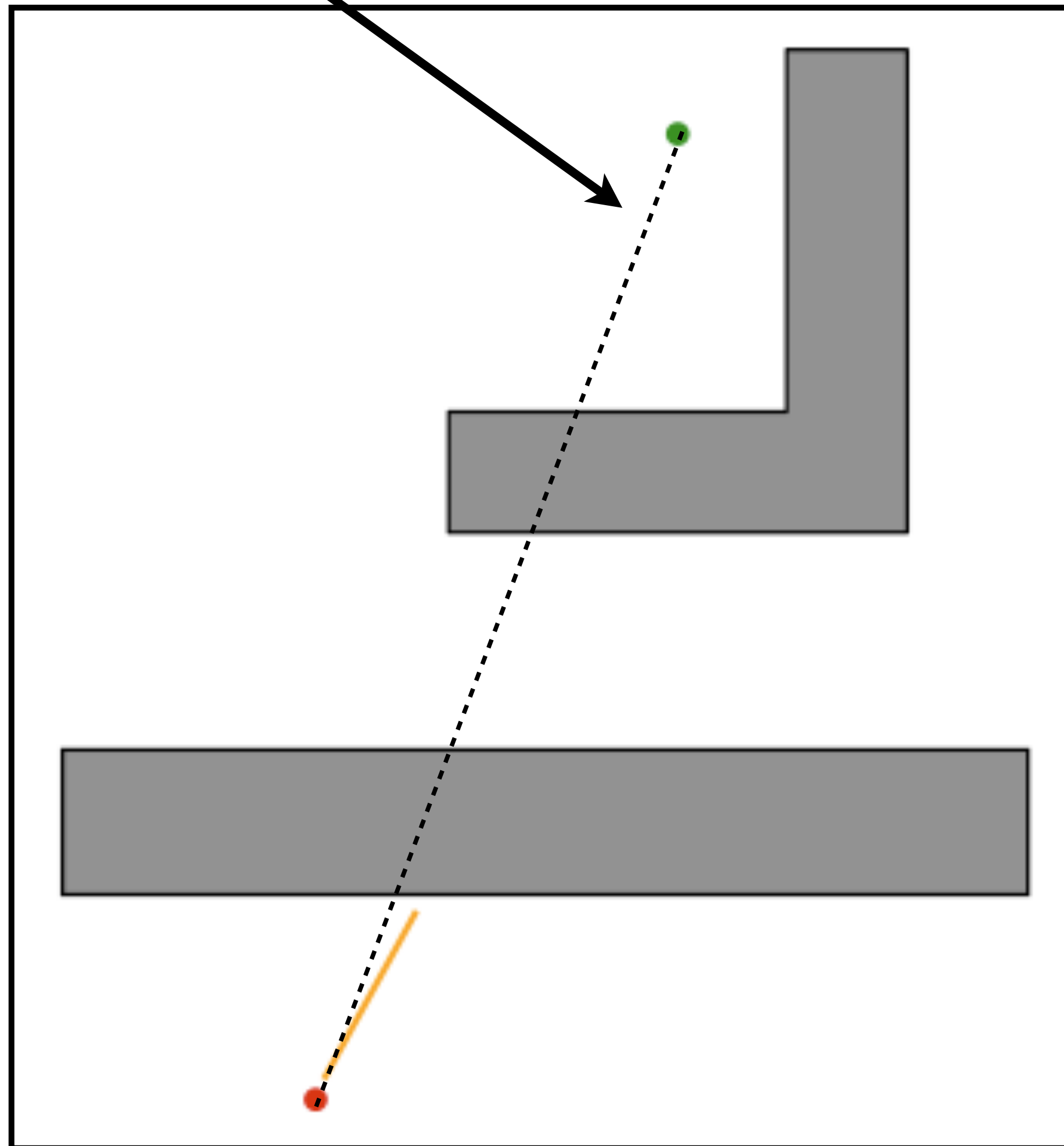
Best case:  $D$

Worst case:  $D + 1.5 \sum_i P_i$

Is there a faster bug?

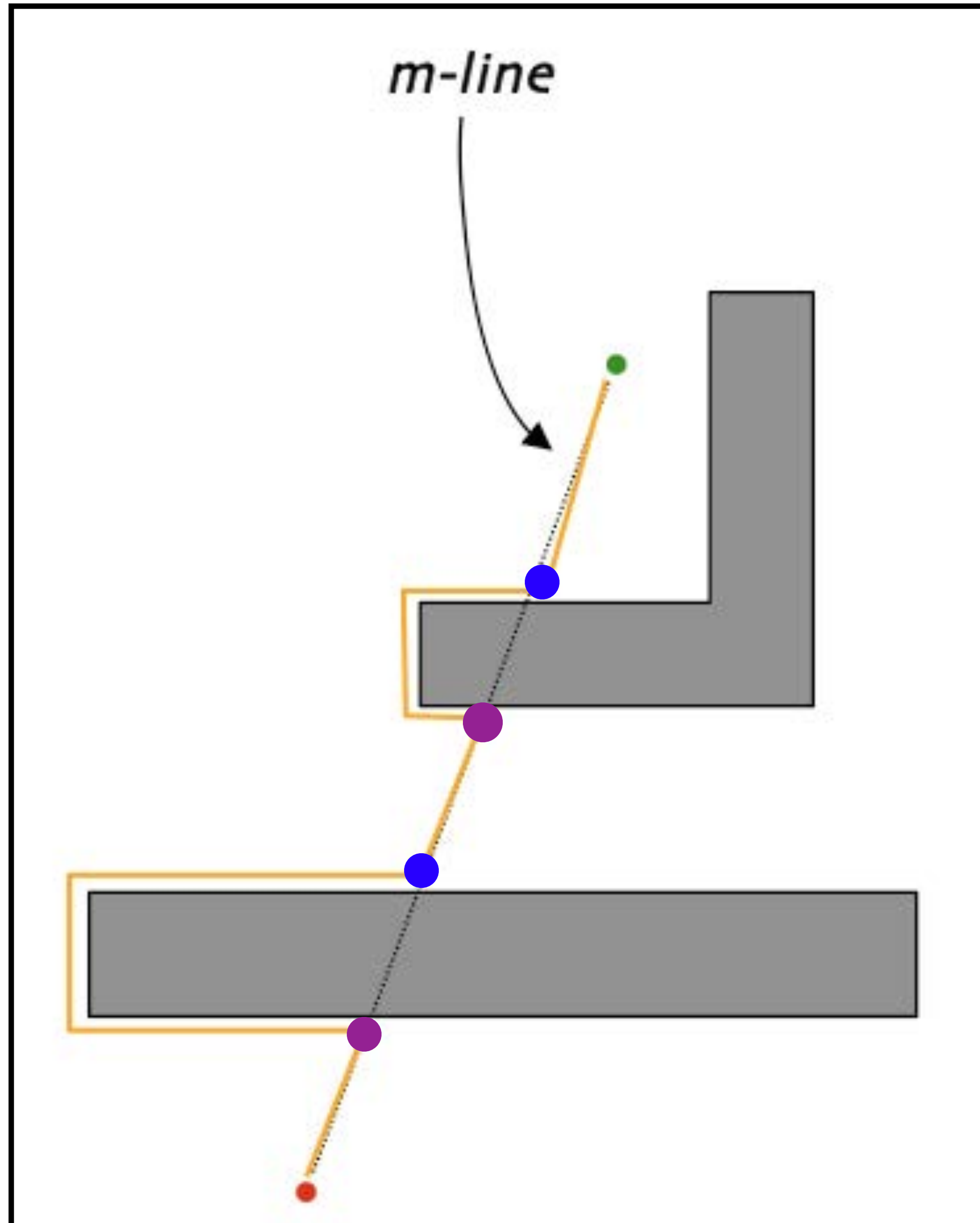
# Bug 2

**m-line:** straight line path to goal



- 1) Head towards goal on m-line
- 2) When hit point set, traverse obstacle until m-line is encountered
- 3) set leave point and exit obstacle
- 4) continue from (1)

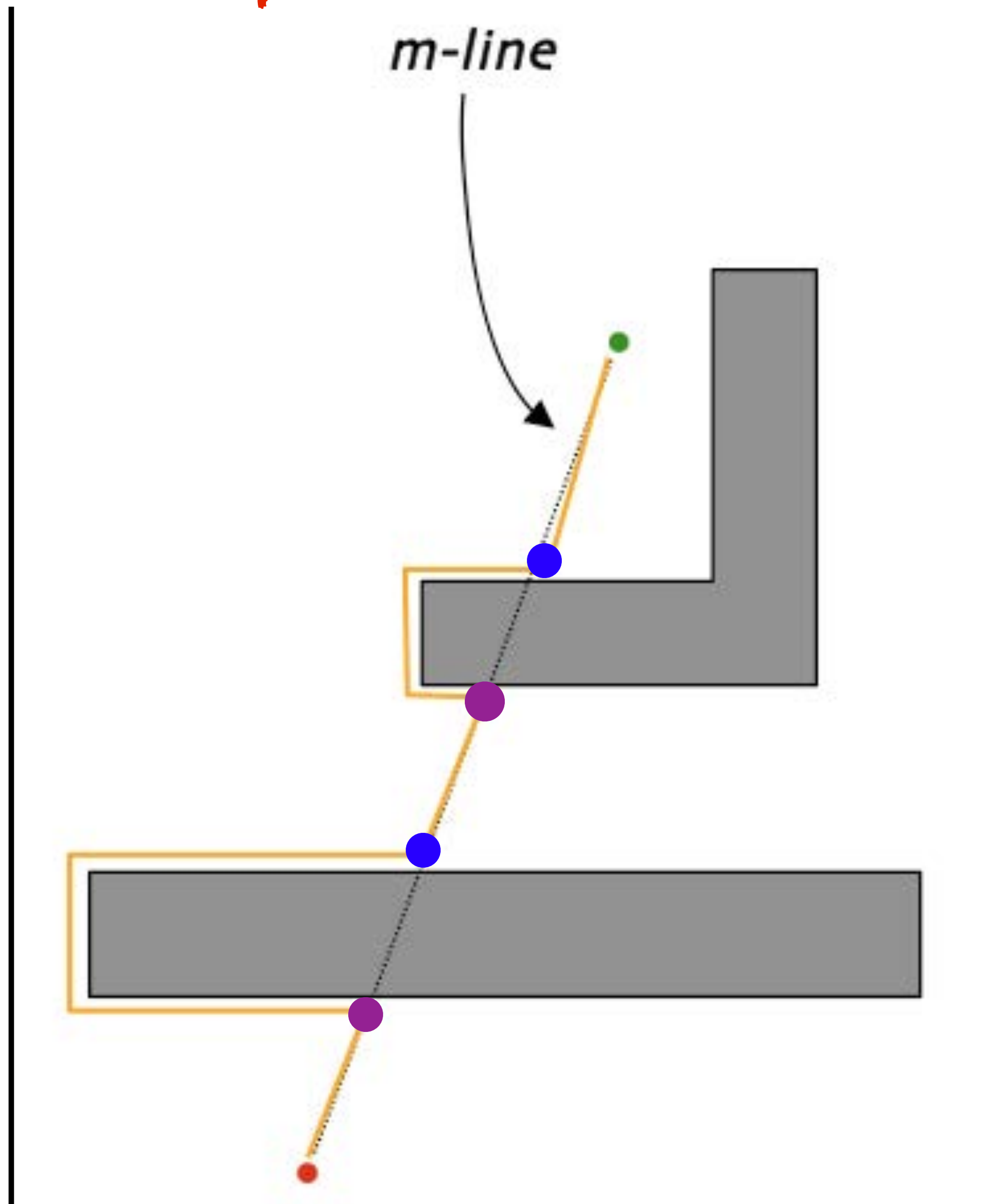
# Bug 2



- 1) Head towards goal on m-line
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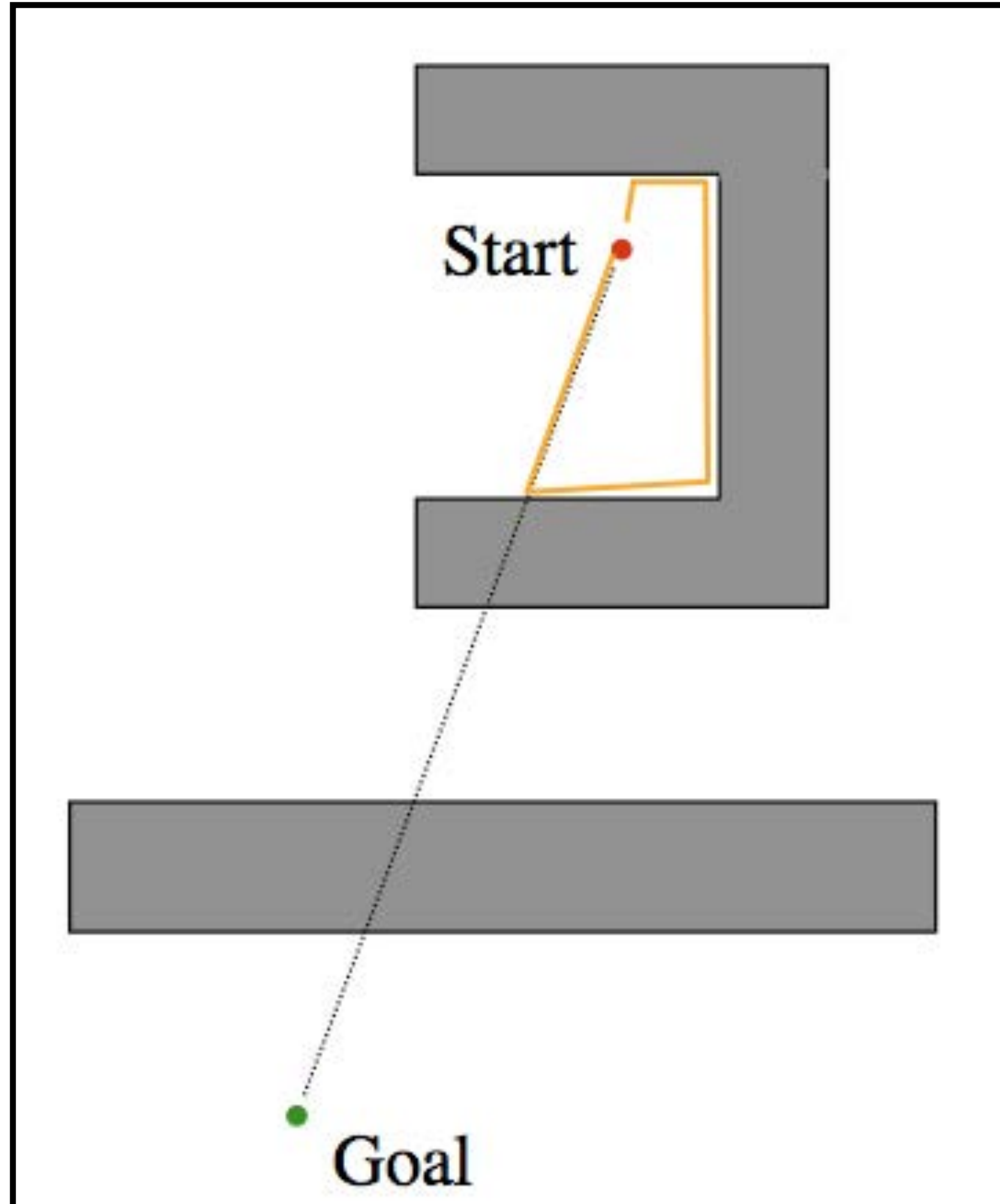
# What map would foil Bug 2?

## Bug 2



- 1) Head towards goal on m-line
- 2) When hit point set, traverse obstacle until m-line is encountered
- 3) set leave point and exit obstacle
- 4) continue from (1)

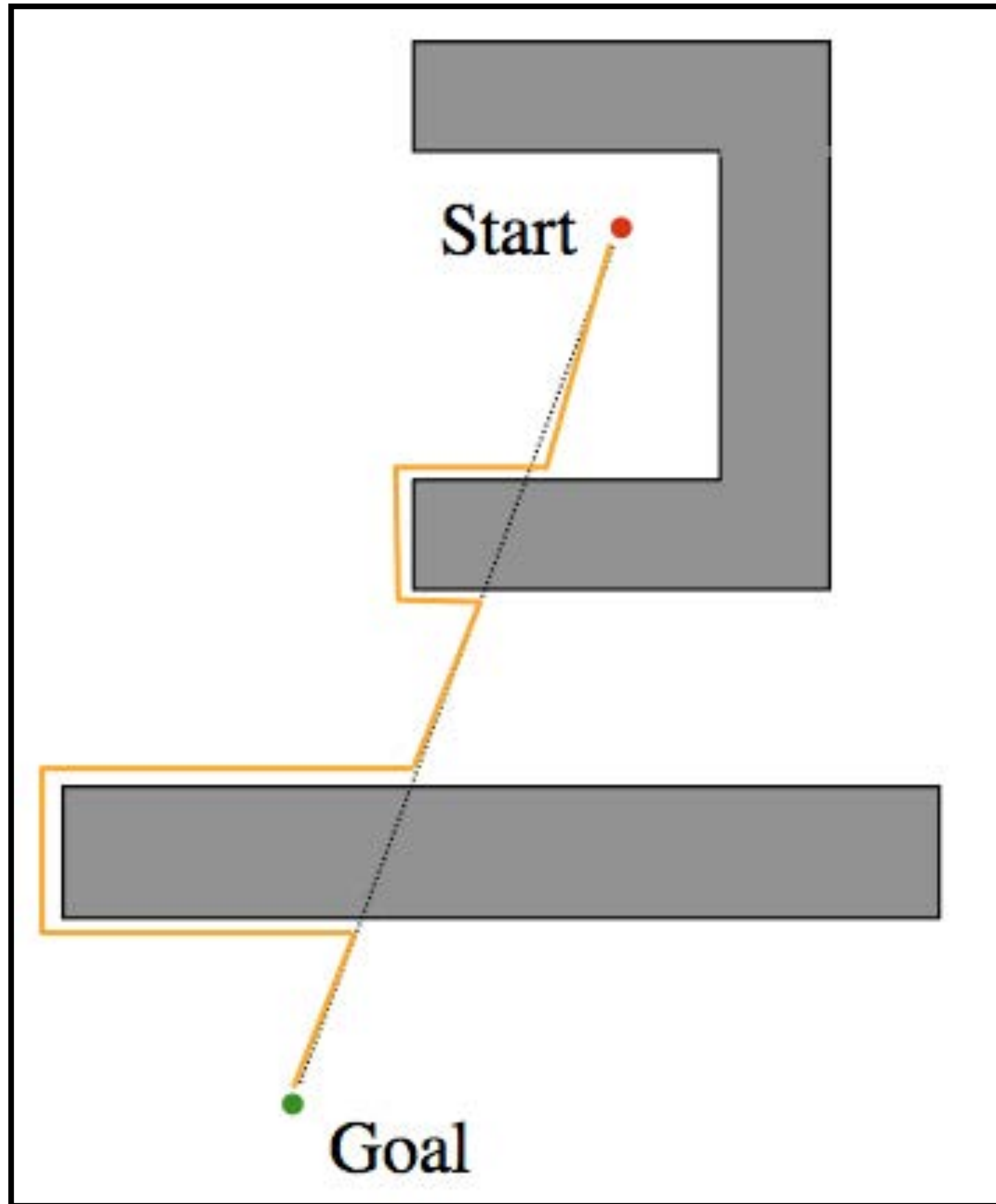
# Bug 2



- 1) Head towards goal on m-line
- 2) When hit point set, traverse obstacle until m-line is encountered
- 3) set leave point and exit obstacle
- 4) continue from (1)



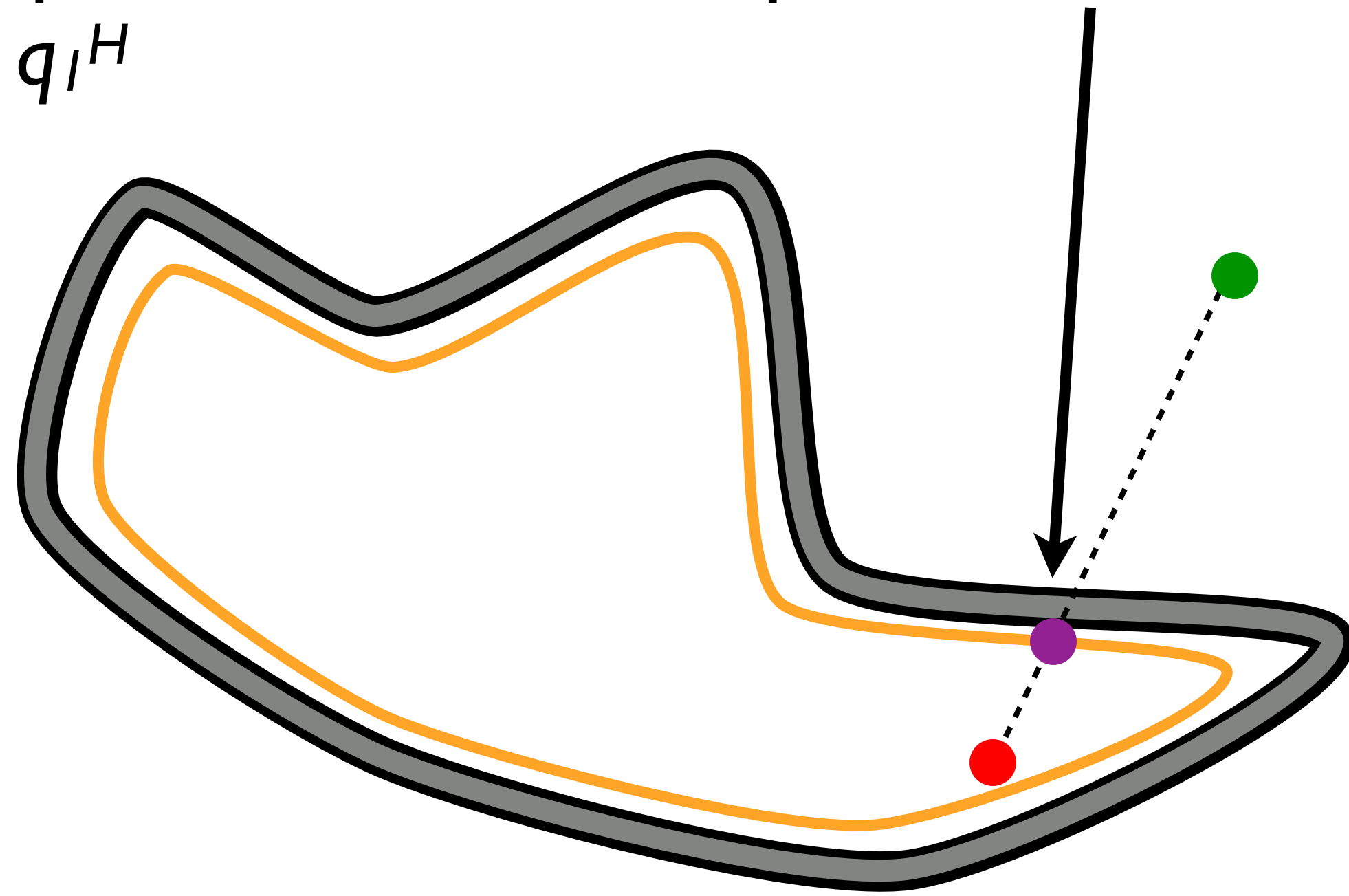
# Bug 2



- 1) Head towards goal on m-line
- 2) When hit point set, traverse obstacle until m-line is encountered **& closer to the goal**
- 3) set leave point and exit obstacle
- 4) continue from (1)

# Bug 2: Detecting Failure

no path exists: no leave point before returning  
to  $q_I^H$

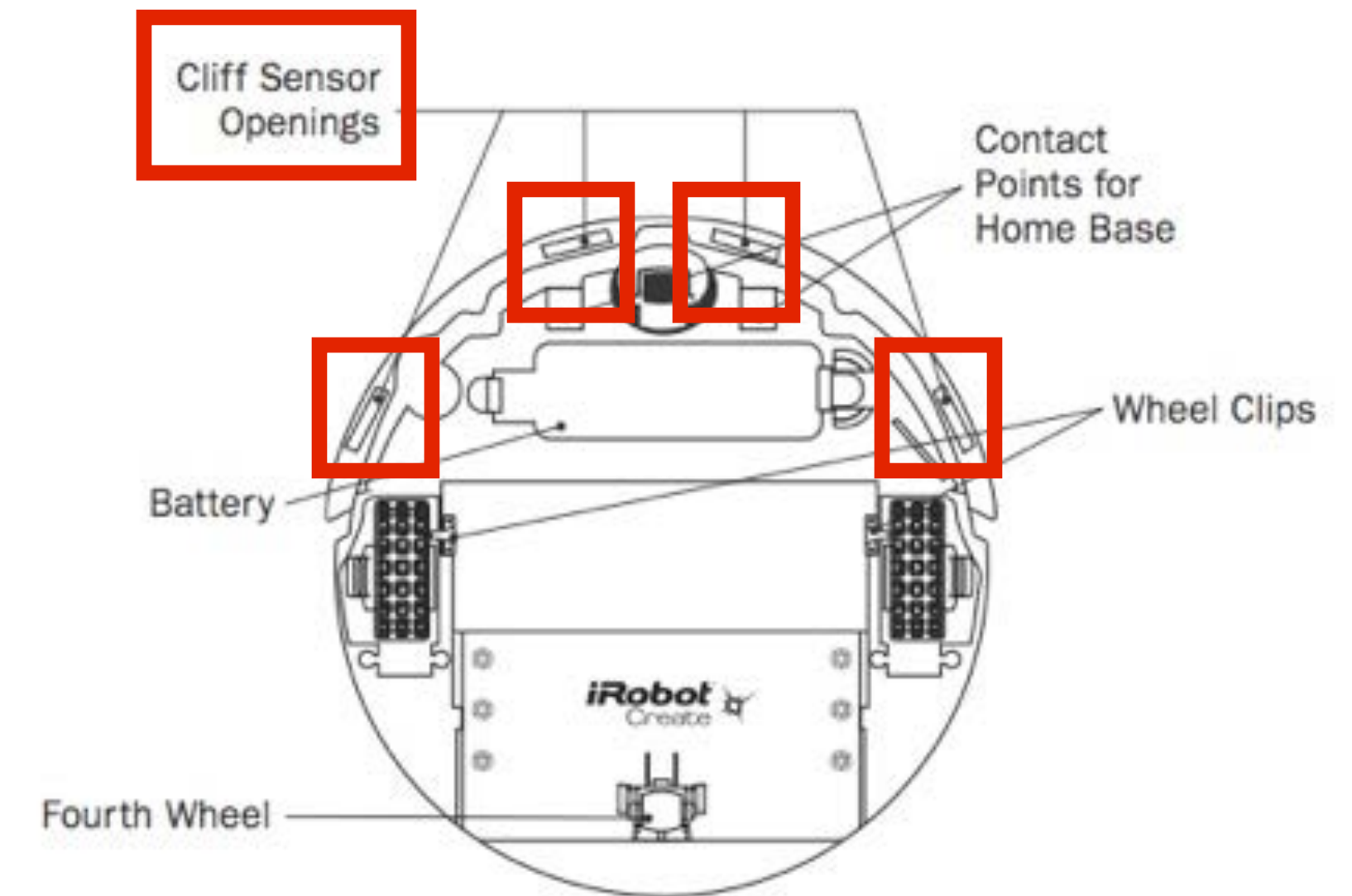


- 1) Head towards goal on m-line
- 2) When hit point set, traverse obstacle until m-line is encountered & closer to the goal  
**or hit point reached**
- 3) **if not  $i^{th}$  hit point**, set leave pt. and exit
- 4) continue from (1)

# Bug 2 in action



m-line drawn on floor  
with tape recognizable by  
Create cliff sensor



Kayle Gishen



Is Bug2 better than Bug1?



# Bug 1 v. Bug 2:

Draw worlds where Bug 2 performs better than Bug 1 (and vice versa)

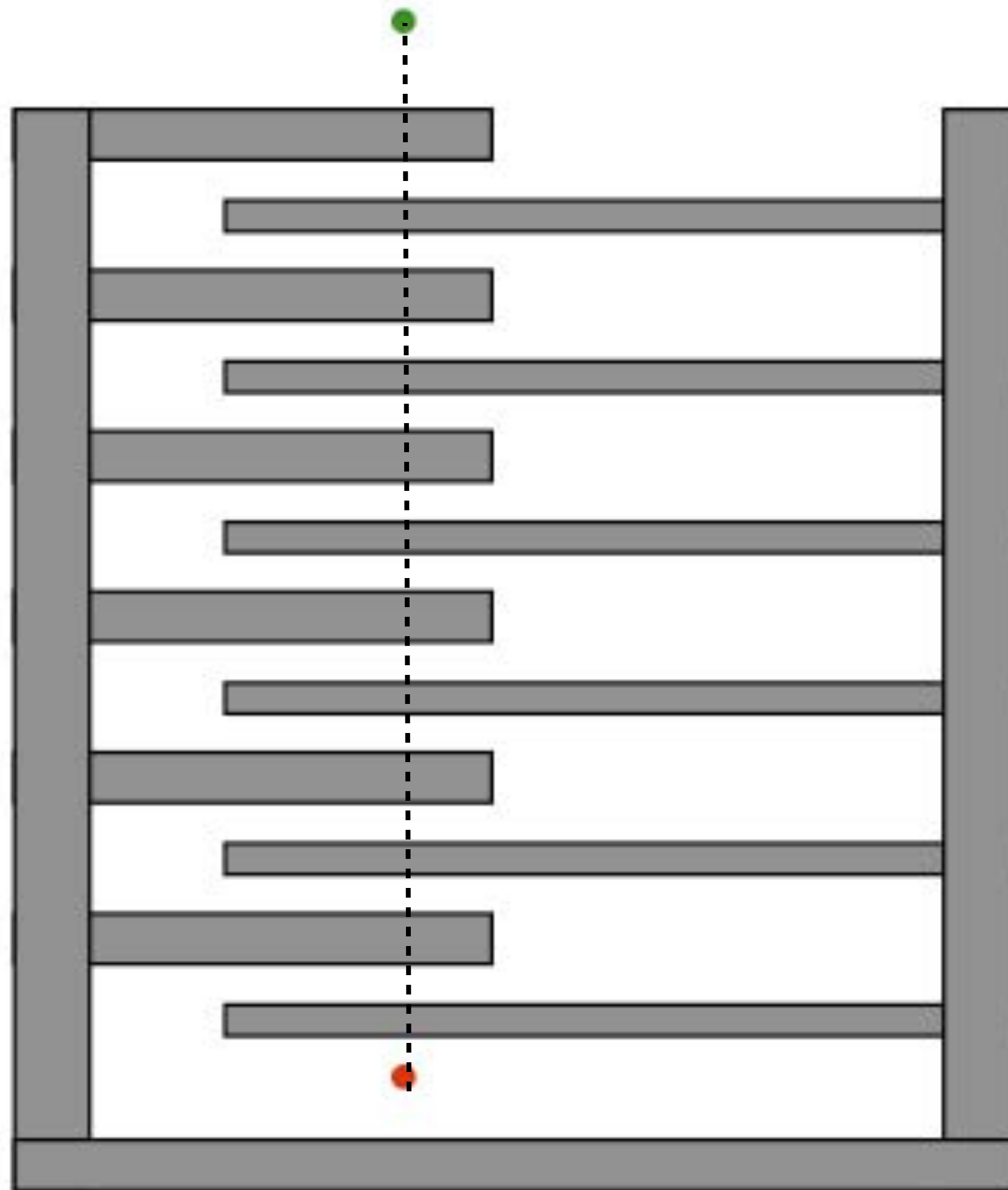
Bug 2 beats Bug 1

Bug 1 beats Bug 2

Home work!



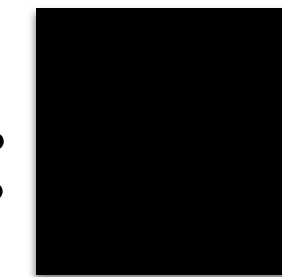
# Bug 2: Search Bounds



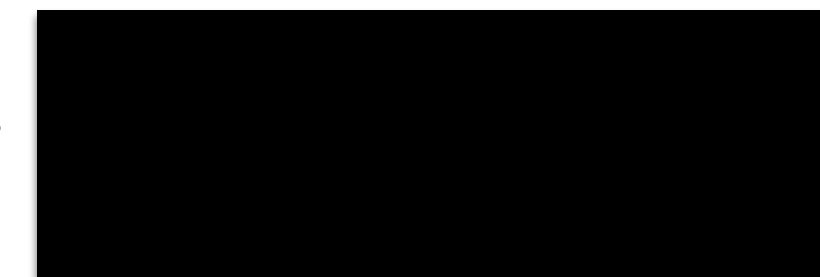
Bounds on path distance, assuming

- $D$ : distance start-to-goal
- $P_i$ : obstacle perimeter
- $n_i$ : number of m-line intersections for  $WO_i$

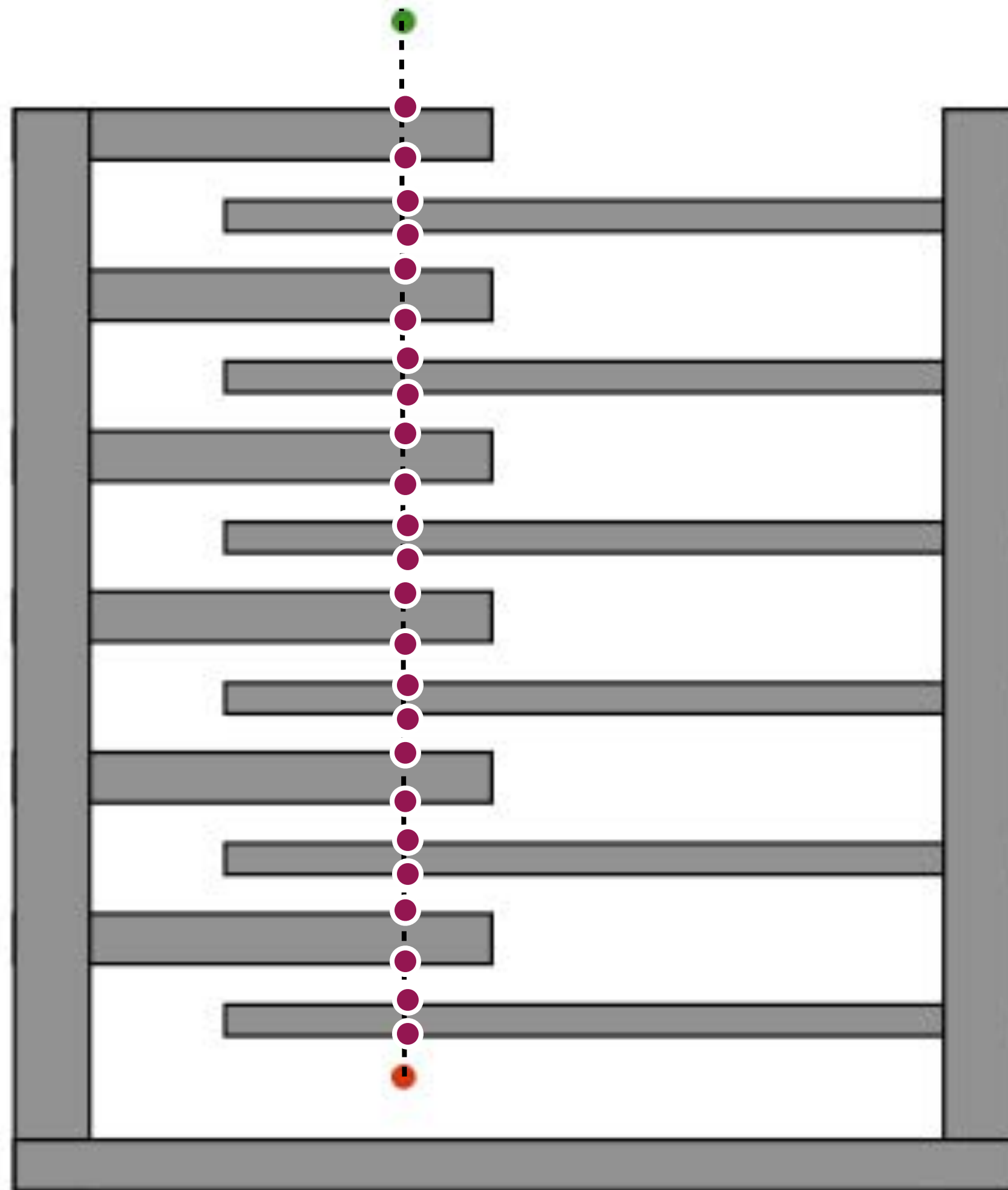
Best case:



Worst case:



# Bug 2: Search Bounds

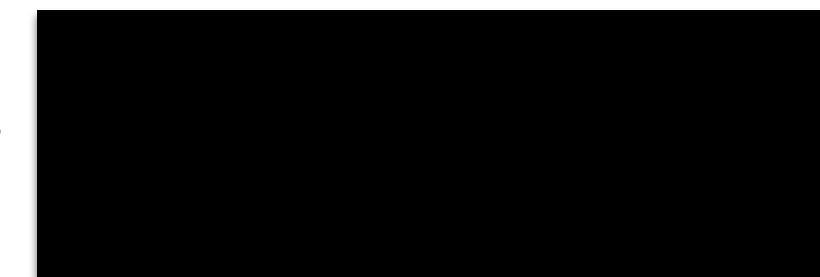


Bounds on path distance, assuming

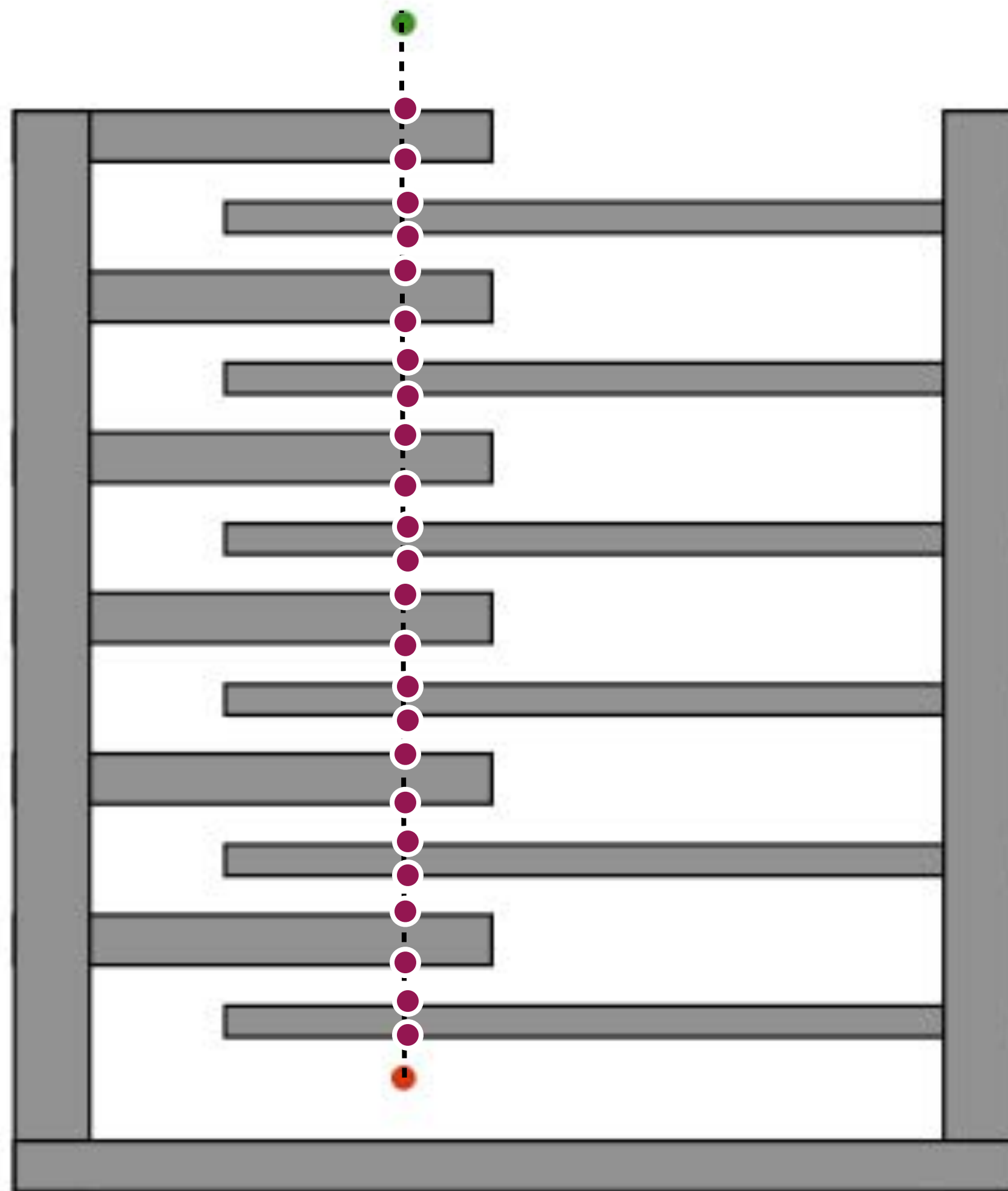
- $D$ : distance start-to-goal
- $P_i$ : obstacle perimeter
- $n_i$ : number of m-line intersections for  $WO_i$

Best case:  $D$

Worst case:



# Bug 2: Search Bounds



Bounds on path distance, assuming

- $D$ : distance start-to-goal
- $P_i$ : obstacle perimeter
- $n_i$ : number of m-line intersections for  $WO_i$

Best case:  $D$

Worst case:  $D + \sum_i (n_i/2)P_i$

Why?



Why?

Consider all leave points on m-line;  
only half are valid



Each leave pt might require traversing entire  
obstacle perimeter, including the outside

# Bug 2: Search Bounds

Bounds on path distance, assuming

- $D$ : distance start-to-goal
- $P_i$ : obstacle perimeter
- $n_i$ : number of m-line intersections for  $WO_i$

Best case:  $D$

Worst case:  $D + \sum_i (n_i/2)P_i$

# Search Bounds:

## Bug 1

Bounds on path distance, assuming

$D$ : distance start-to-goal

$P_i$ : obstacle perimeter

Best case:  $D$

Worst case:  $D + 1.5 \sum_i P_i$

## Bug 2

Bounds on path distance, assuming

- $D$ : distance start-to-goal

- $P_i$ : obstacle perimeter

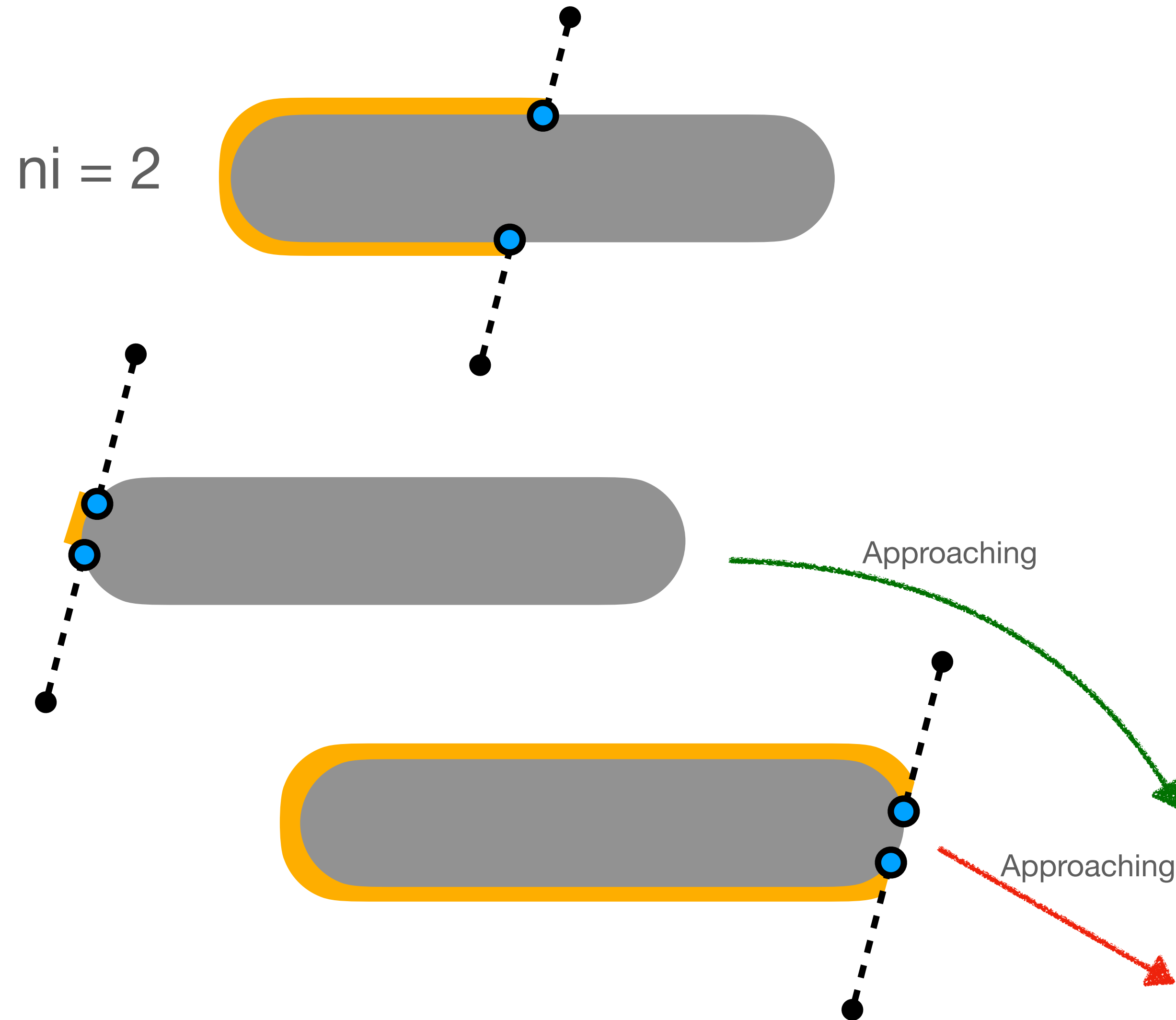
- $n_i$ : number of m-line intersections for  $WO_i$

Best case:  $D$

Worst case:  $D + \sum_i (n_i/2)P_i$



# Search Bounds:



## Bug 2

Bounds on path distance, assuming

- $D$ : distance start-to-goal
- $P_i$ : obstacle perimeter
- $n_i$ : number of m-line intersections for  $WO_i$

Best case:  $D$

Worst case:  $D + \sum_i (n_i/2) P_i$

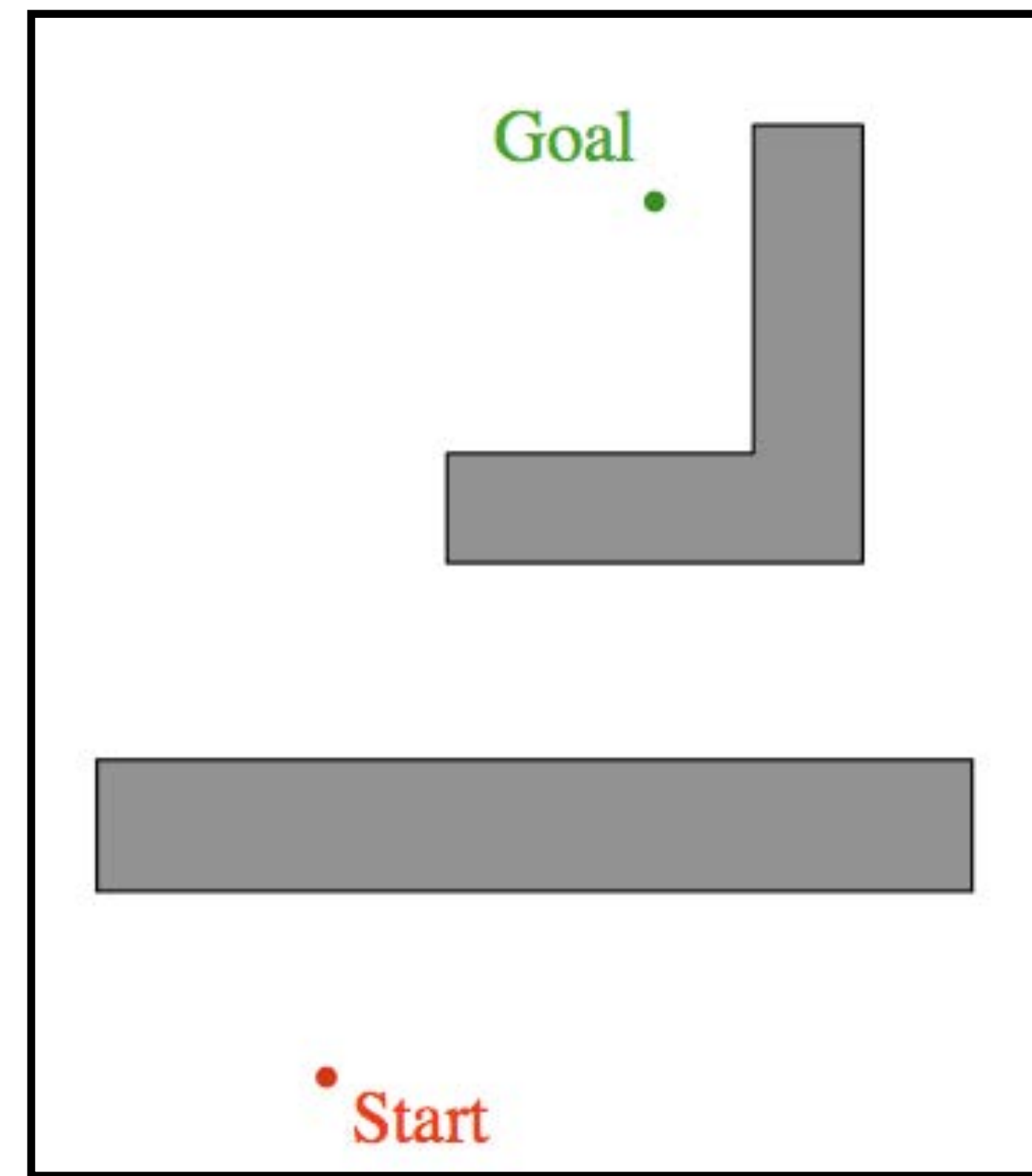
Suppose robot has a range sensor.

Is there a better Bug algorithm?



# Tangent Bug

- Assume bounded world
- Known: global goal
  - measurable distance  $d(x,y)$
- Local sensing
  - **range finding**
  - odometry



>



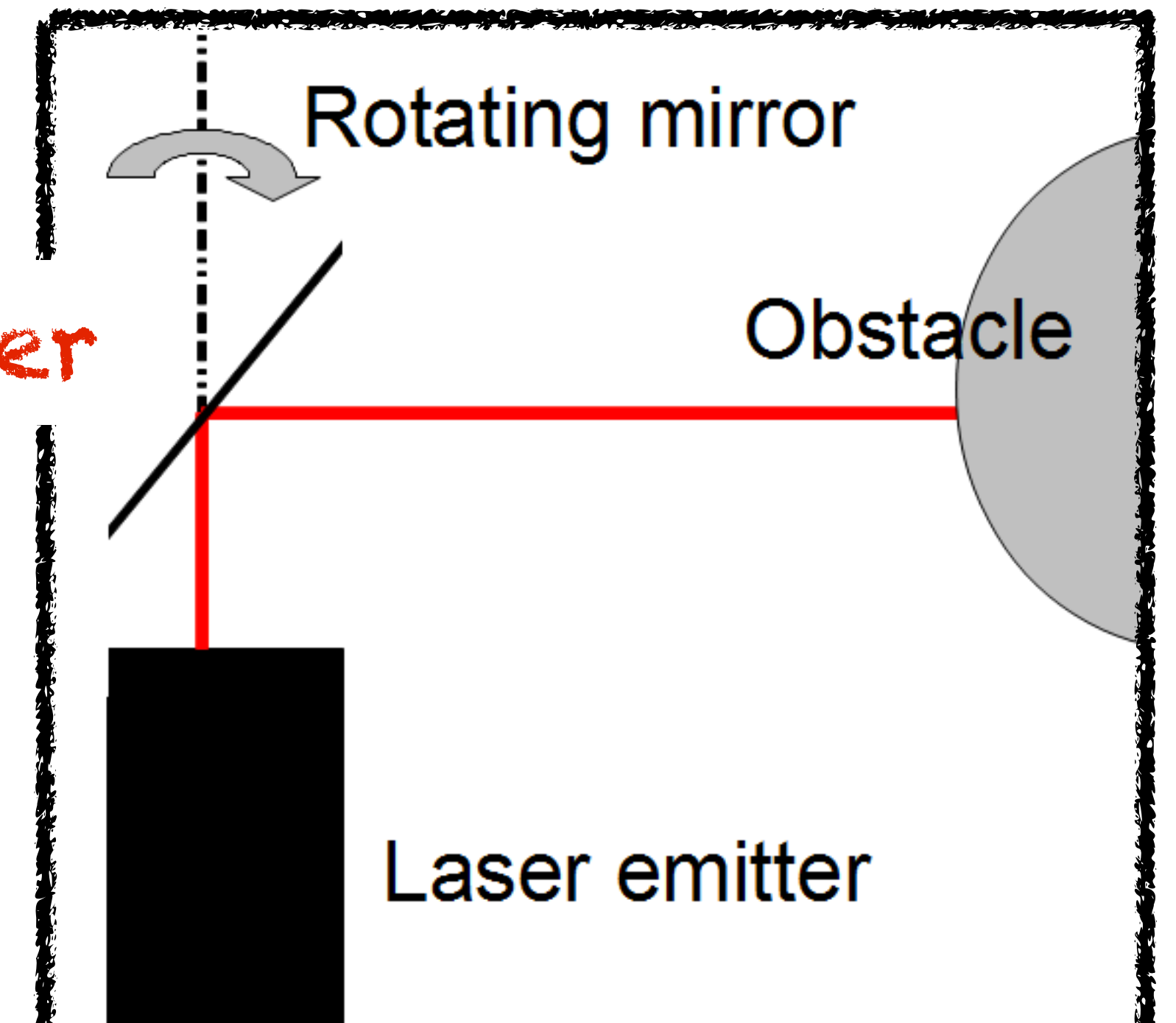
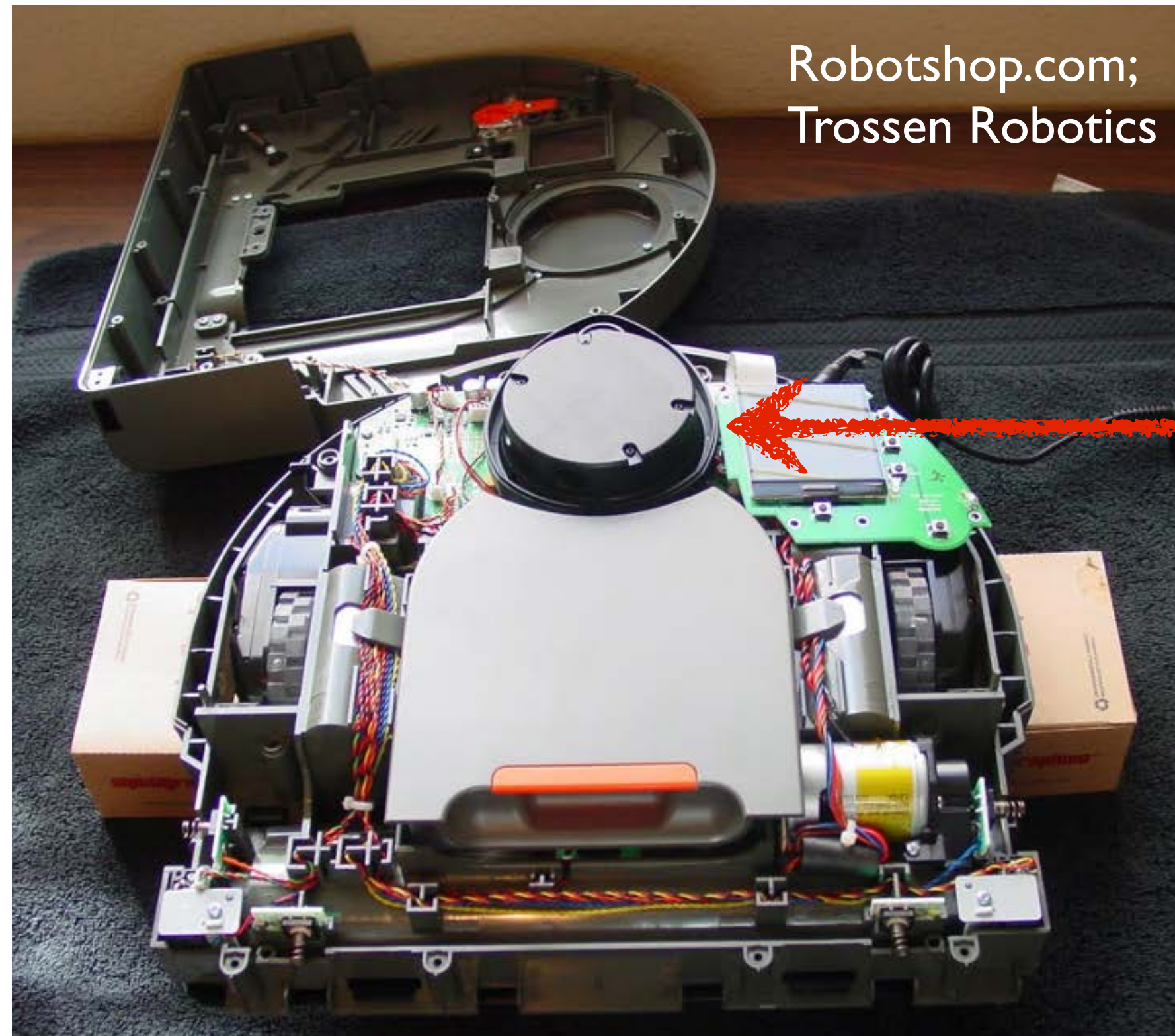
# Laser Rangefinding

(briefly)

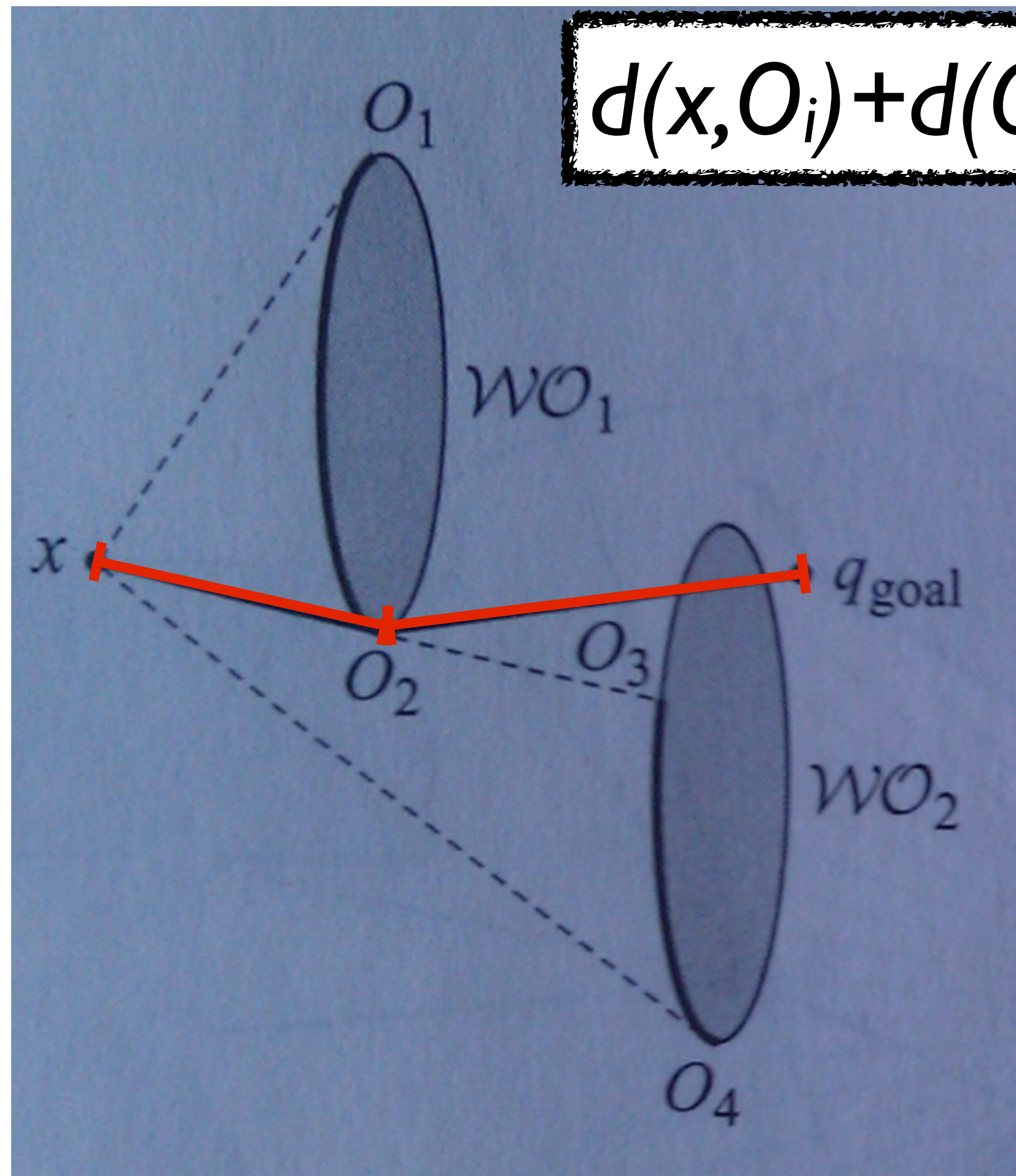
Emit laser beam in a direction

Distance to nearest object related to time from emission to sensing of beam  
(assumes speed of light is known)

Planar range finding : reflect laser on spinning mirror (typically at 10Hz)



# Tangent Bug: Heuristic Distance-to-Goal



$$d(x, O_i) + d(O_i, q_{goal})$$

$O_i$  are visible obstacle extents

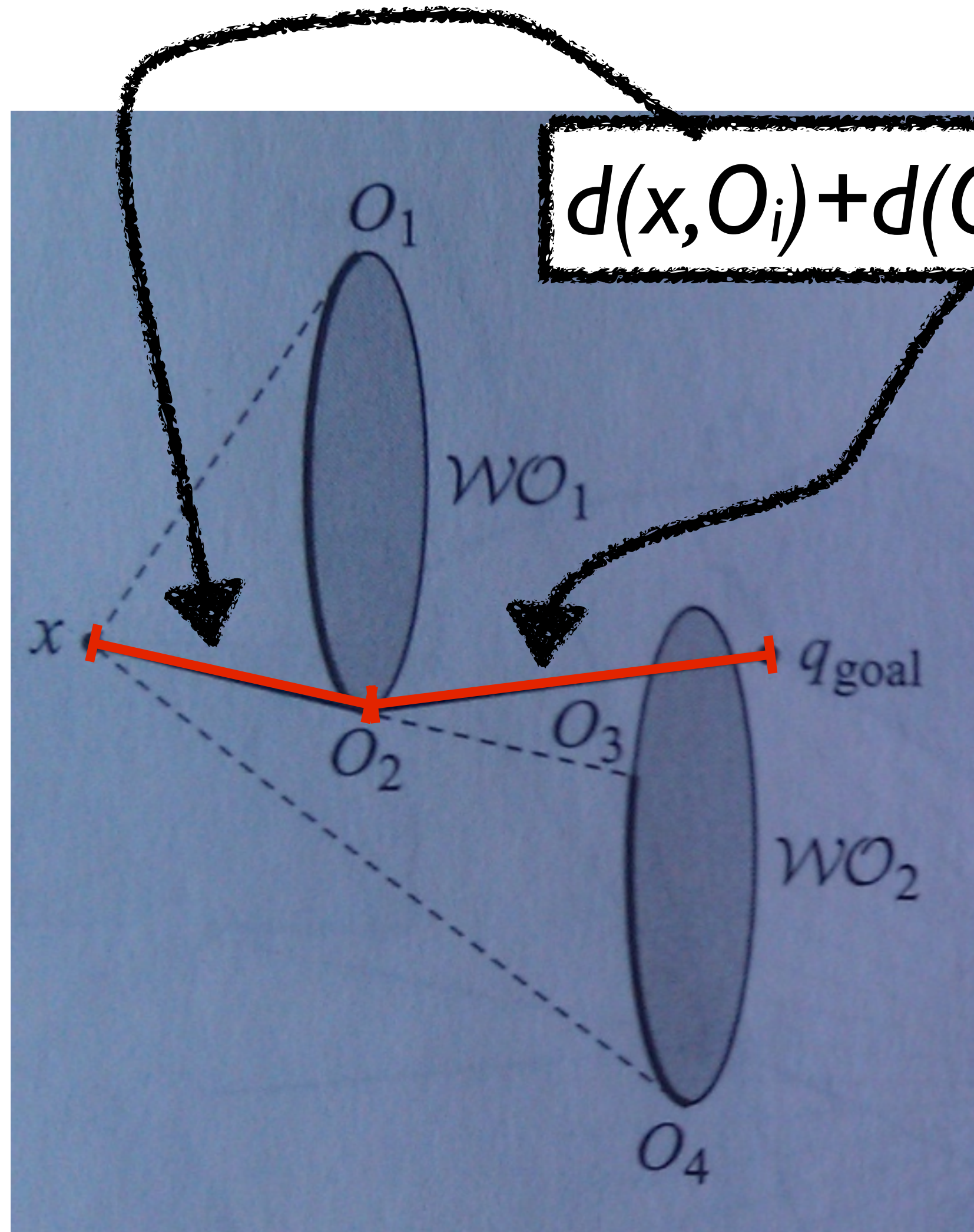
$d(x, O_i)$ : robot can see

$d(O_i, q_{goal})$ : best path robot cannot see

Continually move robot such that distance to goal is decreased

Note similarity to  $A^*$  search heuristic

# Tangent Bug: Heuristic Distance-to-Goal



$O_i$  are visible obstacle extents

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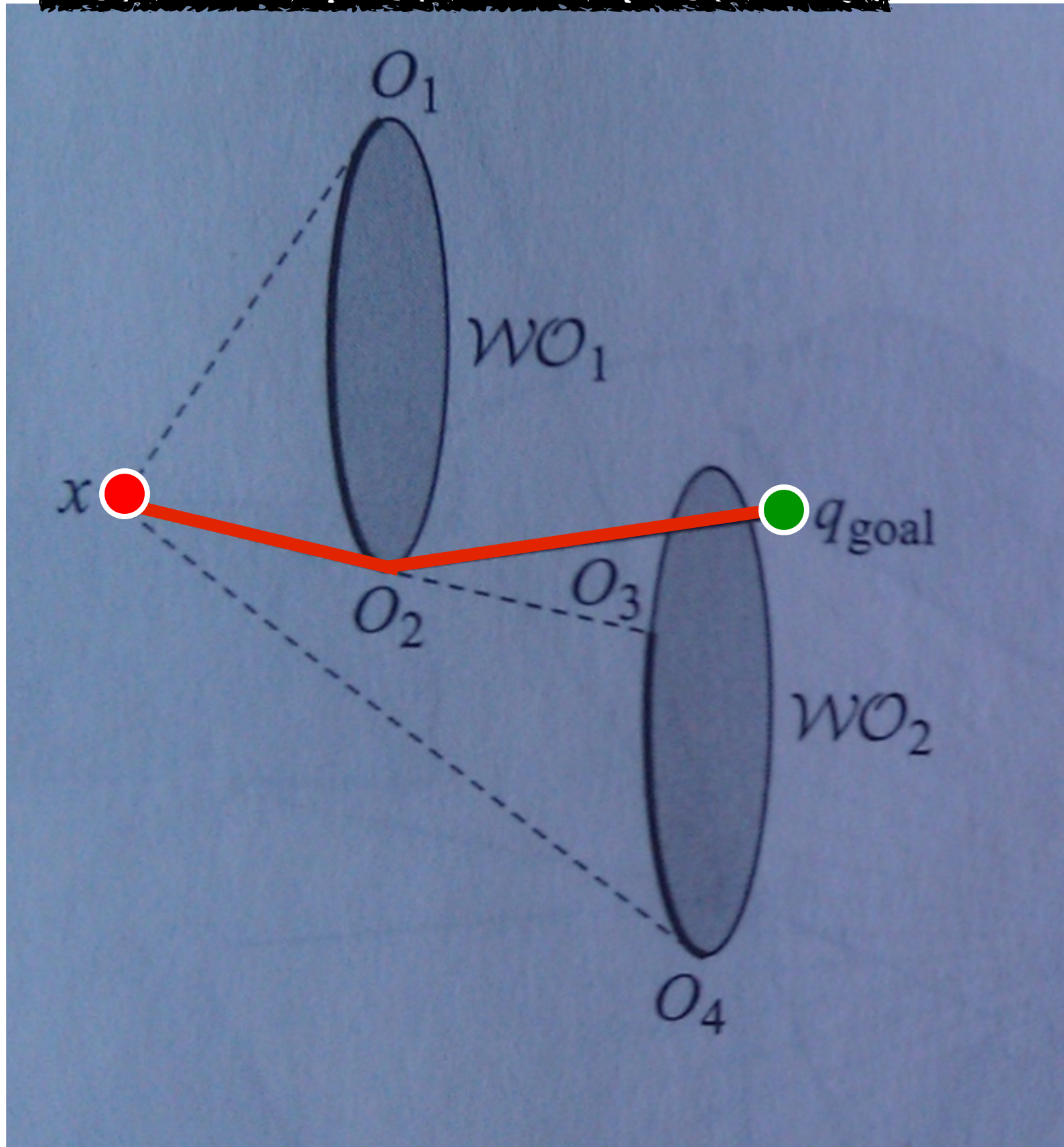
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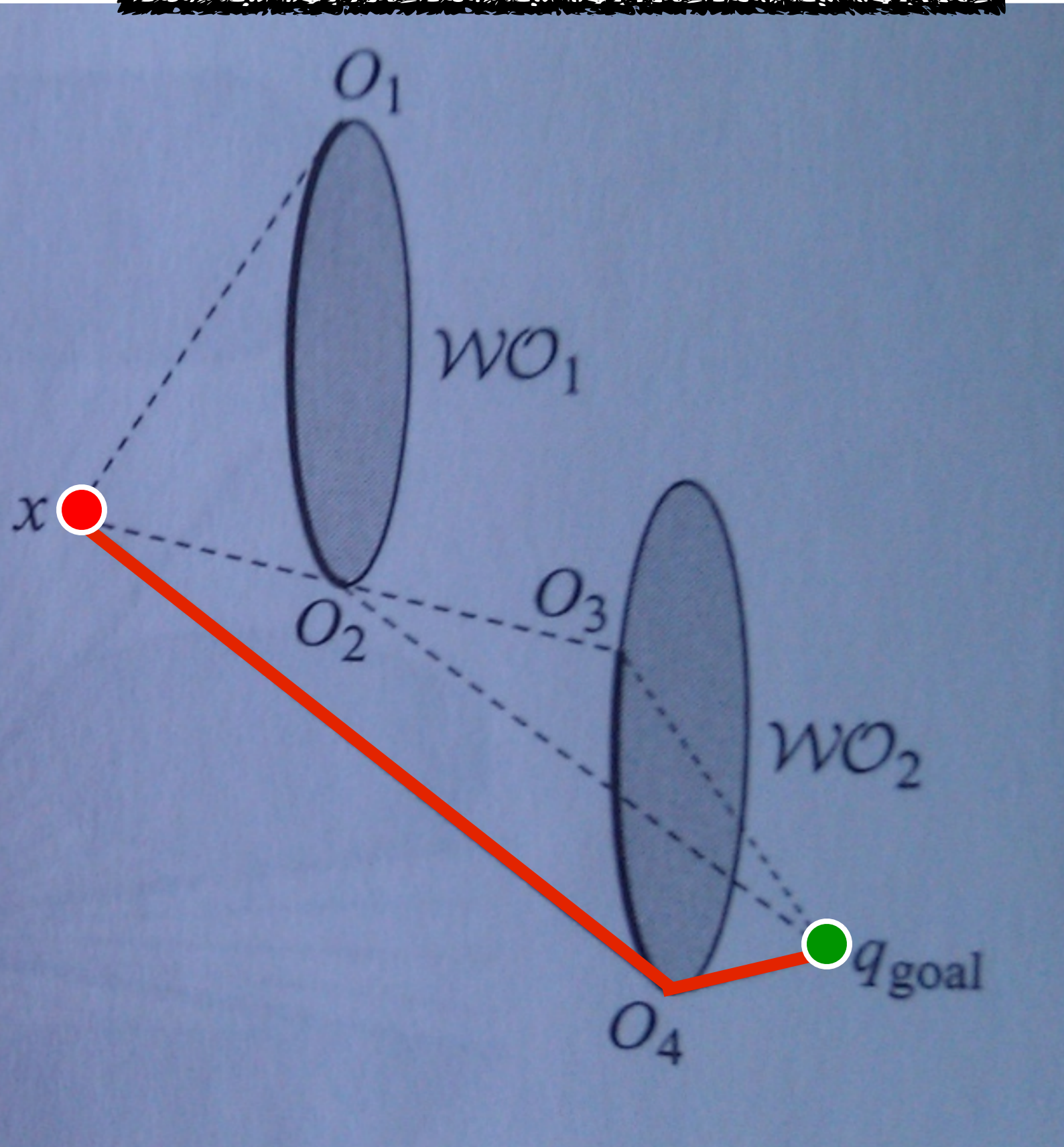
Note similarity to  $A^*$  search heuristic



$$d(x, O_2) + d(O_2, q_{goal})$$



$$d(x, O_4) + d(O_4, q_{goal})$$

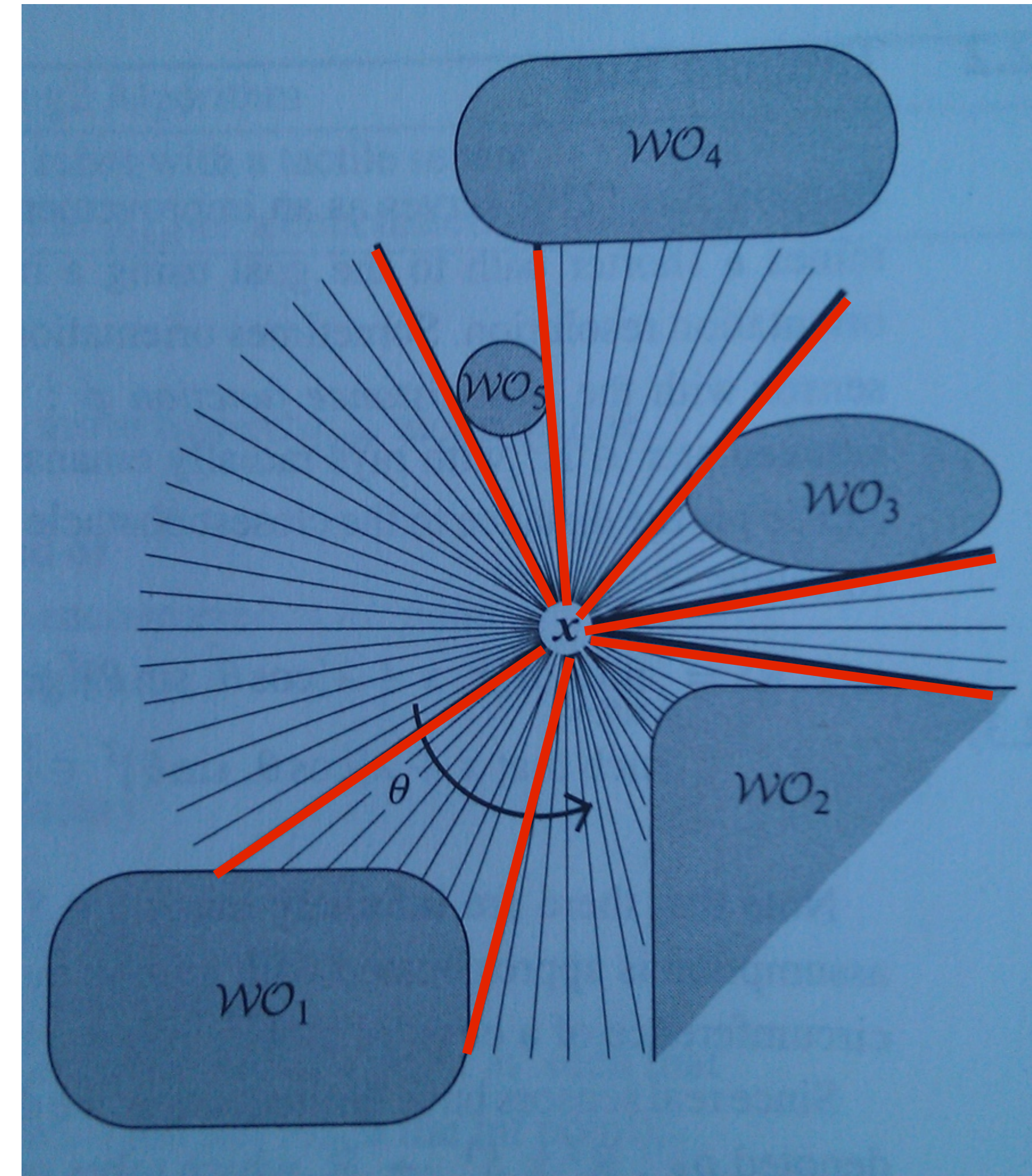


# Range Segmentation

range scan  $\rho(x, \Theta)$ : sensed distance along ray at angle  $\Theta$  within limit  $R$

discontinuities  $\{O_i\}$  in scan result from obstacles

$\{O_i\}$  segments scan into intervals continuity, with obstacles and free space

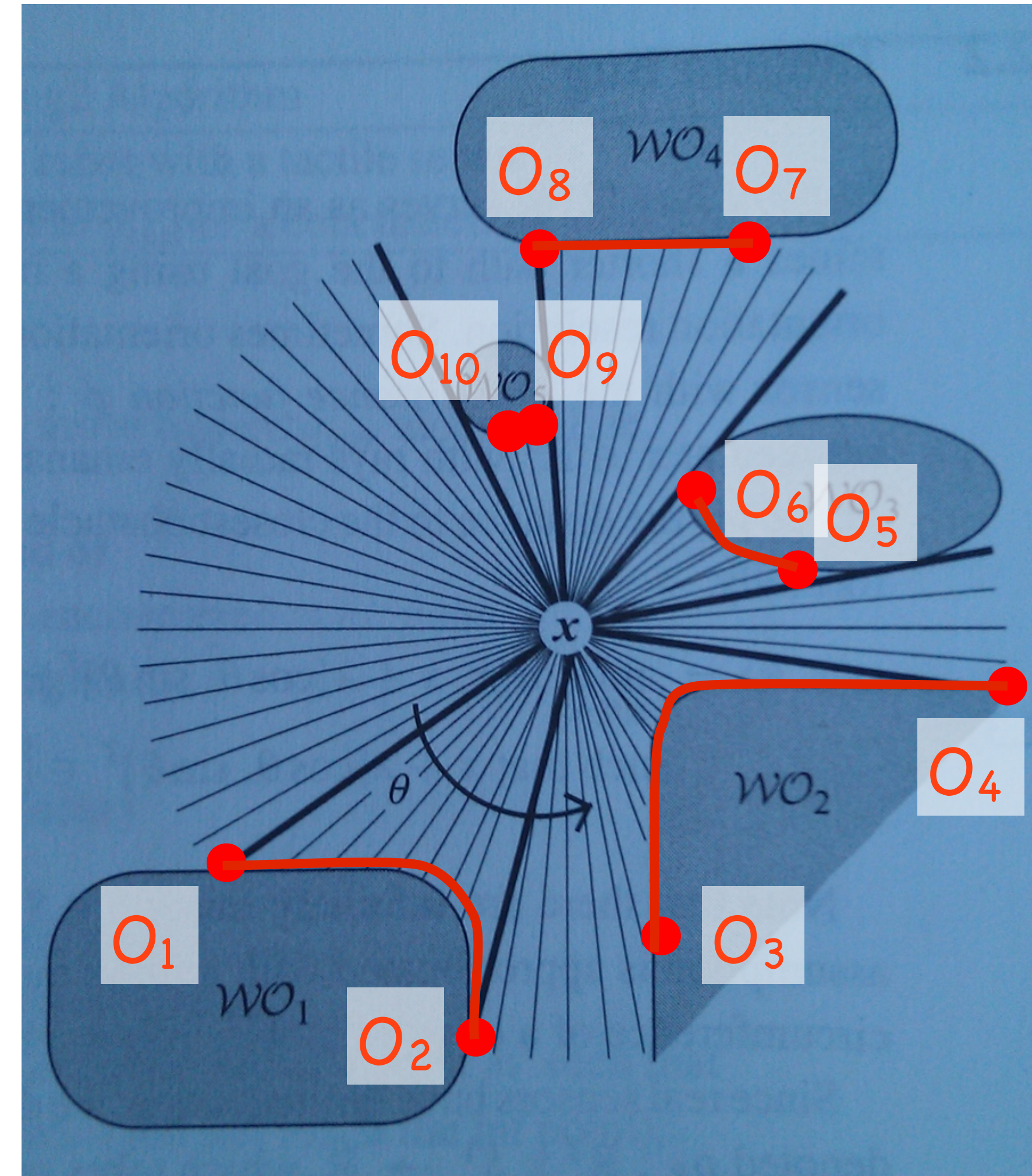


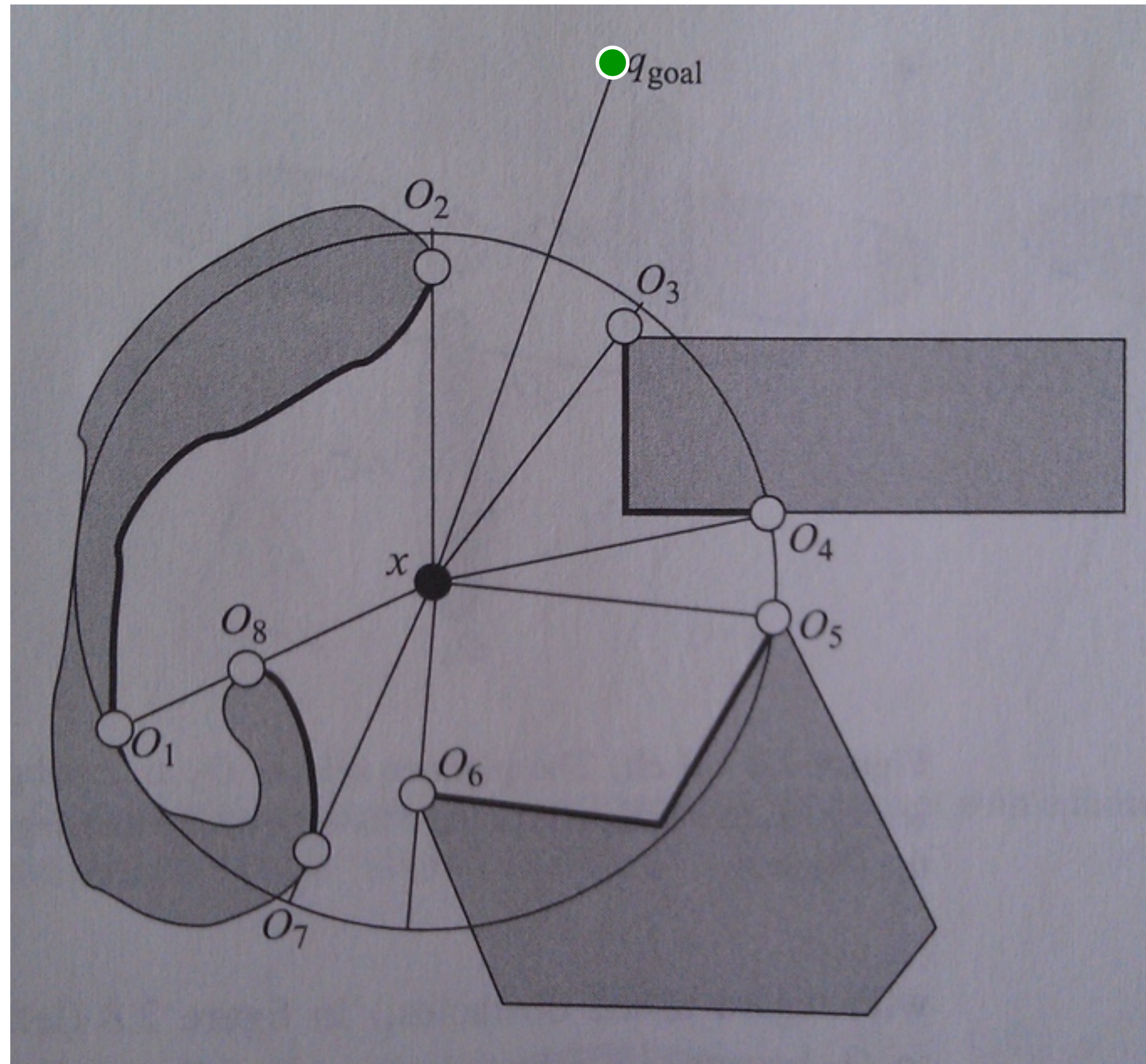
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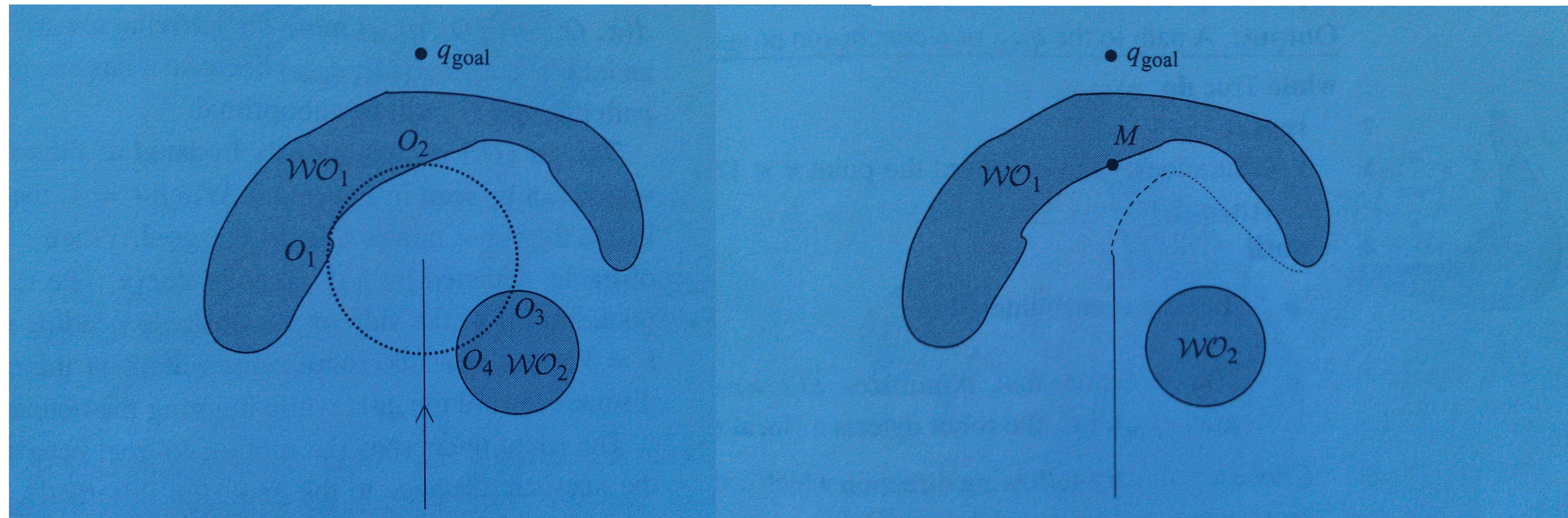
$\{O_i\}$  segments scan into intervals continuity, with obstacles and free space





# Tangent Bug Behaviors

Similar to other bug algorithms, Tangent Bug uses two behaviors:

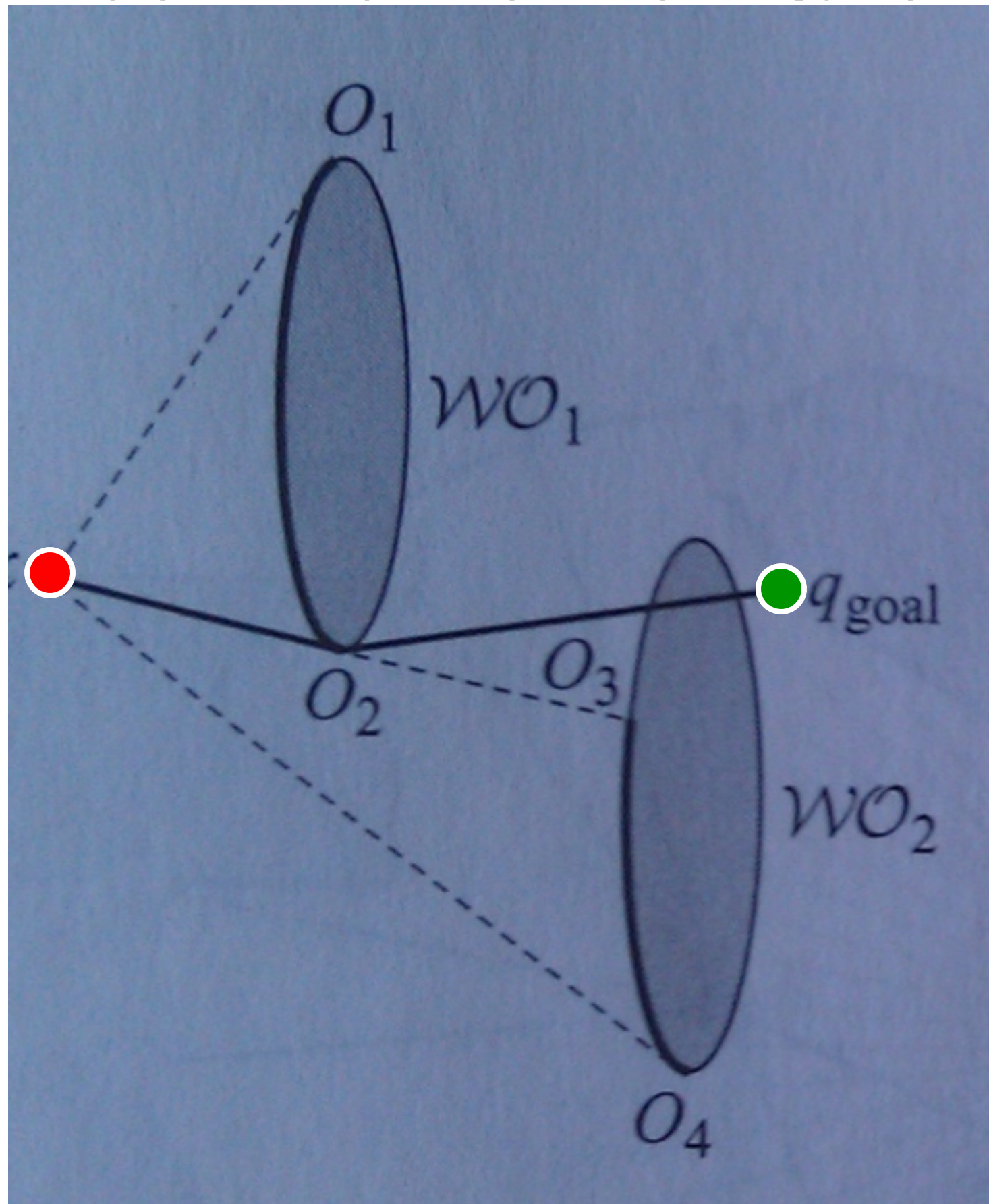


motion-to-goal

boundary-follow

# Tangent Bug

$$G(x) = d(x, O_i) + d(O_i, q_{goal})$$



1) motion-to-goal: Move to current  $O_i$  to minimize  $G(x)$ , until goal (success) or  $G(x)$  increases (local minima)

2) boundary-follow: move in while loop:

a) repeat updates

$$d_{reach} = \min d(q_{goal}, \{\text{visible } O_i\})$$

$$d_{follow} = \min d(q_{goal}, \text{sensed}(WO_j))$$

$$O_i = \operatorname{argmin}_i d(x, O_i) + d(O_i, q_{goal})$$

b) until

goal reached, (**success**)

robot cycles around obstacle, (**fail**)

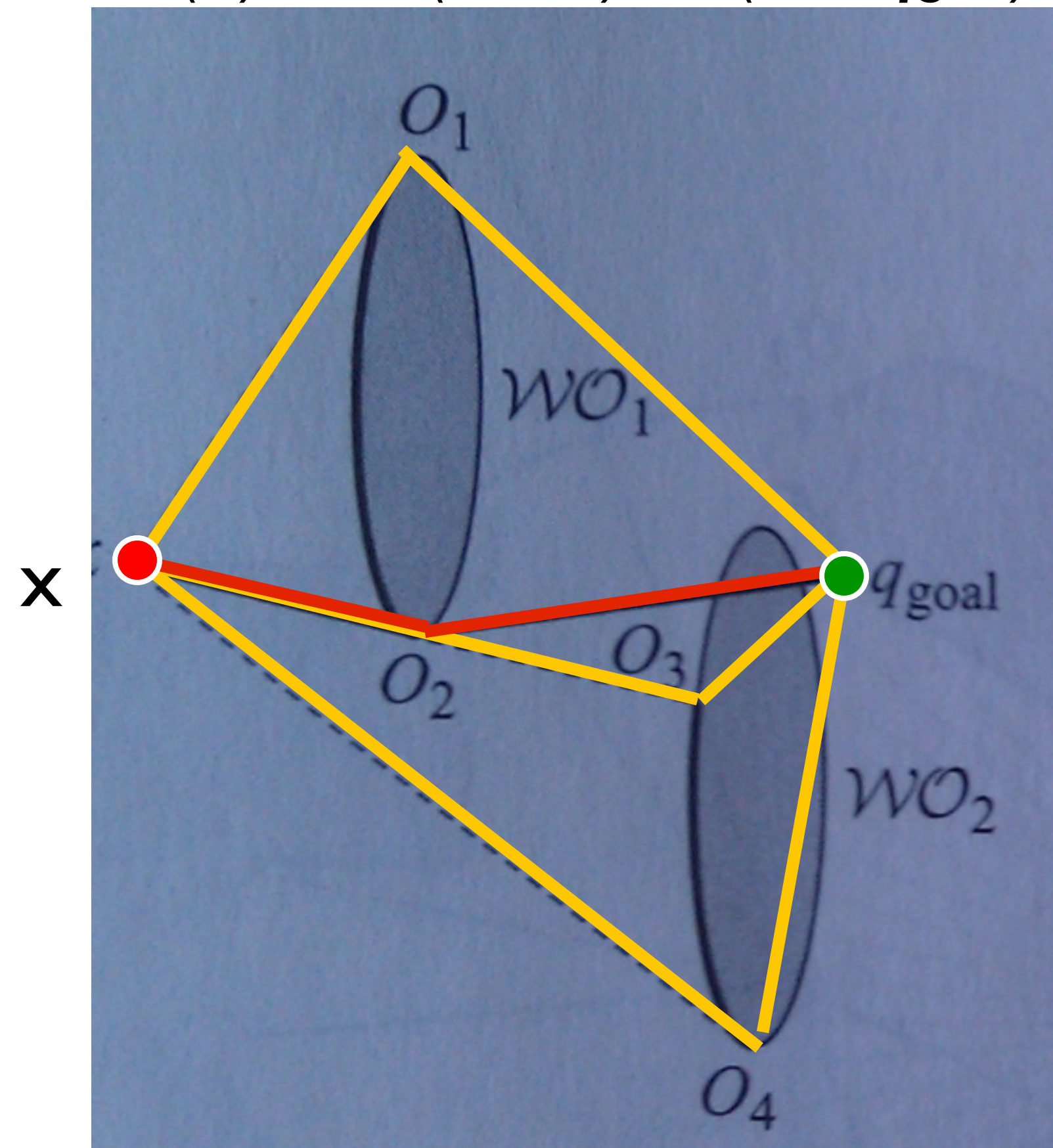
$$d_{reach} < d_{follow},$$

(**cleared obstacle or local minima**)

3) continue from (1)

# Tangent Bug

$$G(x) = d(x, O_2) + d(O_2, q_{goal})$$



min  $G(x)$  in red, others in yellow

1) motion-to-goal: Move to current  $O_i$  to minimize  $G(x)$ , until goal (success) or  $G(x)$  increases (local minima)

2) boundary-follow: move in while loop:

a) repeat updates

$$d_{reach} = \min d(q_{goal}, \{\text{visible } O_i\})$$

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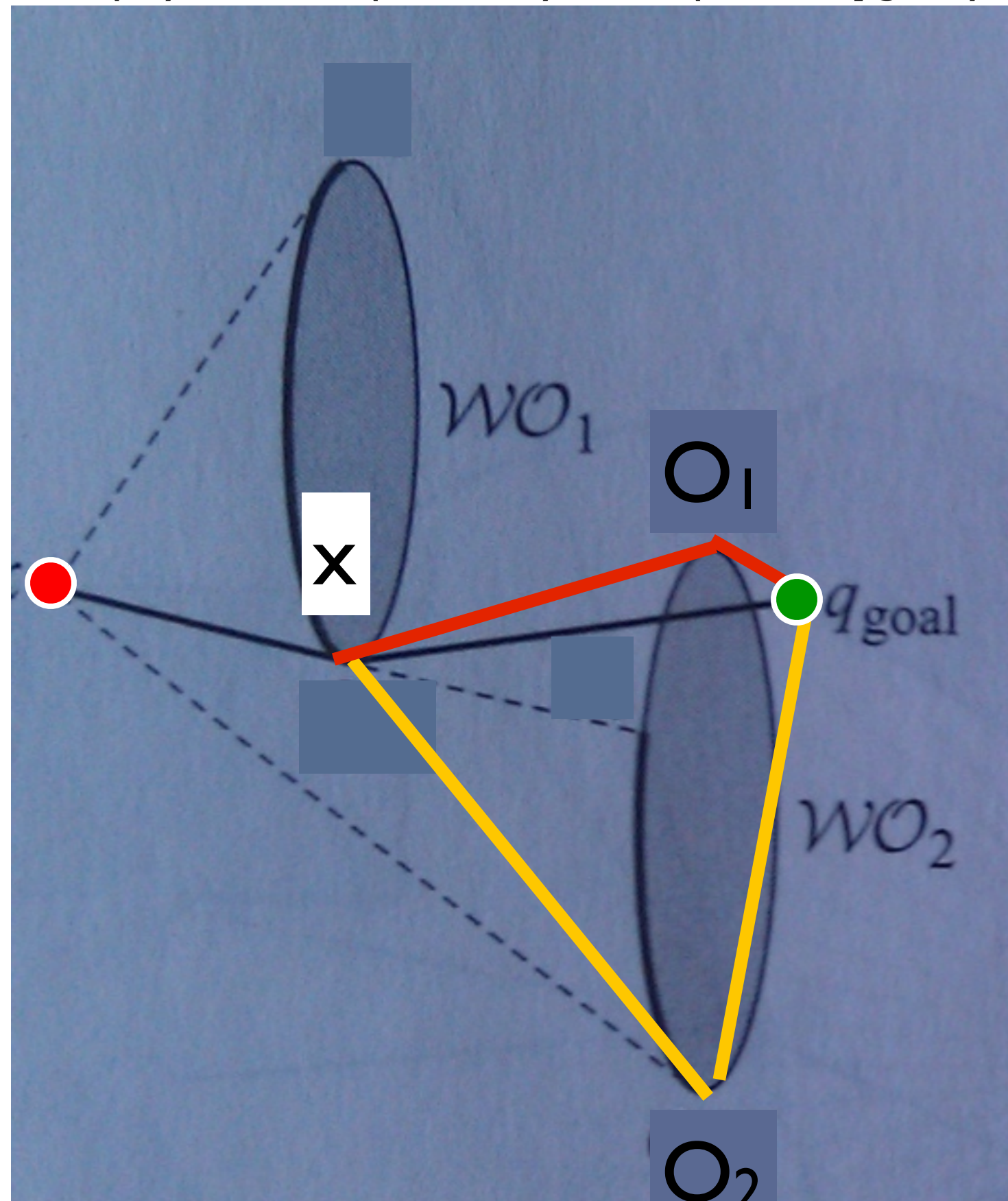
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# Tangent Bug

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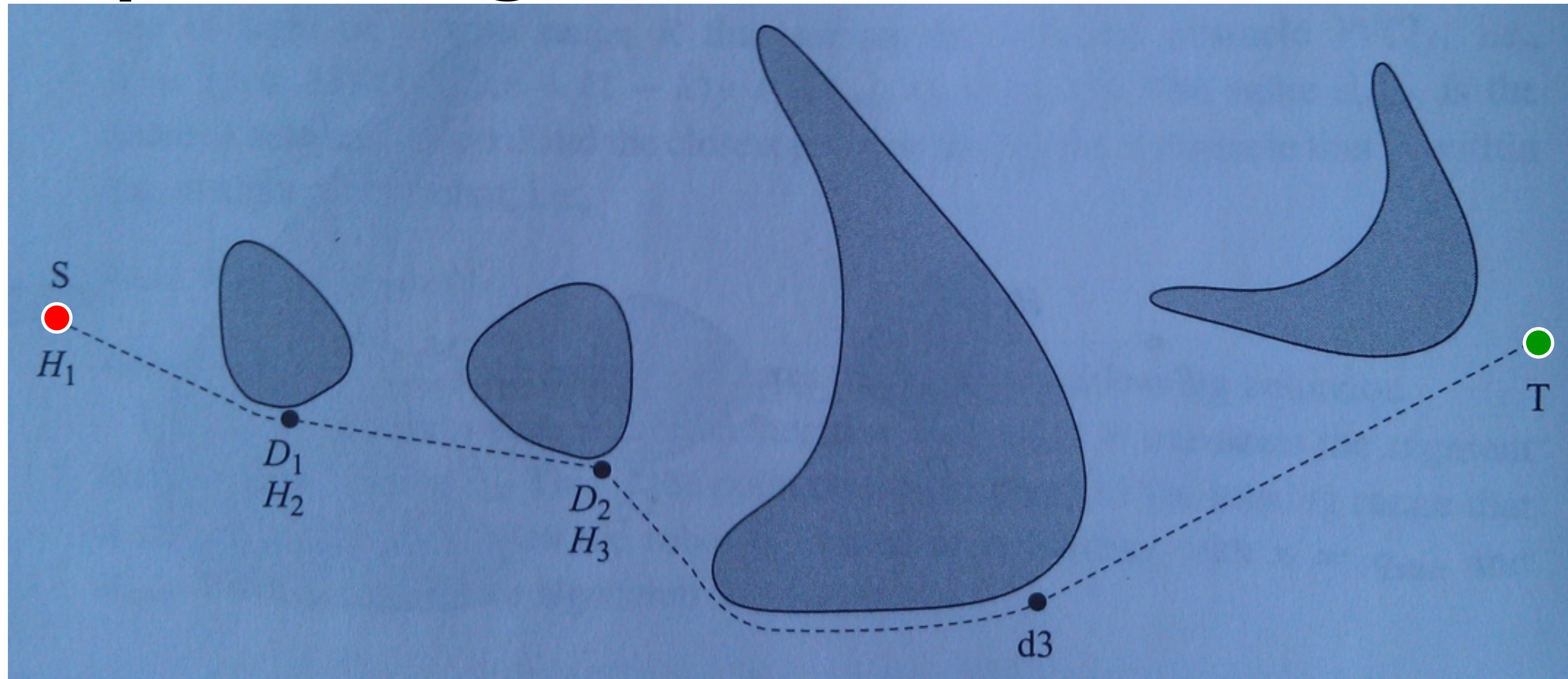
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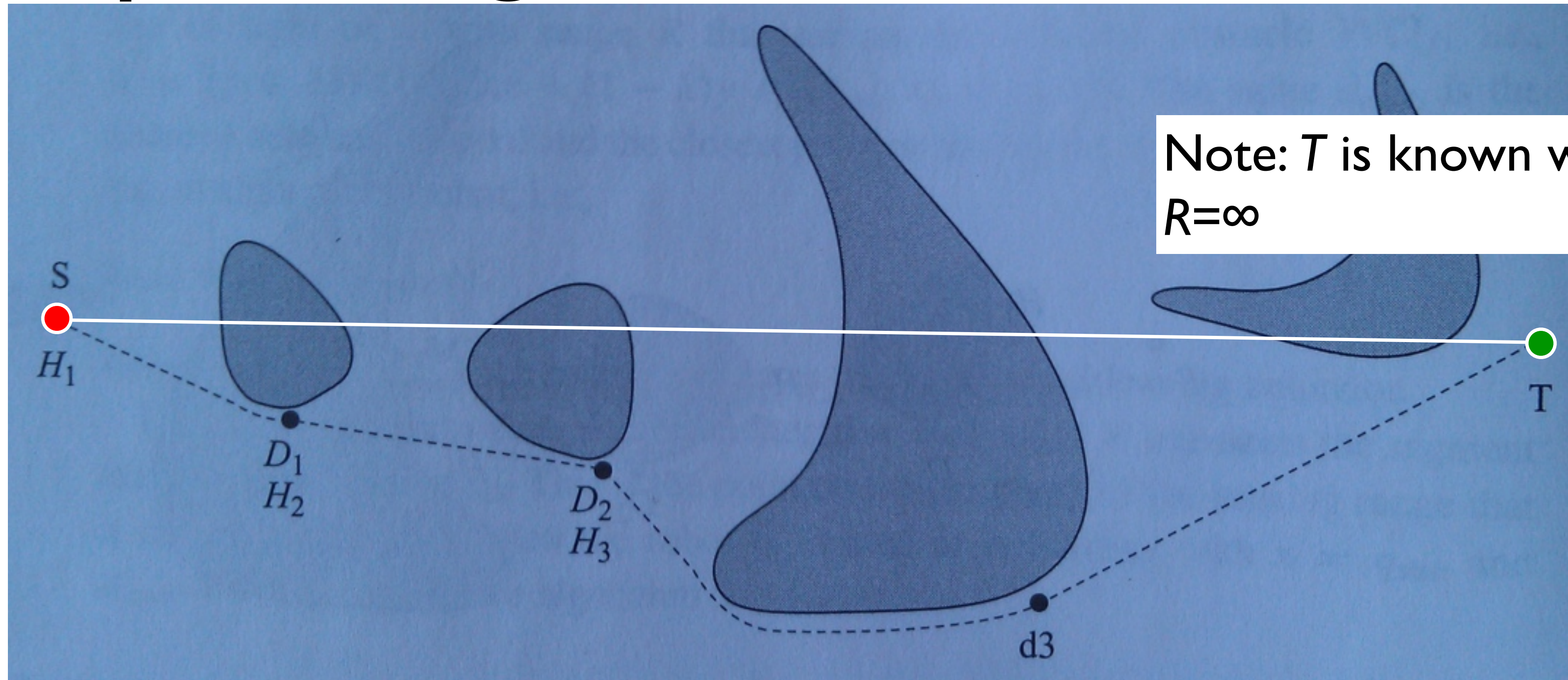


# Example: range $R=\infty$



$H_i$ : hit point  
 $D_i$ : Depart point  
 $L_i$ : Leave point  
 $M_i$ : local minima

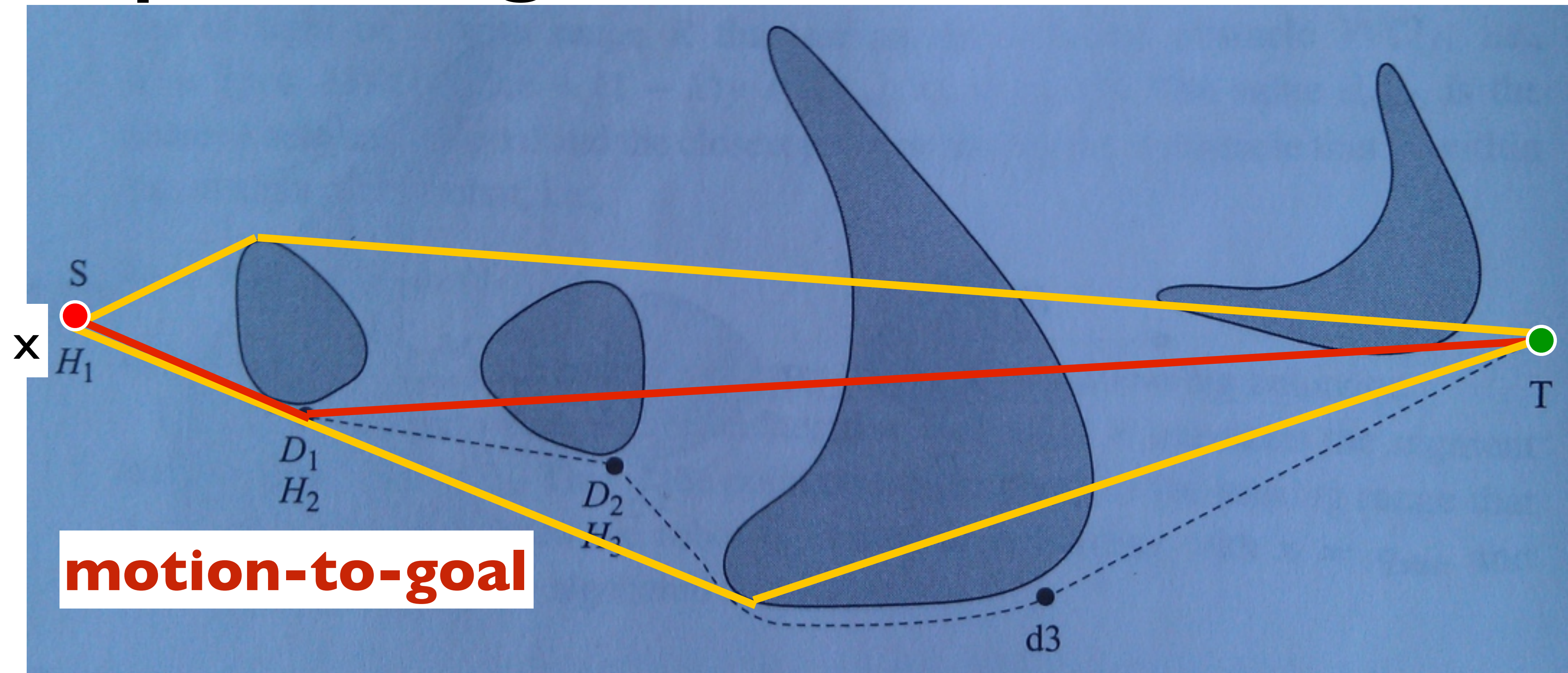
# Example: range $R=\infty$



Note:  $T$  is known when  $R=\infty$

$H_i$ : hit point  
 $D_i$ : Depart point  
 $L_i$ : Leave point  
 $M_i$ : local minima

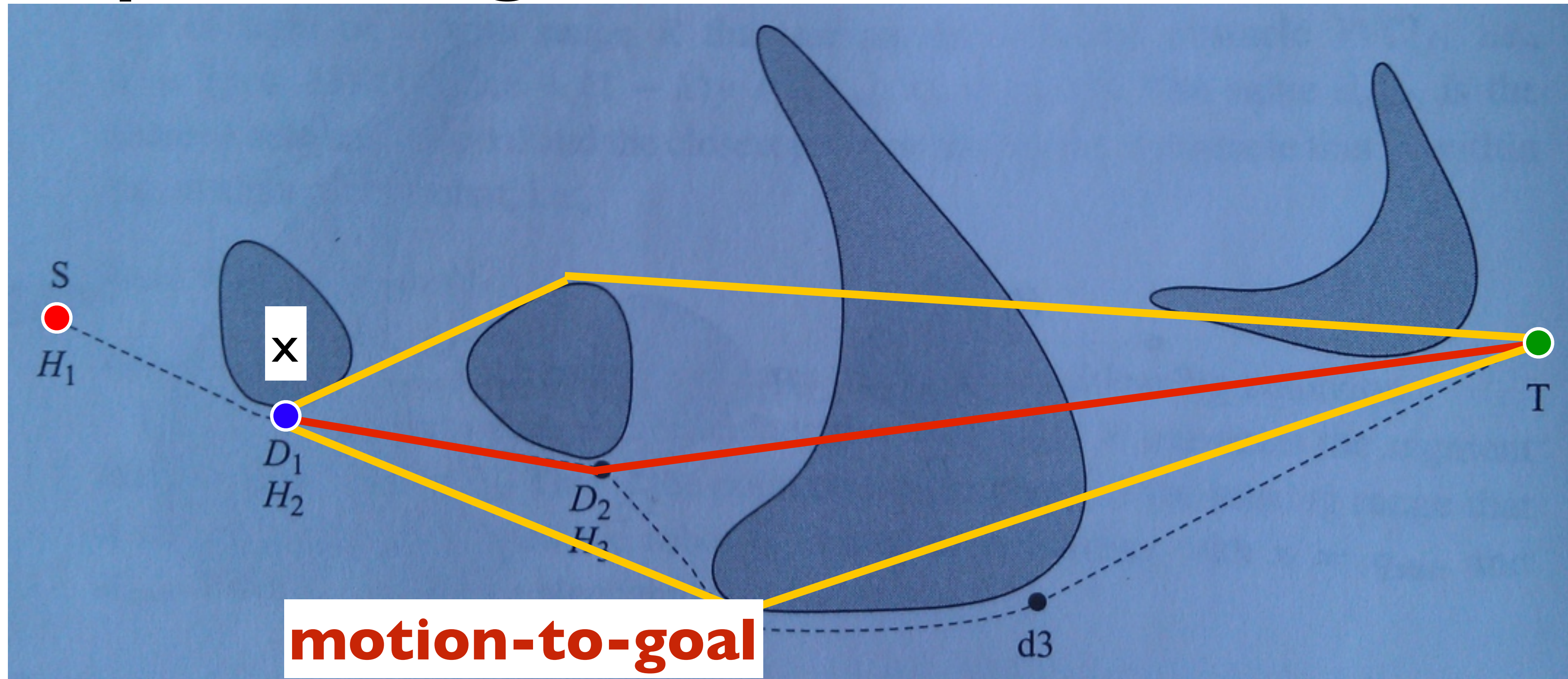
# Example: range $R=\infty$



min  $G(x)$  in red, others in yellow

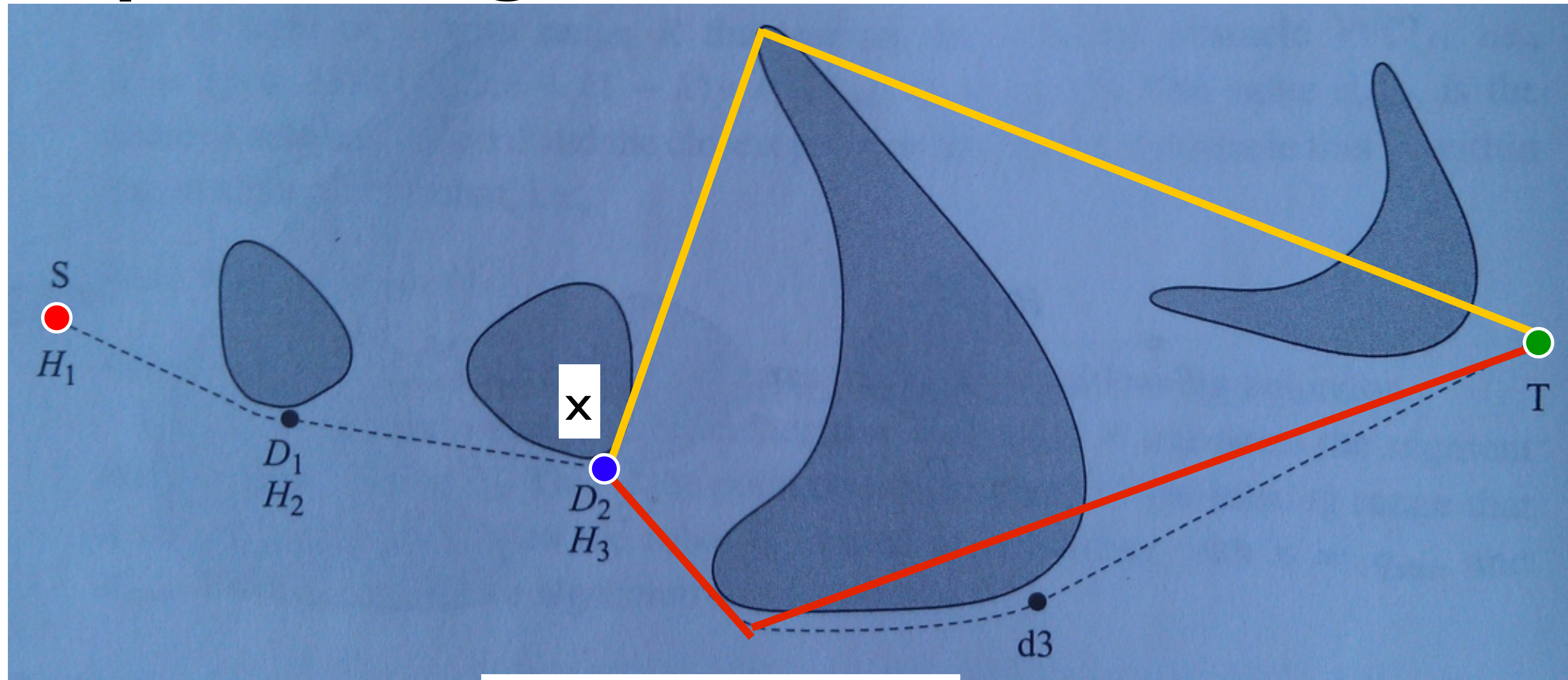
$H_i$ : hit point  
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# Example: range $R=\infty$



$H_i$ : hit point  
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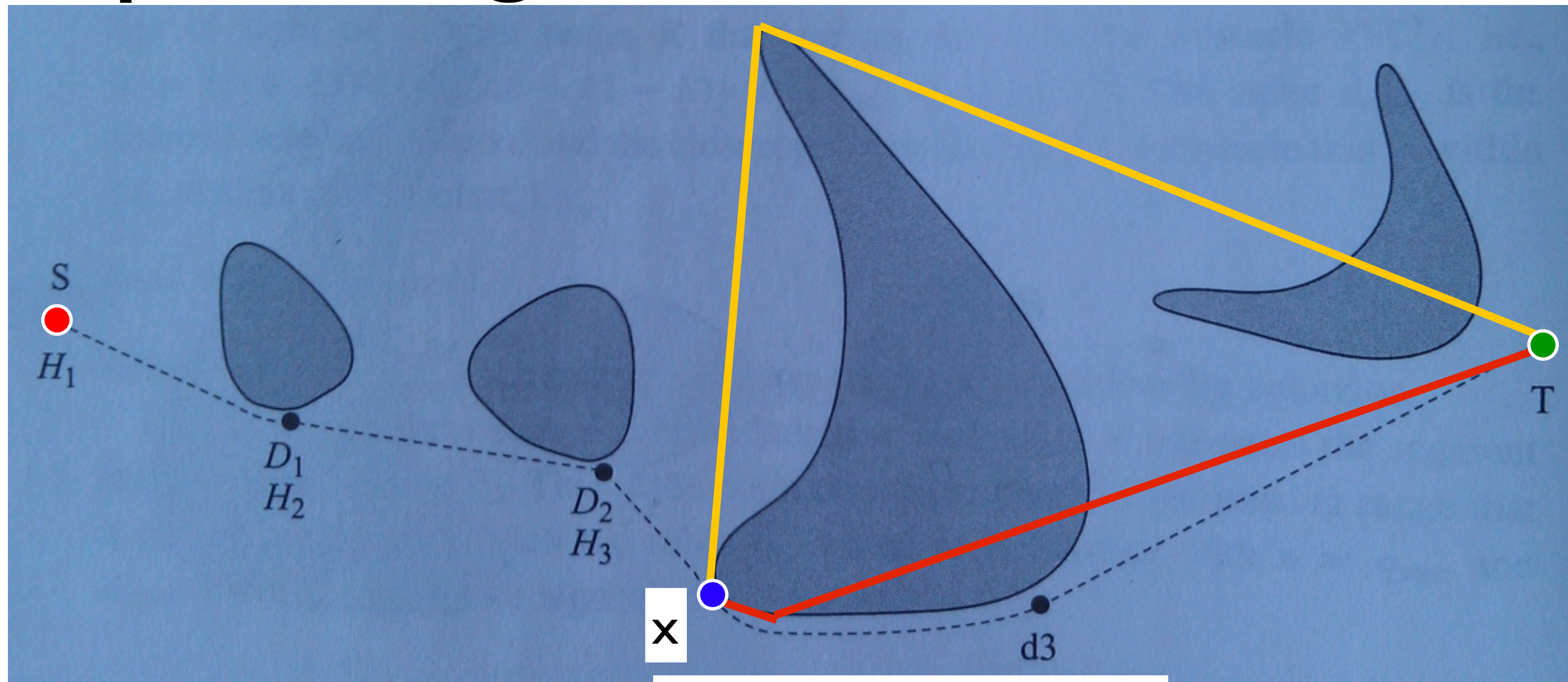
# Example: range $R=\infty$



**motion-to-goal**

$H_i$ : hit point  
 $D_i$ : Depart point  
 $L_i$ : Leave point  
 $M_i$ : local minima

# Example: range $R=\infty$



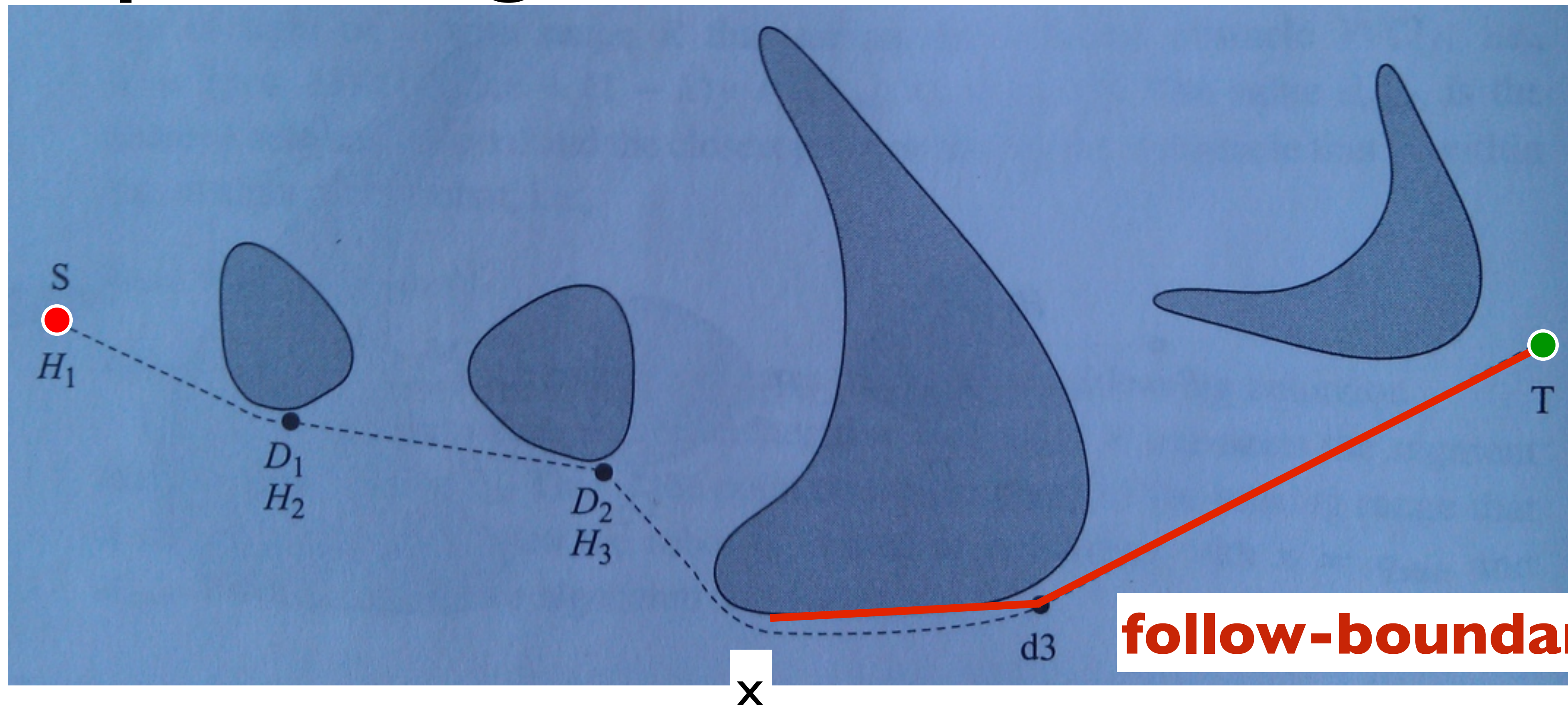
**follow-boundary**

start following:

$$\min d(q_{goal}, \{\text{visible } O_i\}) < \min d(q_{goal}, \text{sensed}(WO_j))$$

$H_i$ : hit point  
 $D_i$ : Depart point  
 $L_i$ : Leave point  
 $M_i$ : local minima

# Example: range $R=\infty$



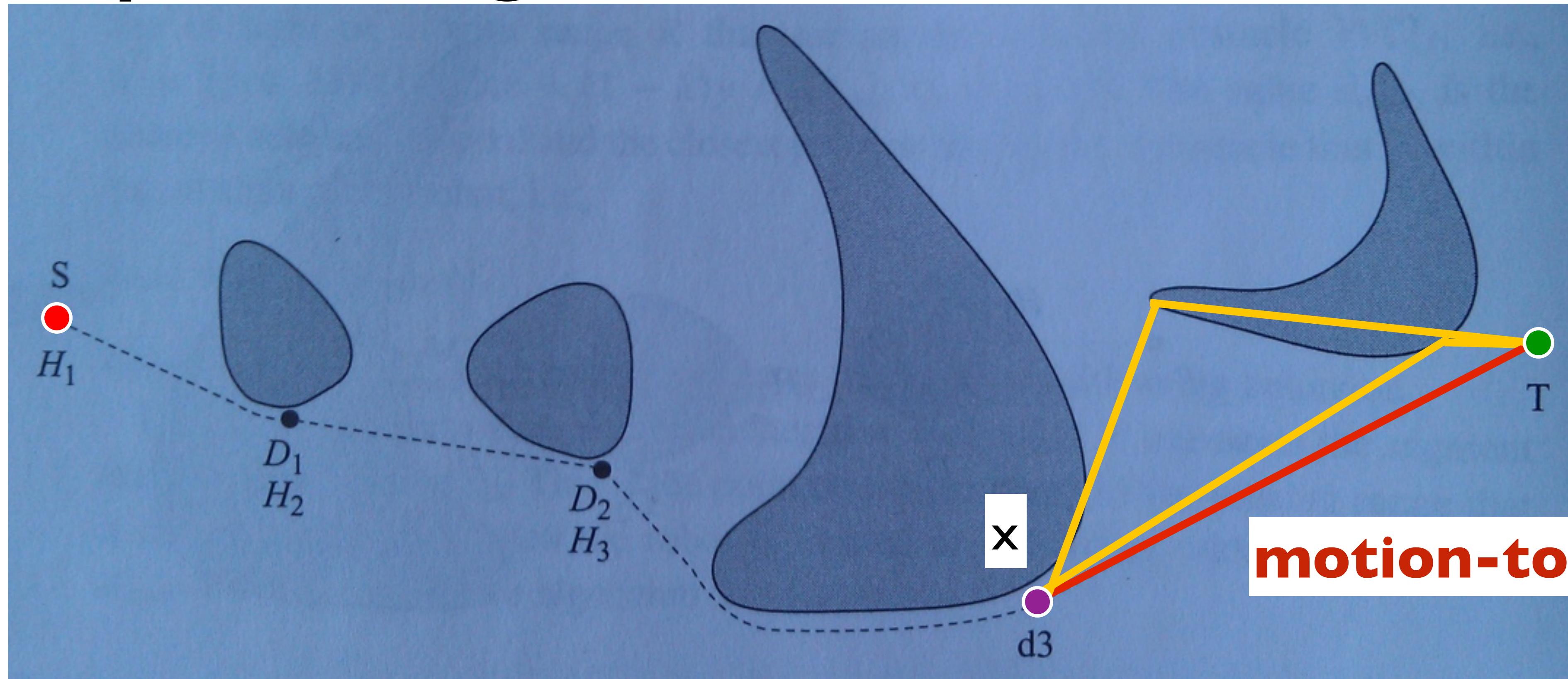
end following:

$$\min d(q_{goal}, \{\text{visible } O_i\}) < \min d(q_{goal}, \text{sensed}(WO_j))$$

**follow-boundary**

- $H_i$ : hit point
- $D_i$ : Depart point
- $L_i$ : Leave point
- $M_i$ : local minima

# Example: range $R=\infty$

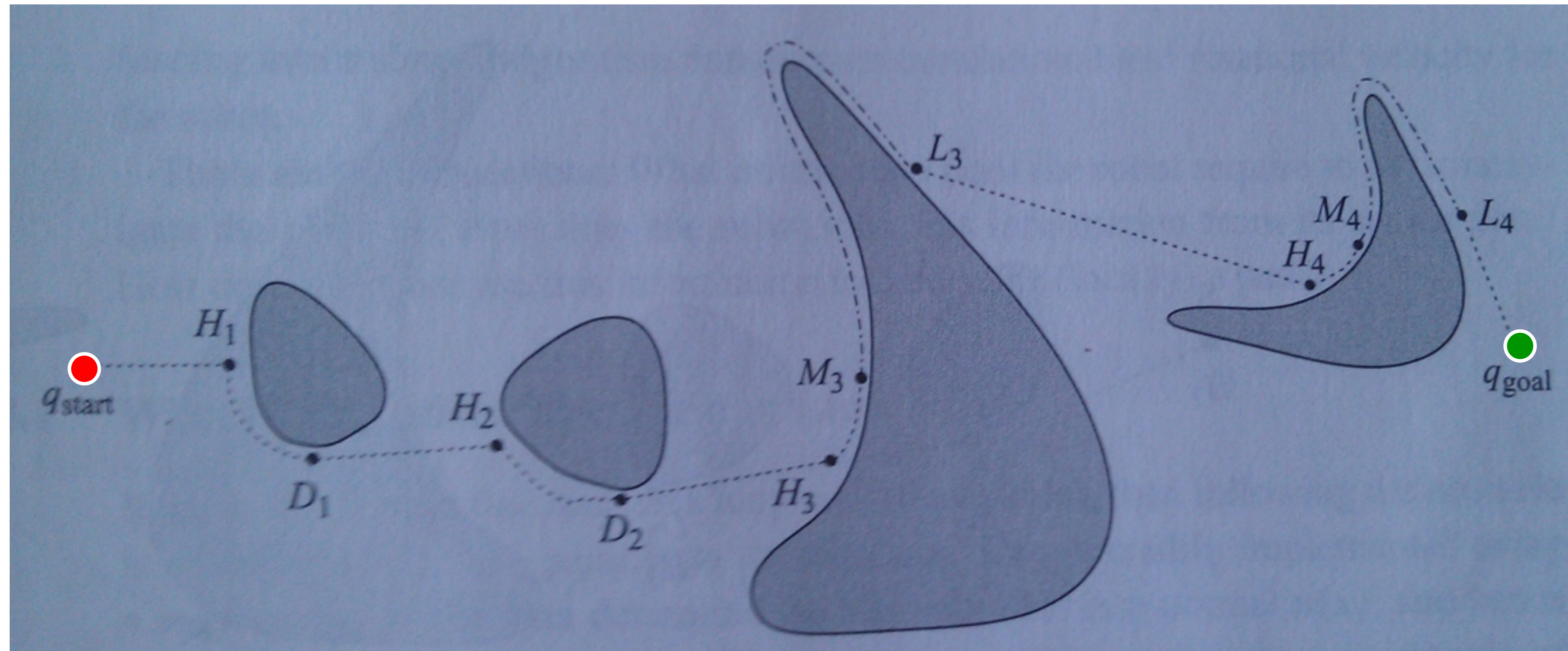


$H_i$ : hit point  
 $D_i$ : Depart point  
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 $M_i$ : local minima



# Example: range $R=0$

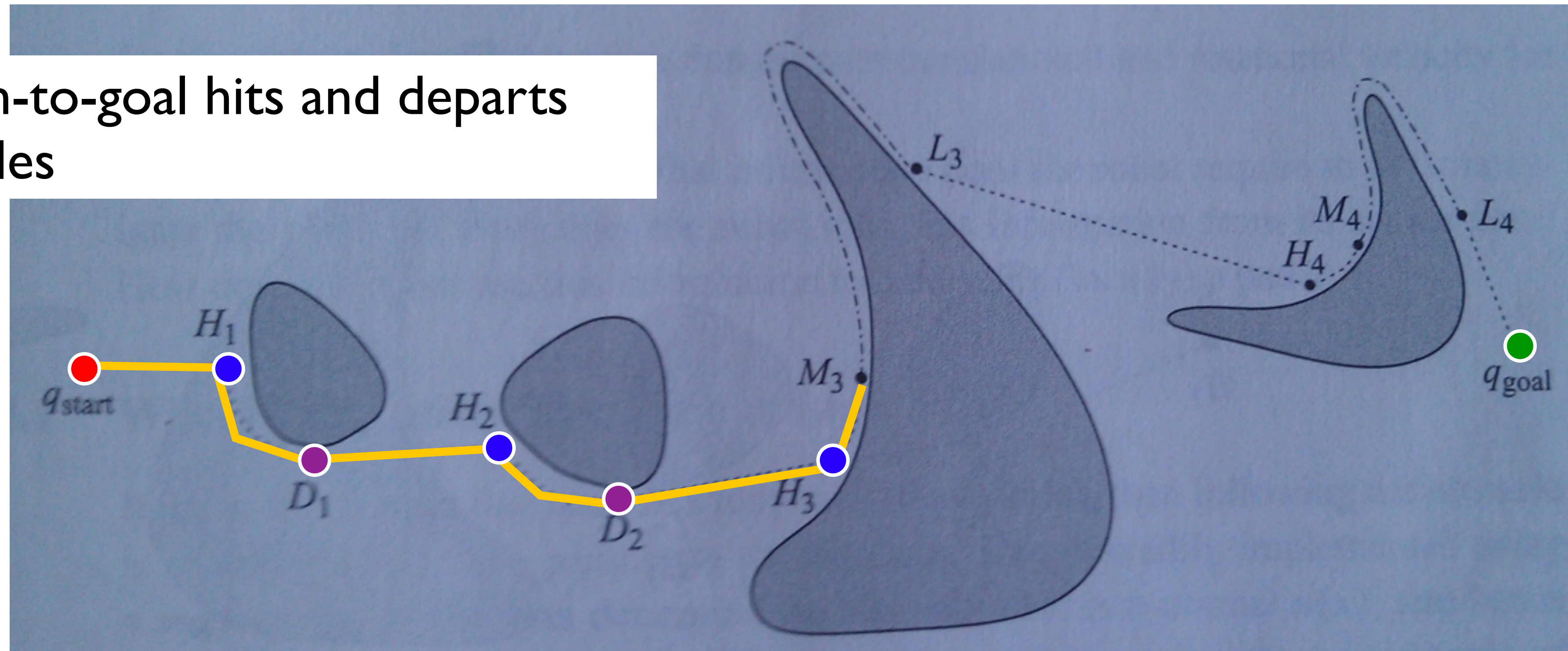
$H_i$ : hit point  
 $D_i$ : Depart point  
 $L_i$ : Leave point  
 $M_i$ : local minima



# Example: range $R=0$

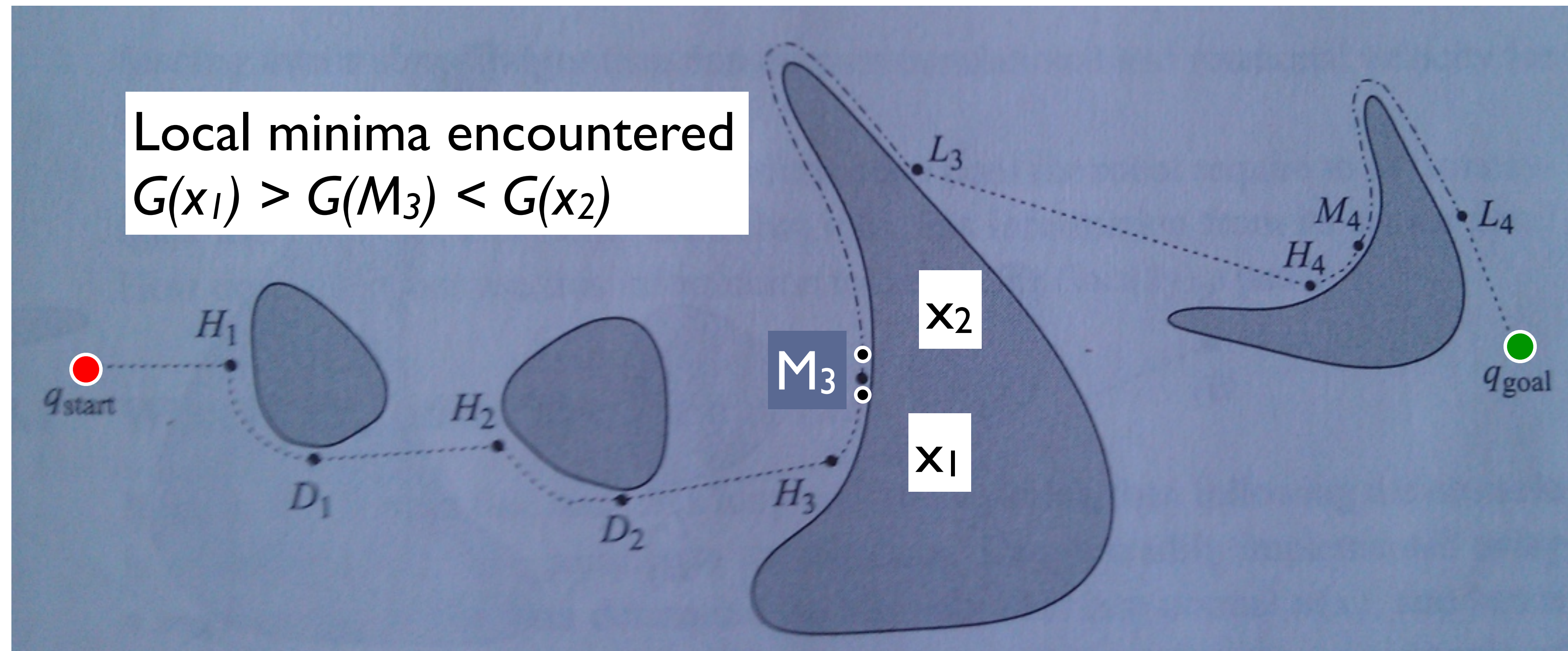
$H_i$ : hit point  
 $D_i$ : Depart point  
 $L_i$ : Leave point  
 $M_i$ : local minima

Motion-to-goal hits and departs  
obstacles



# Example: range $R=0$

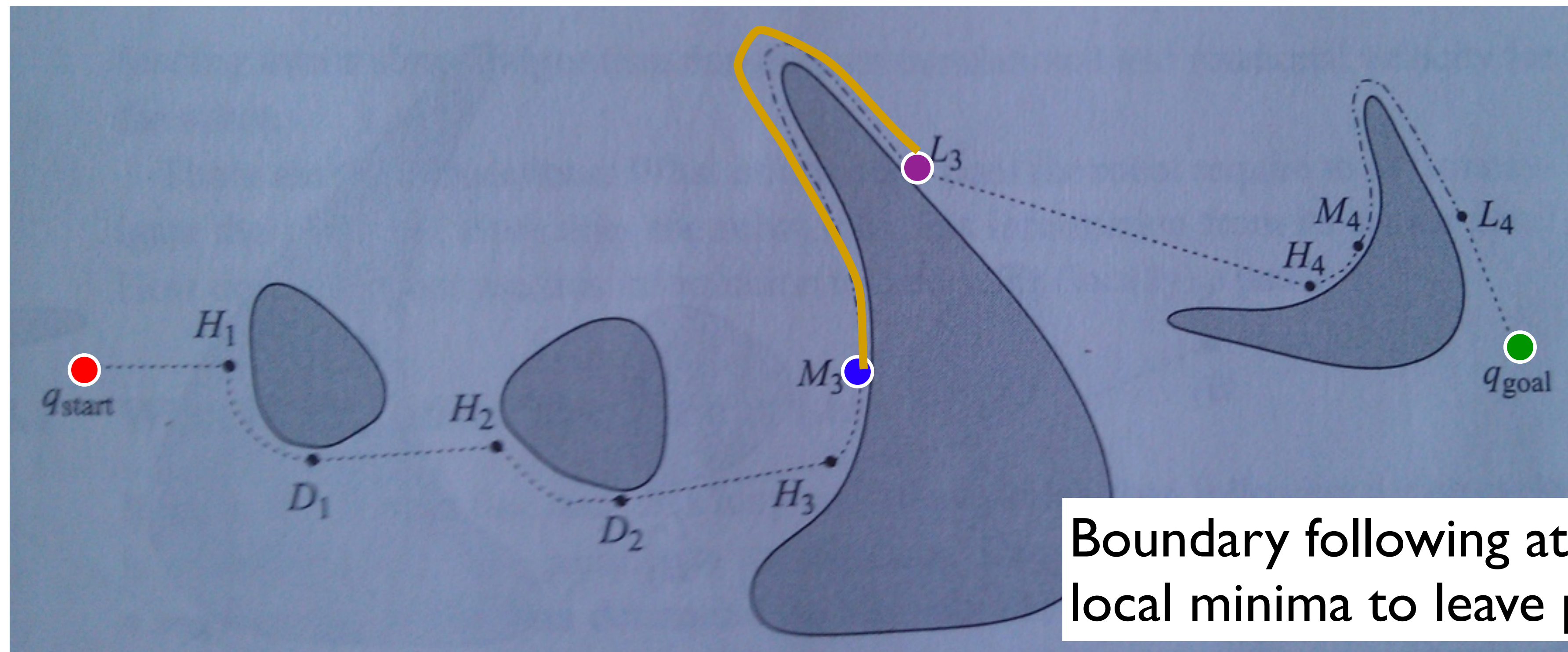
$H_i$ : hit point  
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Local minima at increase of  $G(x) = d(x, O_i) + d(O_i, q_{goal})$

# Example: range $R=0$

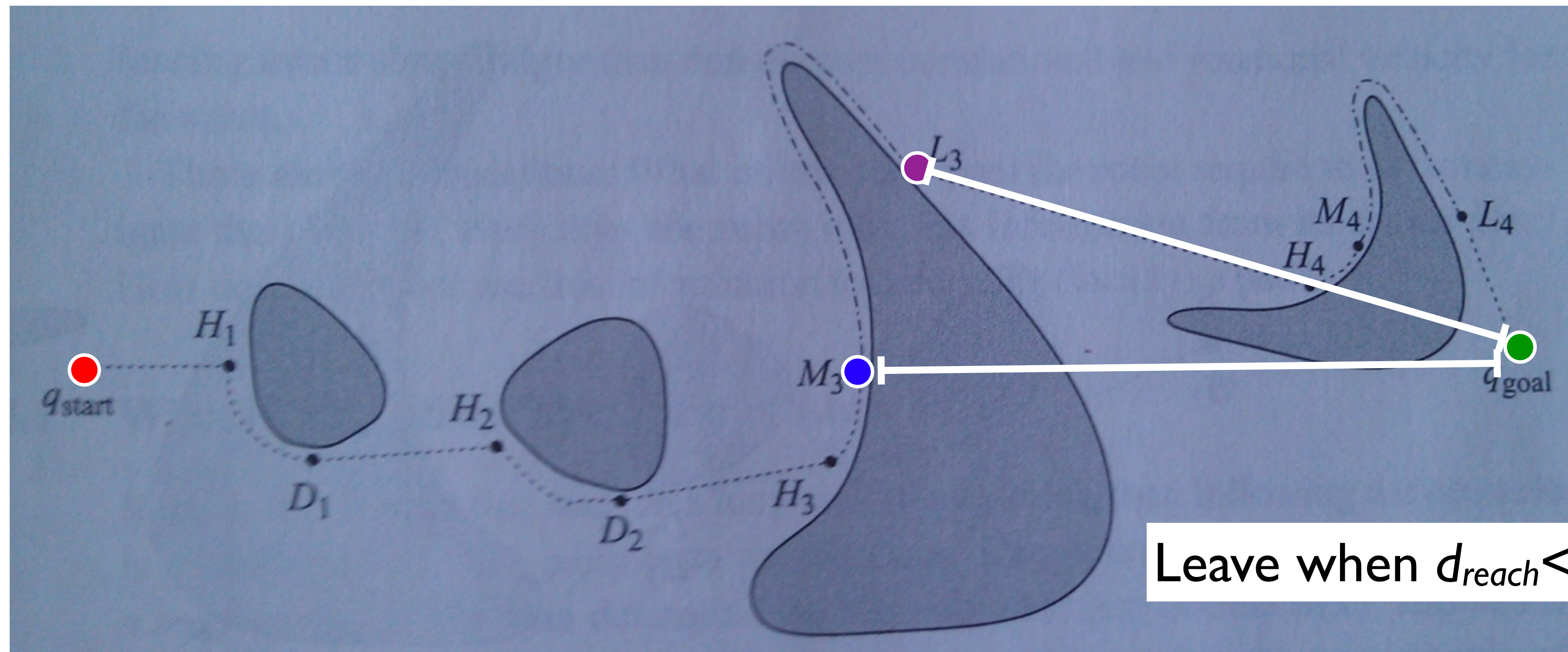
$H_i$ : hit point  
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Local minima at increase of  $G(x) = d(x, O_i) + d(O_i, q_{goal})$

# Example: range $R=0$

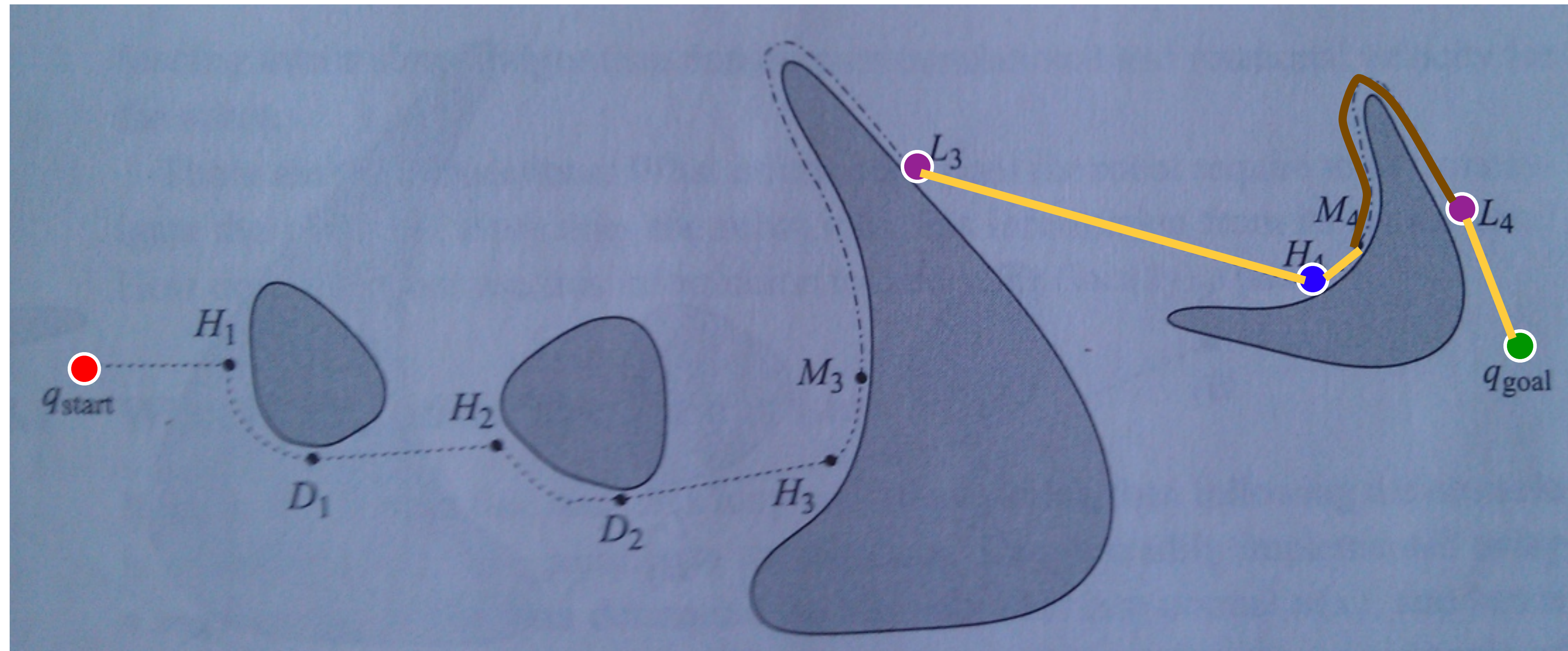
$H_i$ : hit point  
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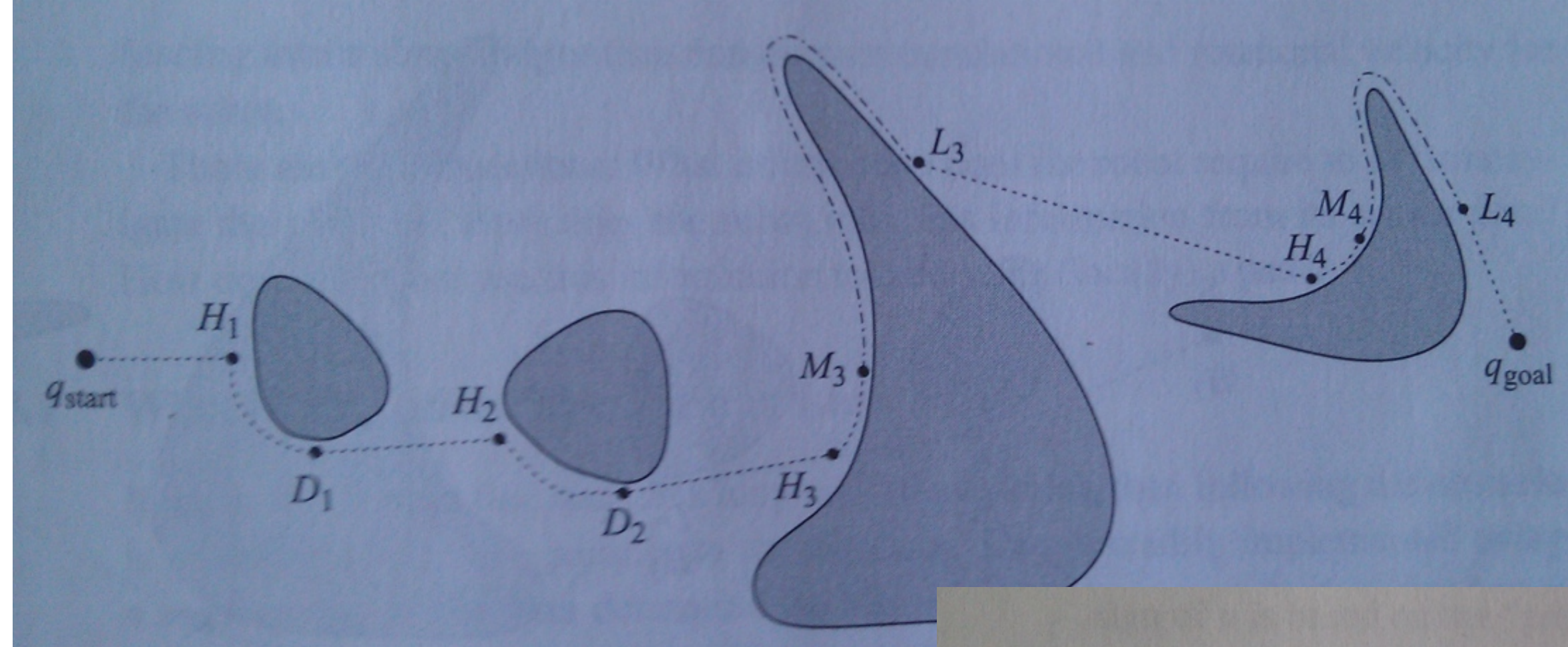
Local minima at increase of  $G(x) = d(x, O_i) + d(O_i, q_{goal})$

# Example: range $R=0$

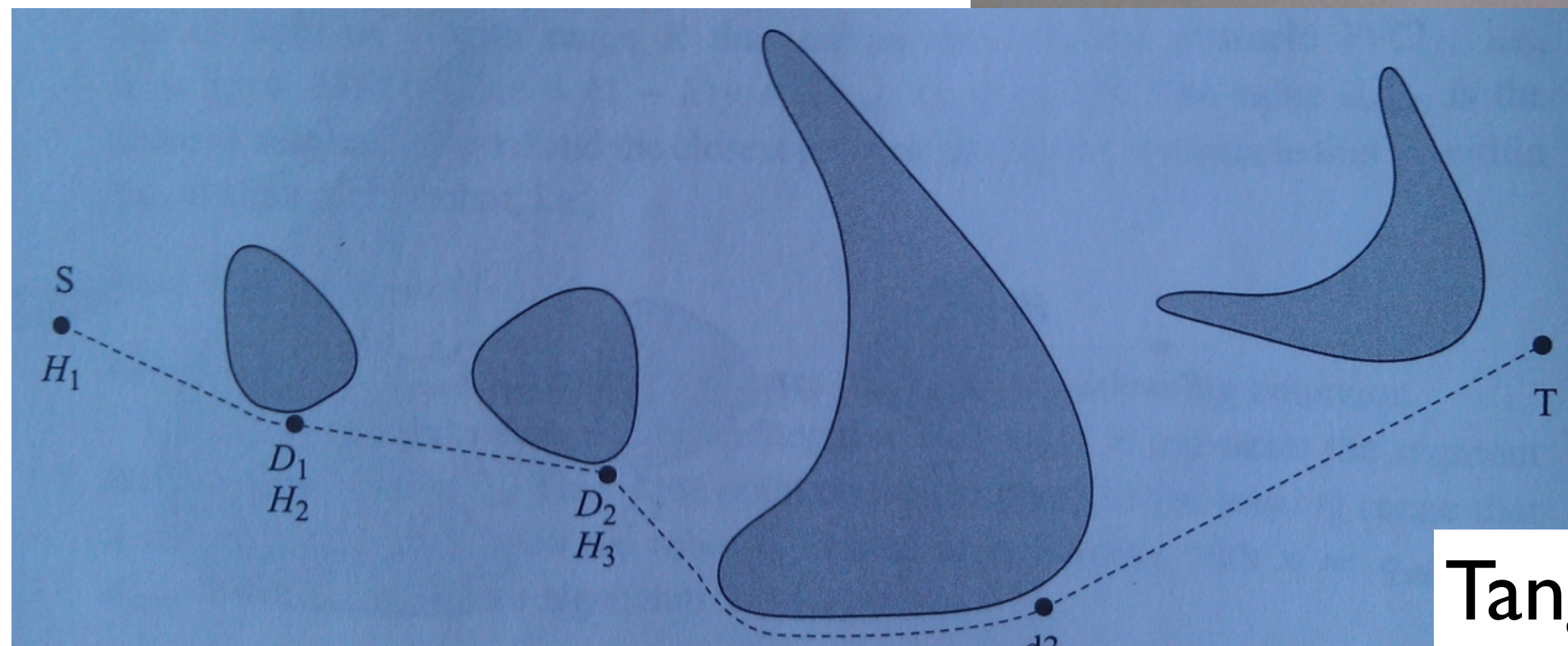
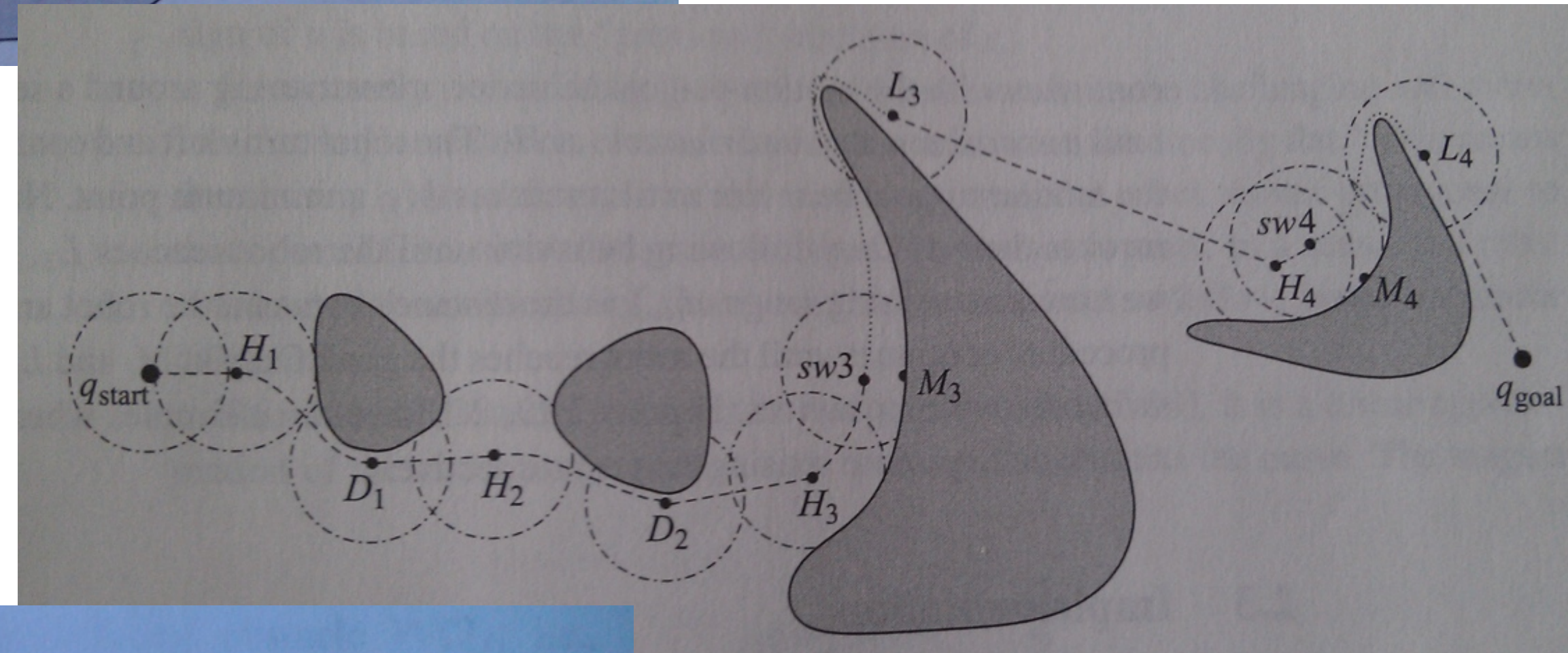
$H_i$ : hit point  
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 $L_i$ : Leave point  
 $M_i$ : local minima



Tangent bug  $R=0$



Tangent bug with limited radius



Tangent bug  $R=\text{infinity}$



What does BugX assume that Random Walk does not?





What does BugX assume that Random Walk does not?

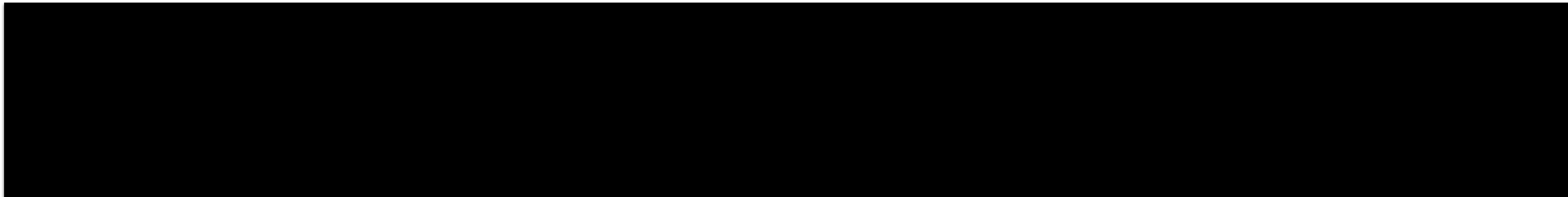
Localization: knowing the robot's location, at least wrt. distance to goal



What does BugX assume that Random Walk does not?

Localization: knowing the robot's location, at least wrt. distance to goal

What do graph search algorithms assume that BugX does not?



What does BugX assume that Random Walk does not?

Localization: knowing the robot's location, at least wrt. distance to goal

What do graph search algorithms assume that BugX does not?

A graph of valid locations that can be traversed



What does BugX assume that Random Walk does not?

Localization: knowing the robot's location, at least wrt. distance to goal

What do graph search algorithms assume that BugX does not?

A graph of valid locations that can be traversed

Suppose we have or can build such a graph...



# Next Lecture

## Planning - III - Configuration Space





"Pick up the butter and hand it over to the robot on your left."

ROBOT STATE

- Joint angles
- Finger positions

ROBOT IMAGES

20 Hz

7-9

7-9

20 Hz

# THINKING, FAST AND SLOW

DANIEL  
KAHNEMAN

WINNER OF THE NOBEL PRIZE IN ECONOMICS

Whole Upper Body Control

Figure - <https://www.youtube.com/watch?v=Z3yQHYNXPws>