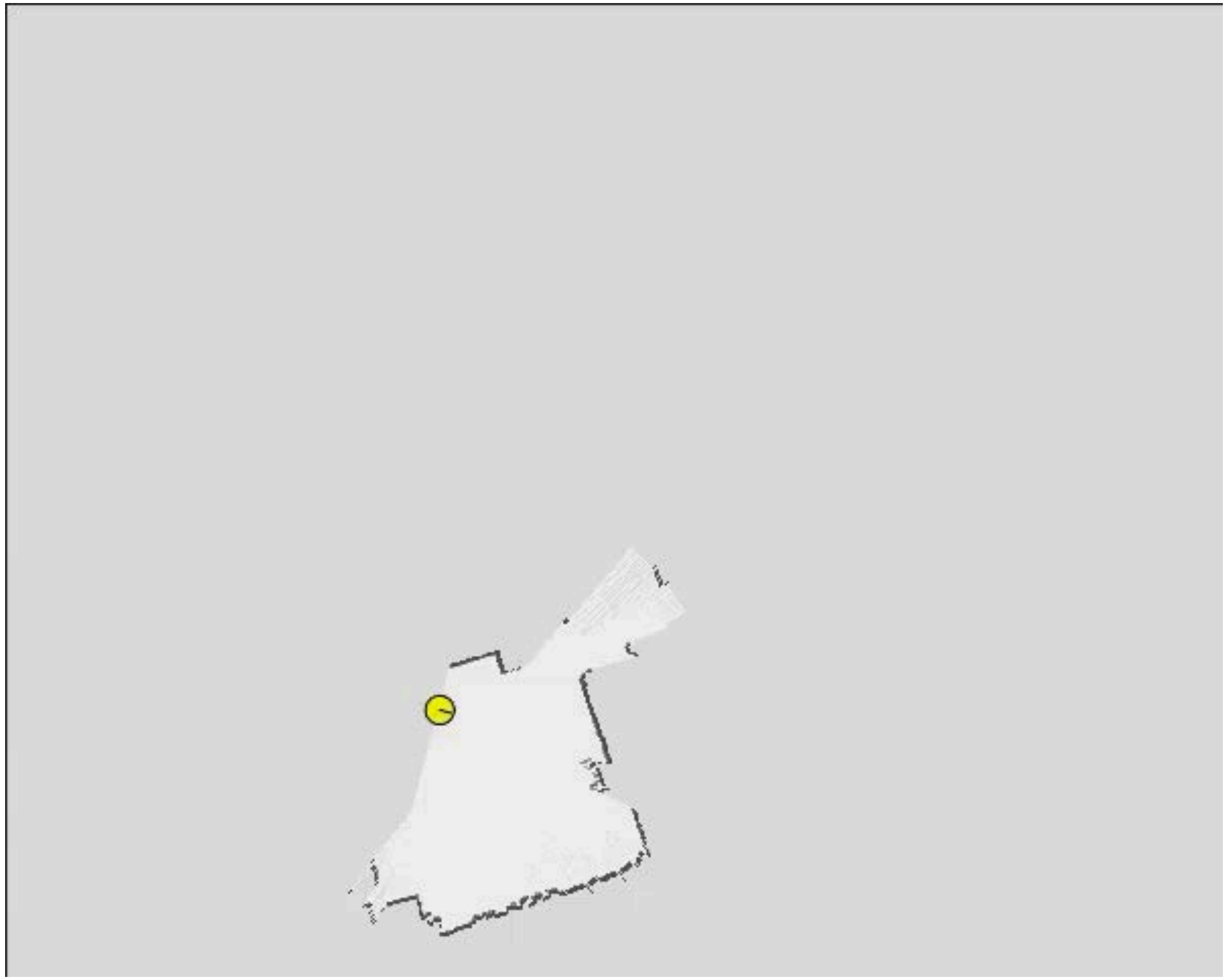
Lecture 22
Mobile Robotics - VII - SLAM



from Cyrill Stachniss

Course logistics

- Quiz 11 will be posted tomorrow and will be due c
- Project 7:
  - Groups are formed.
  - Two parts (~1 hr each) - Instructions will be pr
    1. Beginner's guide.
    2. Real Robot Challenge.
  - Scheduler is shared with the class.
    - Please book your 2 1-hour sessions.
    - Both the parts needs to be completed by 04/15.
- No TA OHs between 03/28 and 04/12.
  - Karthik's OH will be available to discuss final projects.
  - Chahyon and Xun's OH are cancelled between 03/28 and 04/12. They maybe available upon request for the UNITE team.
- **Final Poster Session: 05/04/2024 - Saturday - 1pm - 4pm, Shepherd Labs 164 - mark your calendars**

Location: Shepherd Labs 164 (Drone Lab) - this place will not be available for experiments after the dedicated times shown below.
Note: Talk to your team members and find a slot that works best to do P7 experiments. You will need two sessions as a team to perform the tasks we created for you. Please do not overbook. Start with 2 1-hour sessions.
You will need to come in as a team to finish these tasks.
Course staff will be present to guide you through the process.
Your Group Numbers are available in the next Sheet.

03/28/2024	04/01/2024	04/03/2024	04/04/2024

04/08/2024			04/09/2024			04/11/2024			04/15/2024		
Robot-0	2:30-3:30 pm	Group-4	Robot-0	2:00-3:00 pm	Group-11	Robot-0	2:00-3:00 pm	Available	Robot-0	2:30-3:30 pm	Available
Robot-1	2:30-3:30 pm	Available	Robot-1	2:00-3:00 pm	Group-8	Robot-1	2:00-3:00 pm	Available	Robot-1	2:30-3:30 pm	Available
Robot-2	2:30-3:30 pm	Group-5	Robot-2	2:00-3:00 pm	Available	Robot-2	2:00-3:00 pm	Available	Robot-2	2:30-3:30 pm	Available
Robot-3	2:30-3:30 pm	Available	Robot-3	2:00-3:00 pm	Available	Robot-3	2:00-3:00 pm	Available	Robot-3	2:30-3:30 pm	Available
Robot-4	2:30-3:30 pm	Group-3	Robot-4	2:00-3:00 pm	Available	Robot-4	2:00-3:00 pm	Group-9	Robot-4	2:30-3:30 pm	Available
Robot-0	3:30-4:30 pm	Group-12	Robot-0	3:00-4:00 pm	Available	Robot-0	3:00-4:00 pm	Available	Robot-0	3:30-4:30 pm	Available
Robot-1	3:30-4:30 pm	Available	Robot-1	3:00-4:00 pm	Group-7	Robot-1	3:00-4:00 pm	Group-7	Robot-1	3:30-4:30 pm	Available
Robot-2	3:30-4:30 pm	Available	Robot-2	3:00-4:00 pm	Available	Robot-2	3:00-4:00 pm	Available	Robot-2	3:30-4:30 pm	Available
Robot-3	3:30-4:30 pm	Available	Robot-3	3:00-4:00 pm	Available	Robot-3	3:00-4:00 pm	Available	Robot-3	3:30-4:30 pm	Available
Robot-4	3:30-4:30 pm	Group-3	Robot-4	3:00-4:00 pm	Available	Robot-4	3:00-4:00 pm	Group-9	Robot-4	3:30-4:30 pm	Available

Final (Open) Project timeline

In-class proposal presentation

- Groups 1-4: 04/10
- Groups 5-8: 04/15
- Groups 9-13: 04/17

Sun	Mon	Tue	Wed	Thu	Fri	Sat
7	Today 8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	1	2	3	4

Proposal Slides Due before the class

Poster for printing due

Poster Presentation

Final video due

Final (Open) Project timeline

- Proposal Slides: (template will be provided by 04/03)
  - 1-4 Slides
  - Title, Motivation, Input - Output, Evaluation, Deliverables, Timeline, Who is doing what?
  - Where does your project stand not the 3-axes (robots, objects, tasks)?
  - Backup plan
- In-class proposal presentation (<8mins) :
  - Teams will get feedback from the class
- Final video:
  - Describing the project idea and the outcome.
- Poster presentation: (template will be provided by 04/03)
  - Presenting the project idea and the outcome to audience.

- Final Project: 15%
  - Project proposal slides + presentation: 3%
  - Final project video: 6%
  - Poster presentation (evaluation by judges): 6%

Final Project (Open ended)

Think along these axes to decide your final project!

Evaluating your implementation/system with quantitative results are VERY important!

Objects

Rearrangment of a set of objects

You may use:

- Kineval codebase
- Other sim environments (pybullet, Gazebo, DRAKE, Isaac sim)
- Turtlebot3 (provided only upon compelling proposal, only 5 are available)
- Other robots you may have access to.

Multi-robot task execution

Long horizon tasks

Tasks

Robots

Frontier-based Exploration:

Frontier-based exploration is **the process of repeatedly detecting frontiers and moving towards them, until there are no more frontiers and therefore no more unknown regions.**

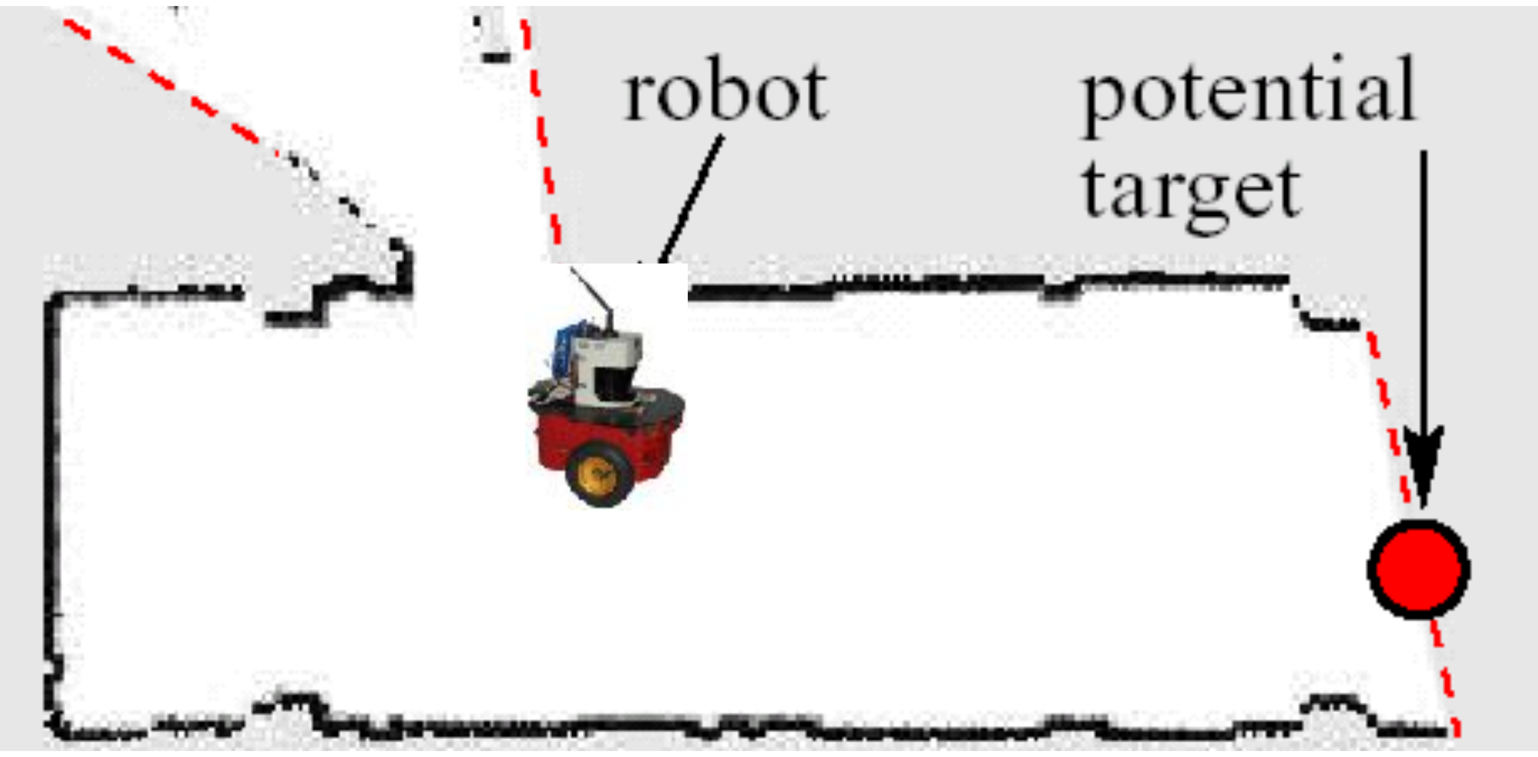
What are frontiers?

Frontier cells define the border between known and unknown space.

Frontier-Based Exploration

Single Robot Exploration

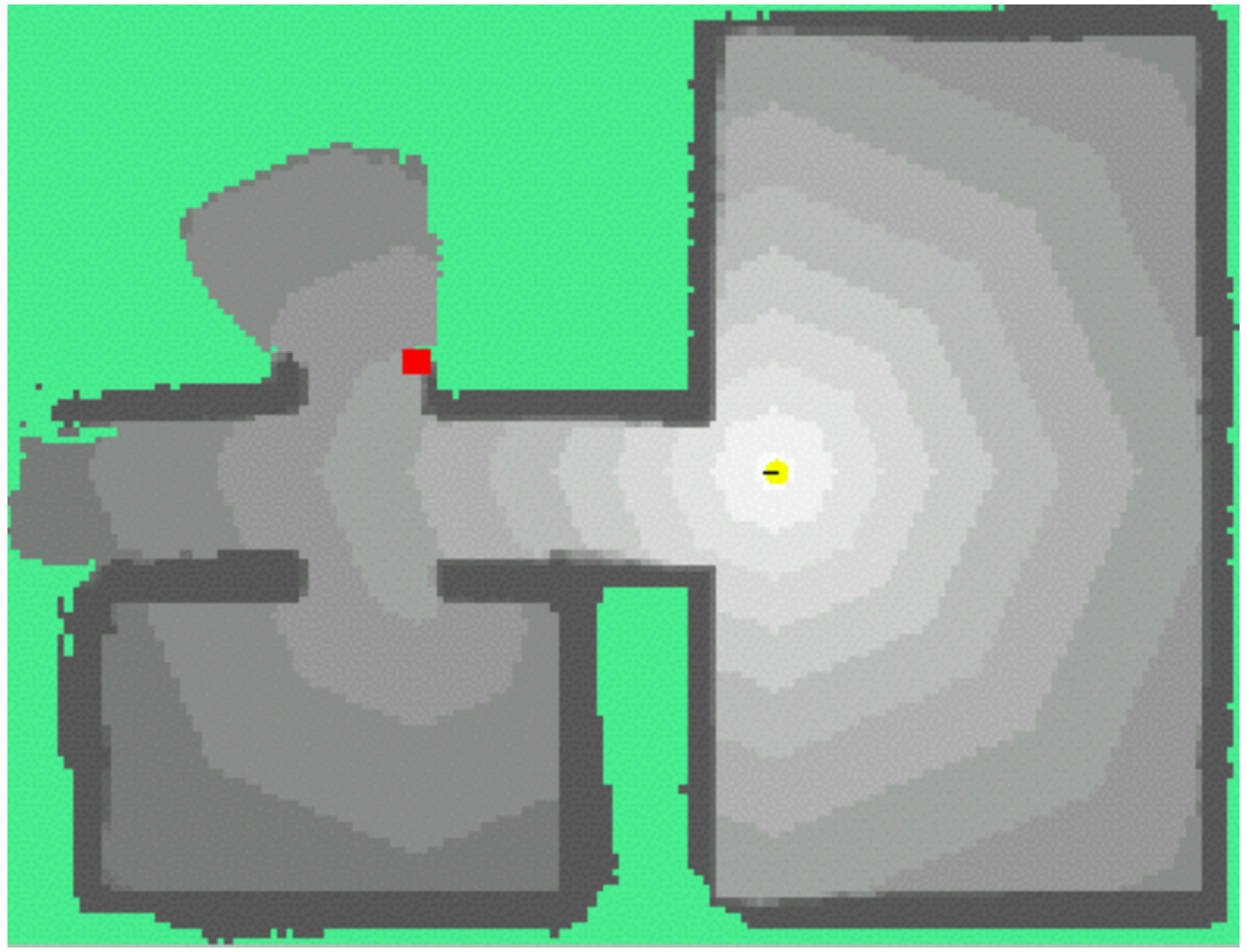
- Frontiers between free space and unknown areas are potential target locations
- Going to frontiers will gain information



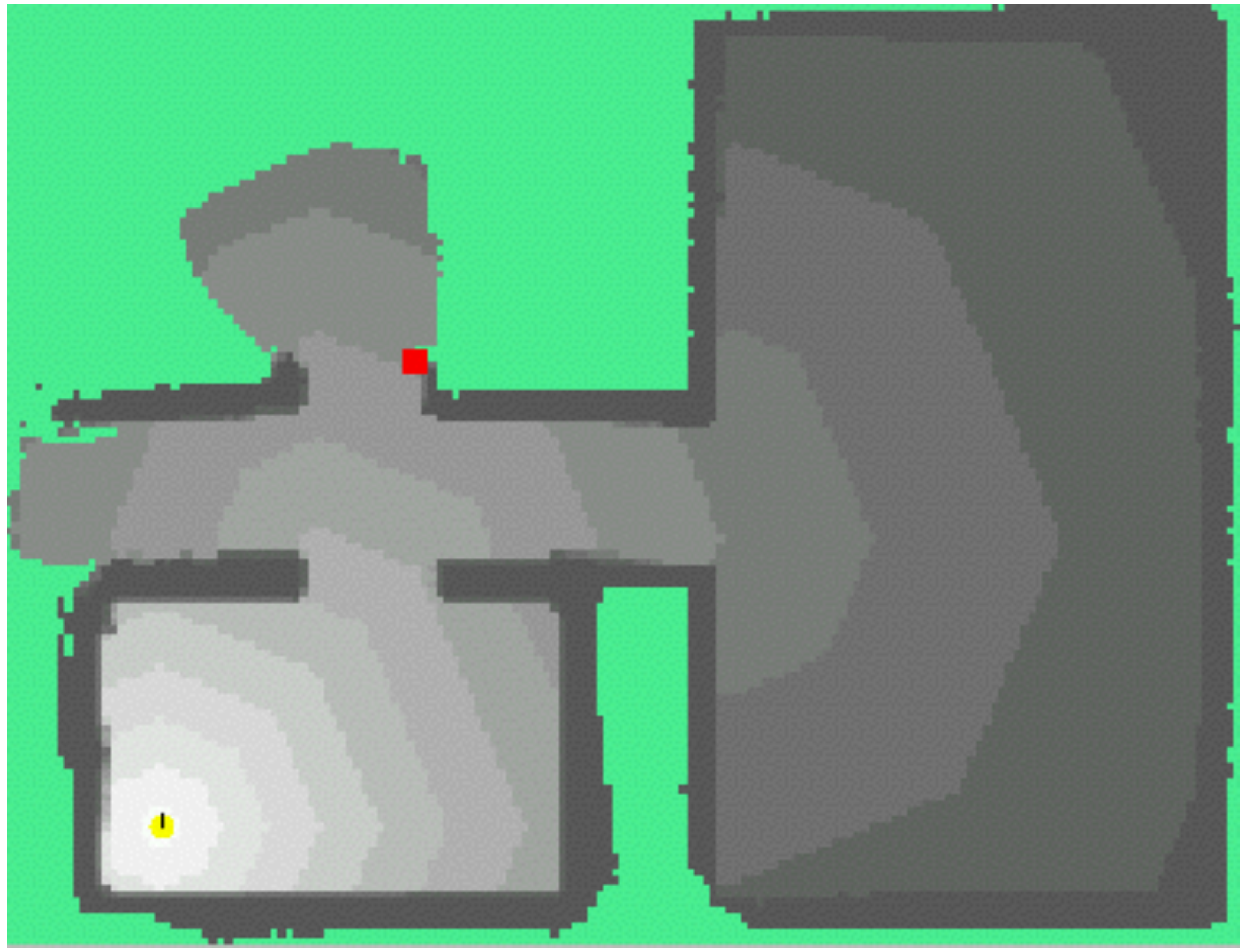
- Select the target that minimizes a cost function (e.g. travel time / distance /…)

Multi-Robot Exploration

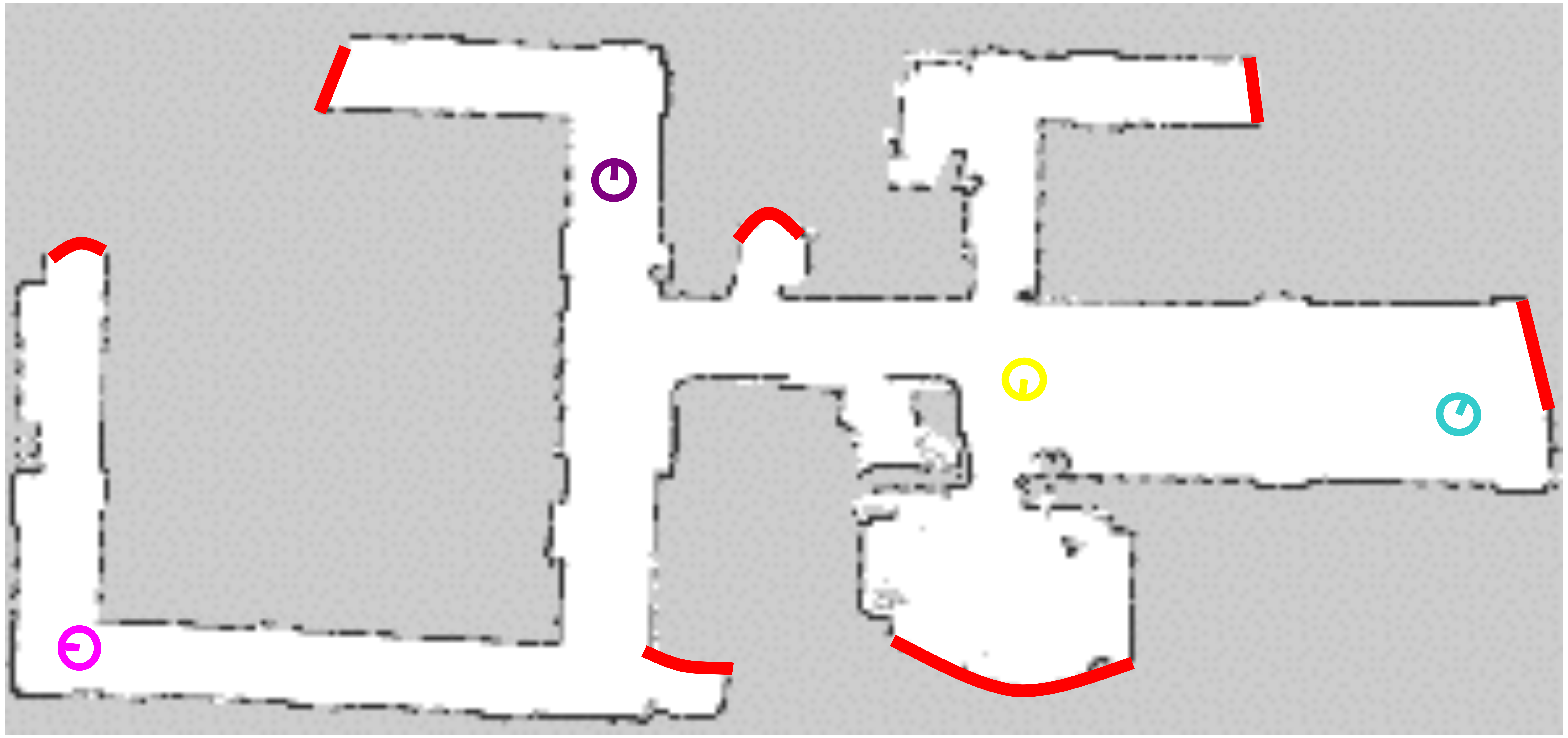
Robot 1:



Robot 2:



Coordinated Exploration



[Burgard et al. 00],
[Simmons et al. 00]

The SLAM Problem

The SLAM Problem

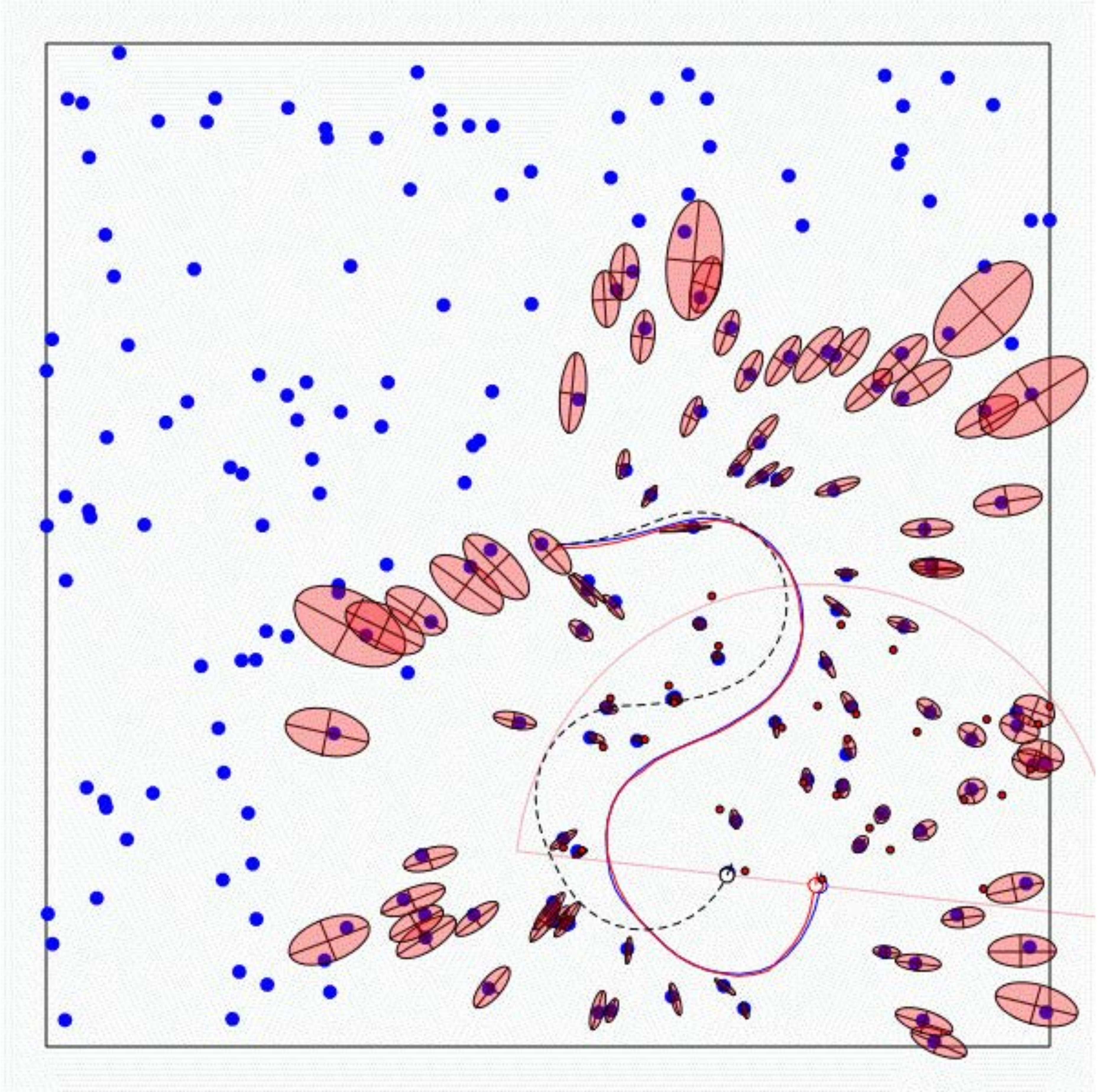
A robot is exploring an unknown, static environment.

Given:

- The robot's controls
- Observations of nearby features

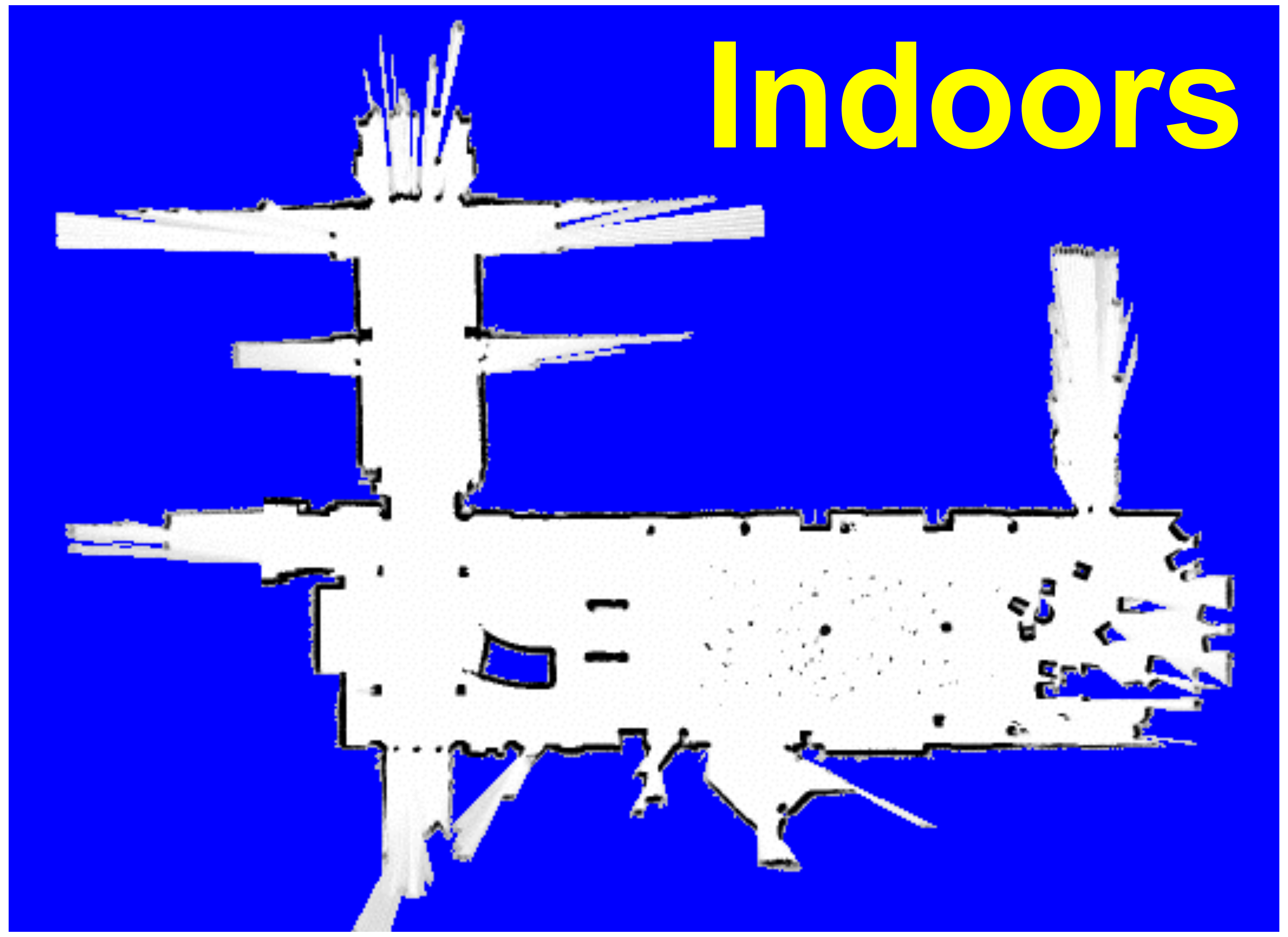
Estimate:

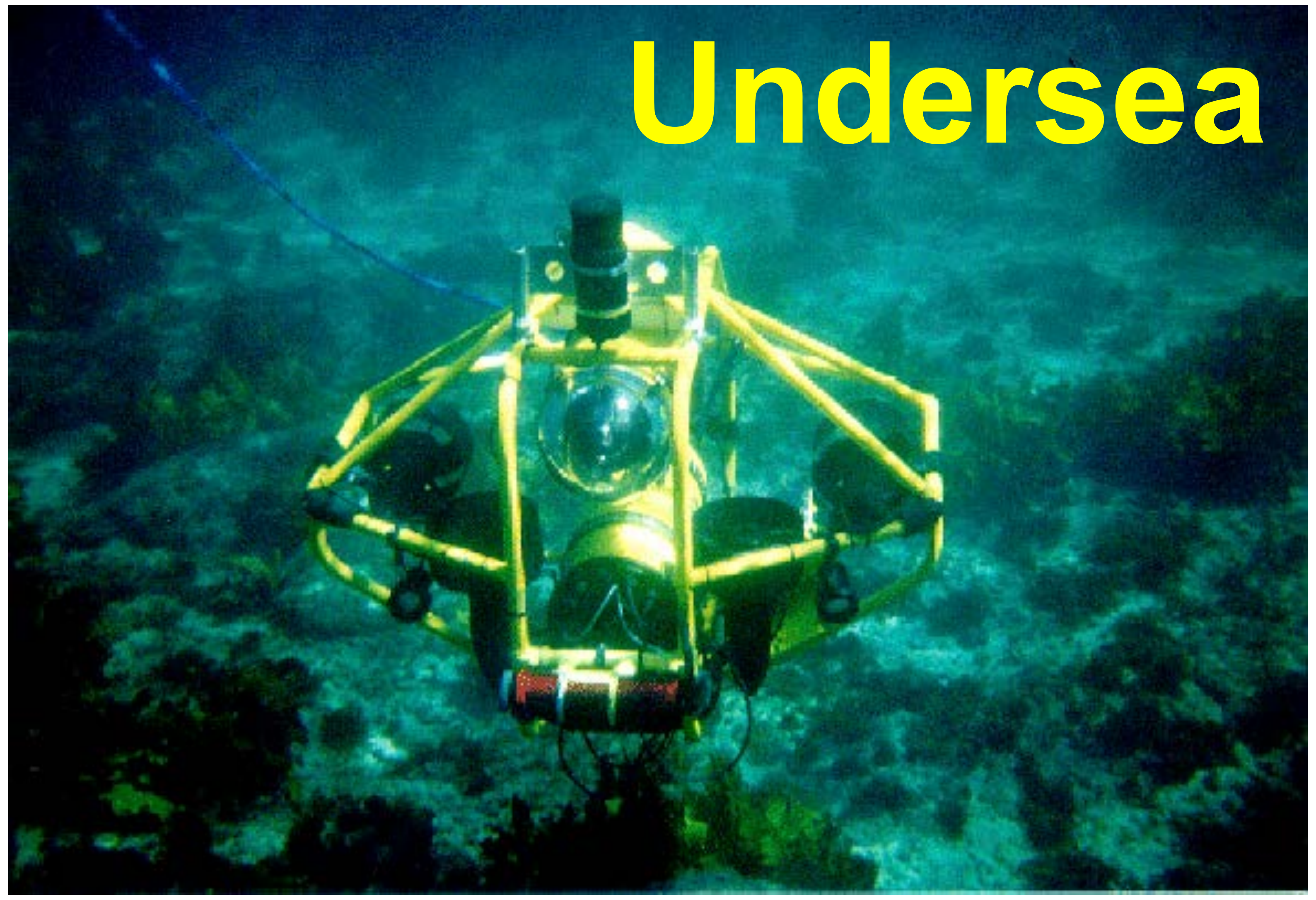
- Map of features
- Path of the robot

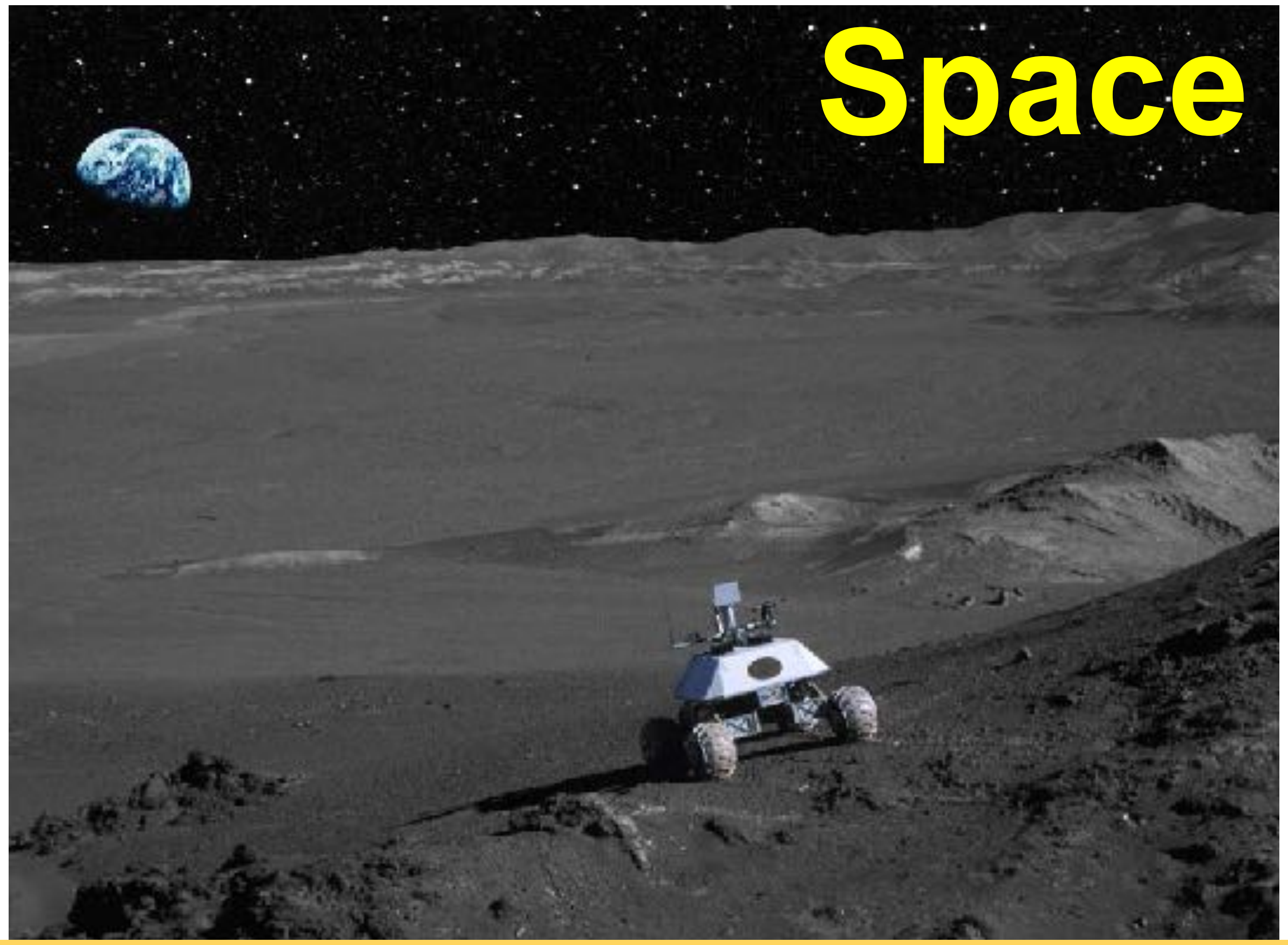


Slide borrowed from Dieter Fox

SLAM Applications

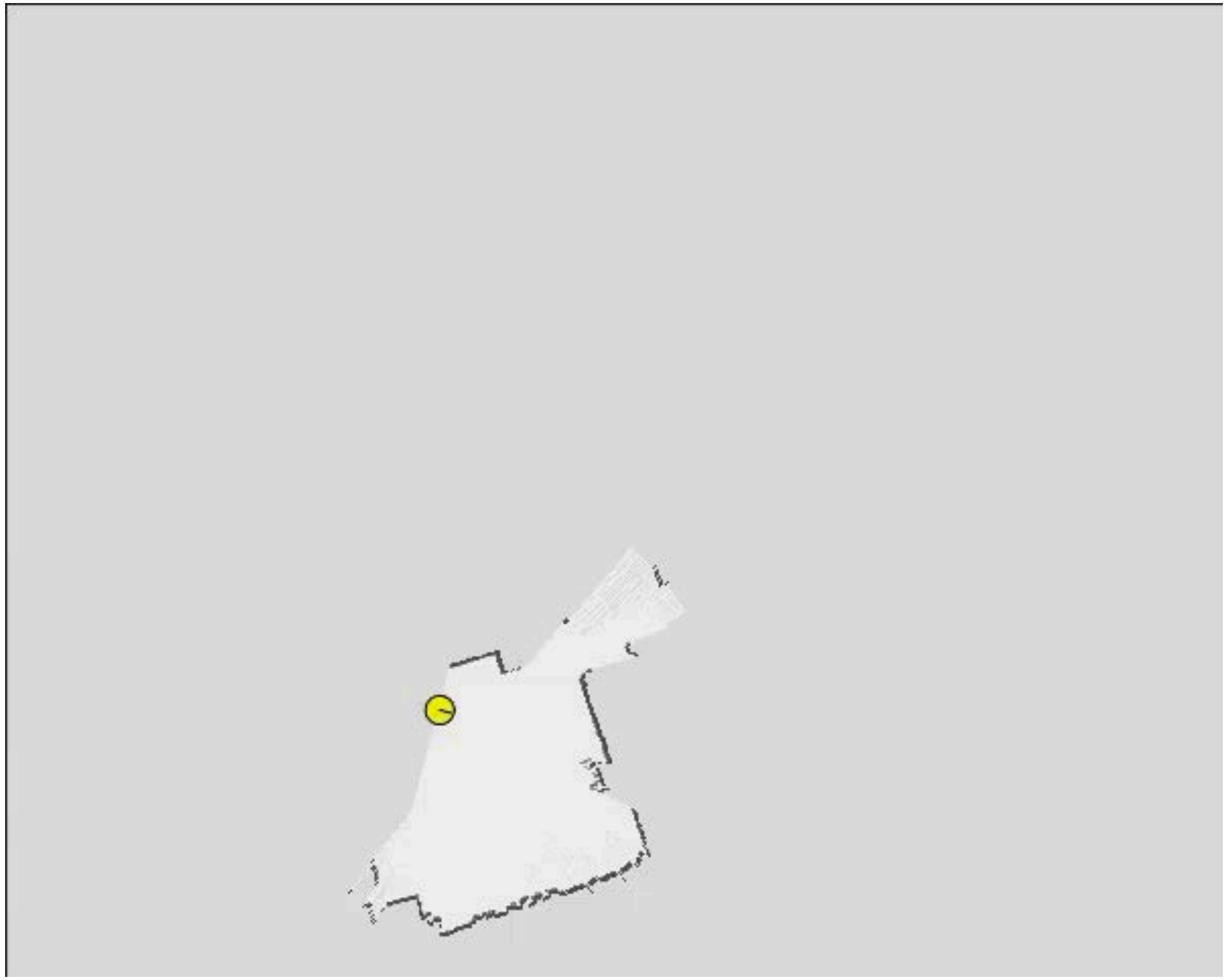






Slide borrowed from Dieter Fox

Mapping with Perfect Odometry



Mapping with Raw Odometry

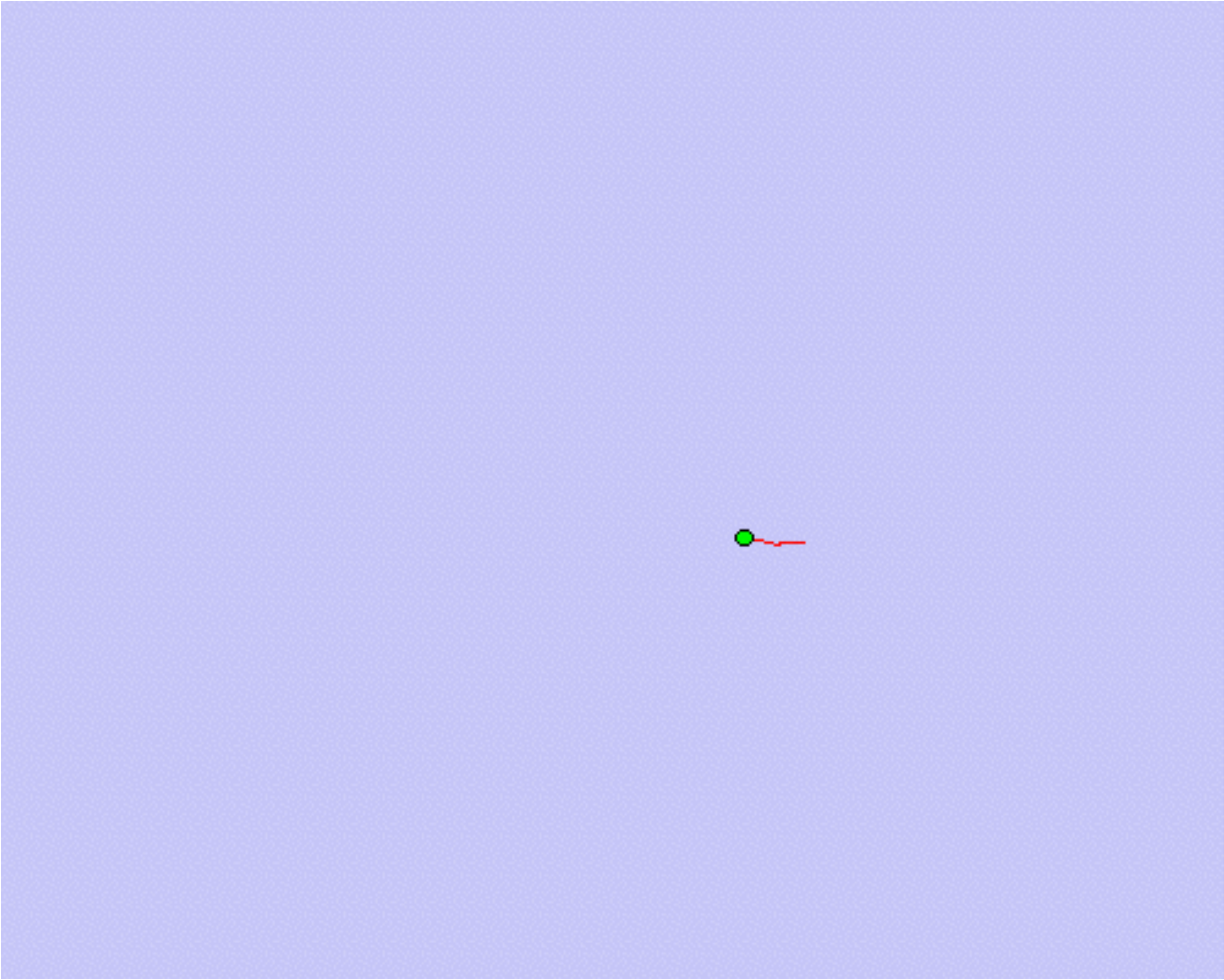
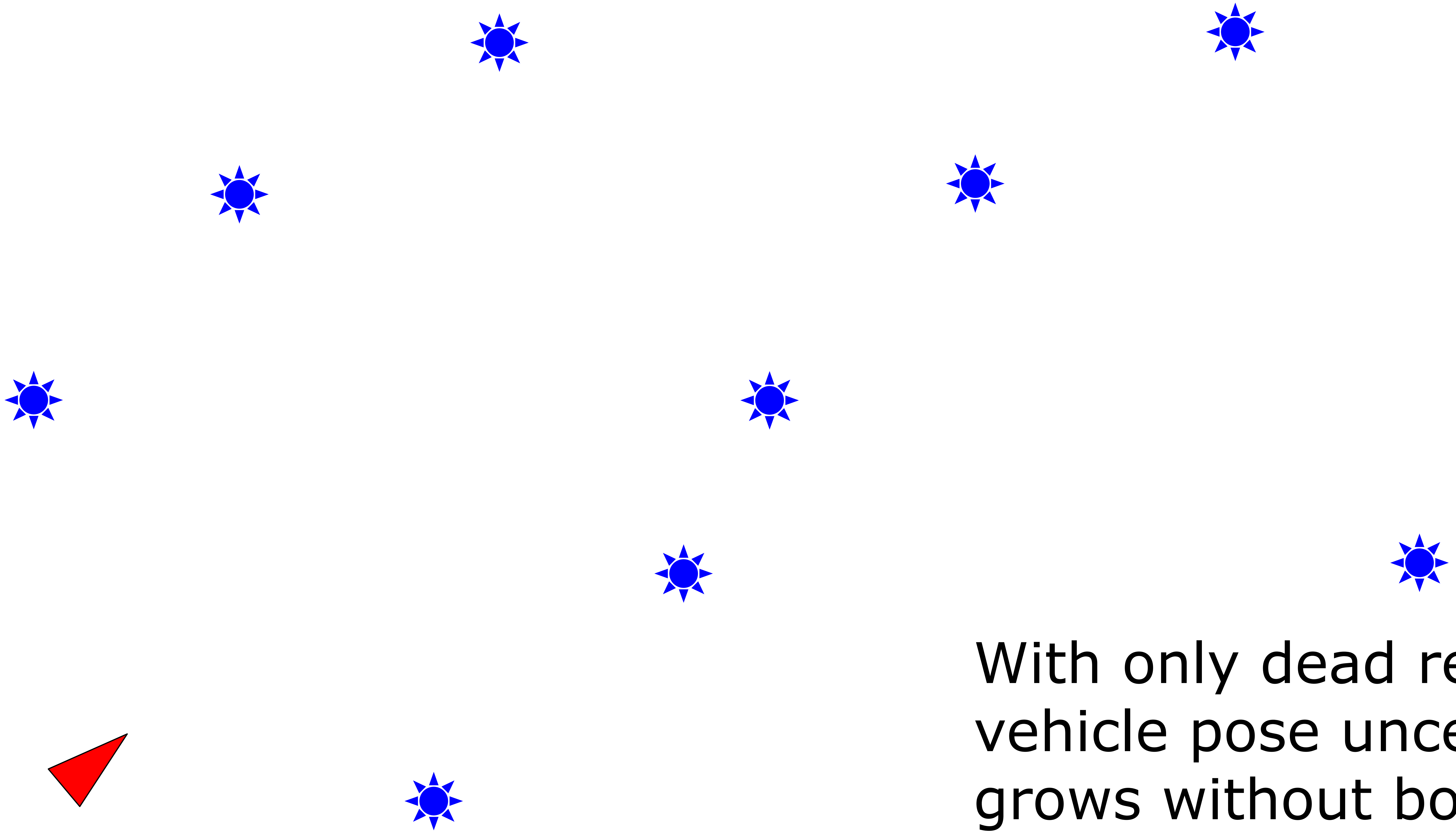


Illustration of SLAM

Illustration of SLAM without Landmarks

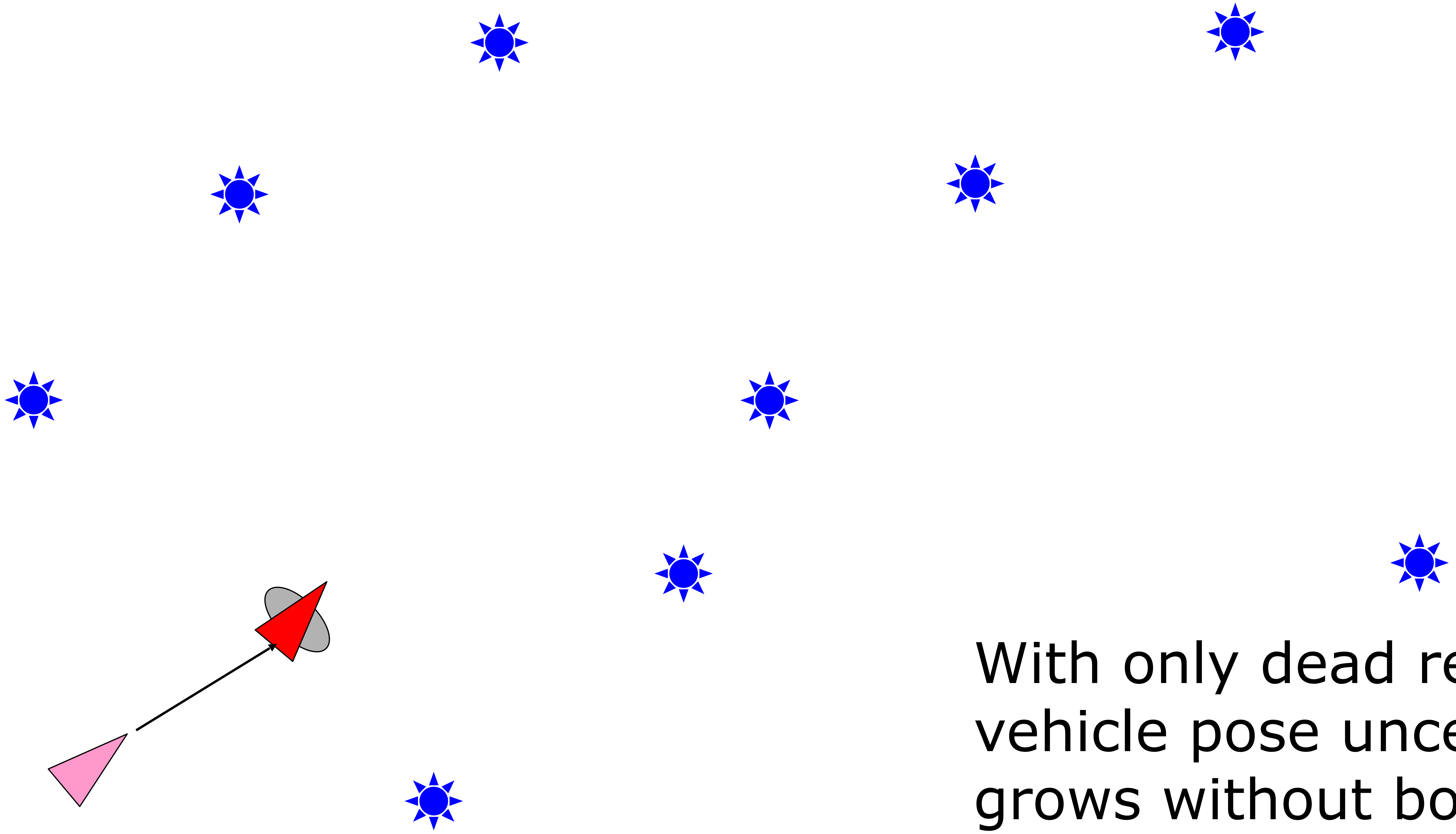


Courtesy J. Leonard

With only dead reckoning, vehicle pose uncertainty grows without bound

Slide borrowed from Dieter Fox

Illustration of SLAM without Landmarks



With only dead reckoning, vehicle pose uncertainty grows without bound

Courtesy J. Leonard

Illustration of SLAM without Landmarks

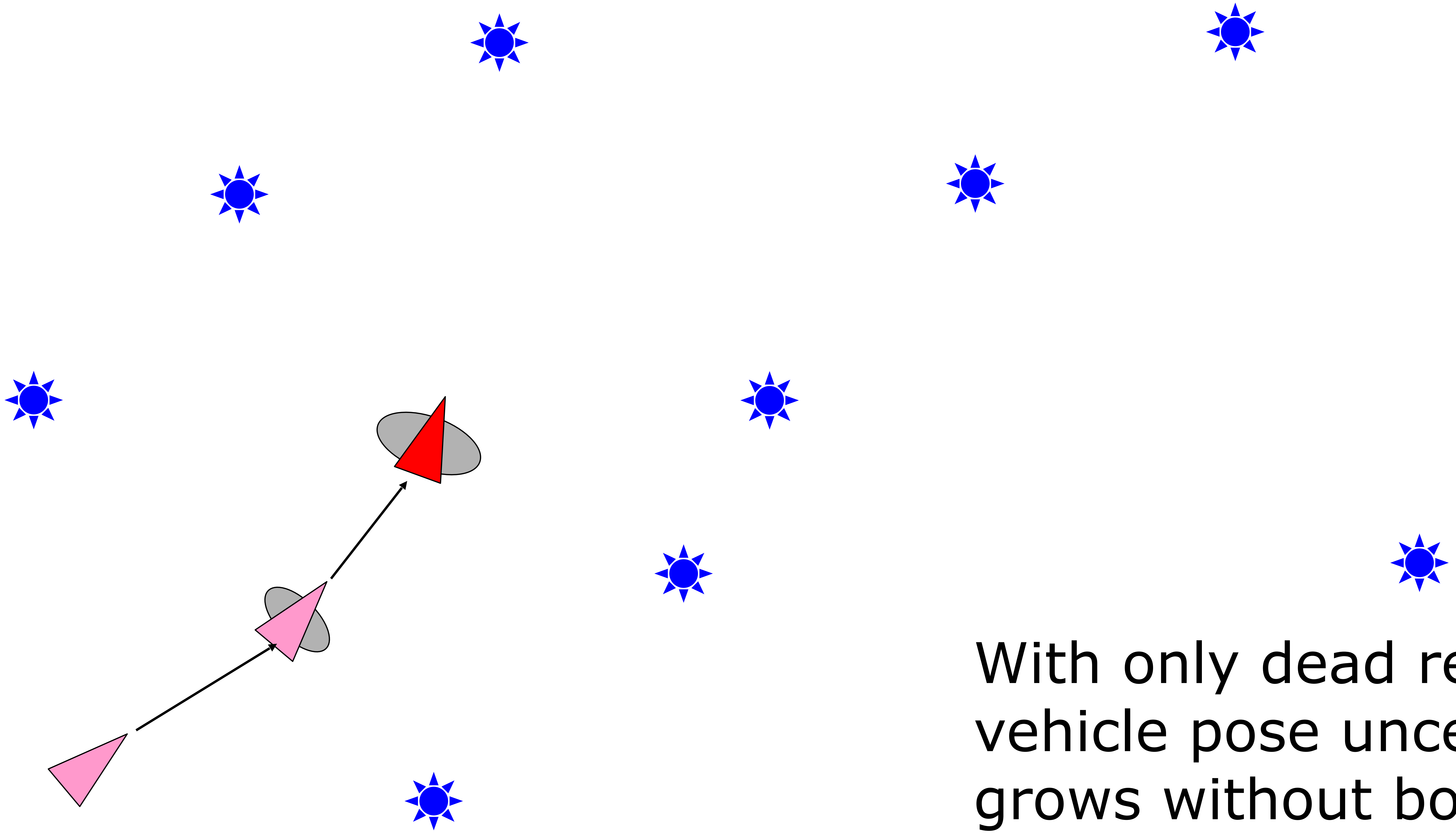


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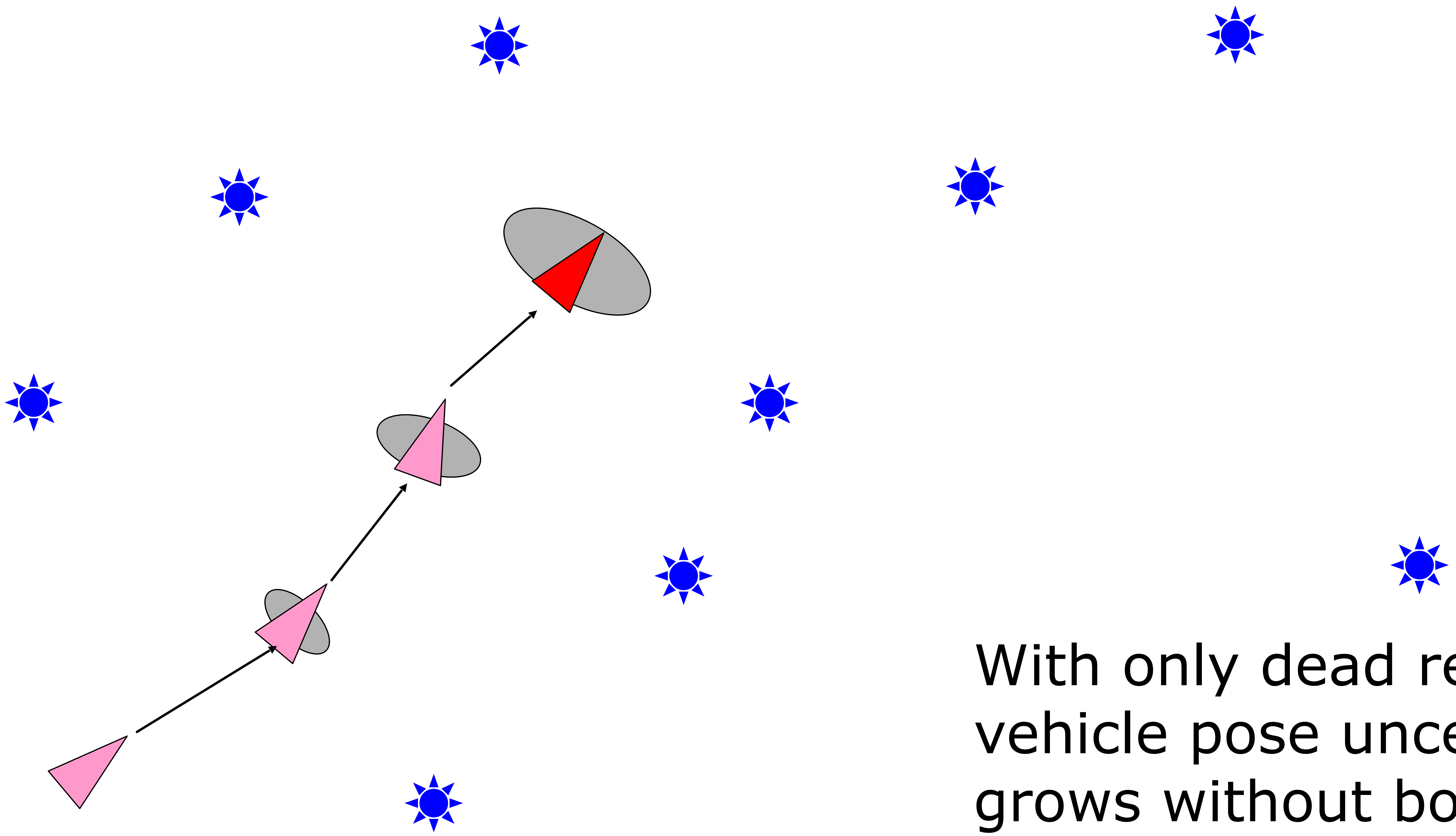
Illustration of SLAM without Landmarks



Courtesy J. Leonard

With only dead reckoning, vehicle pose uncertainty grows without bound

Illustration of SLAM without Landmarks

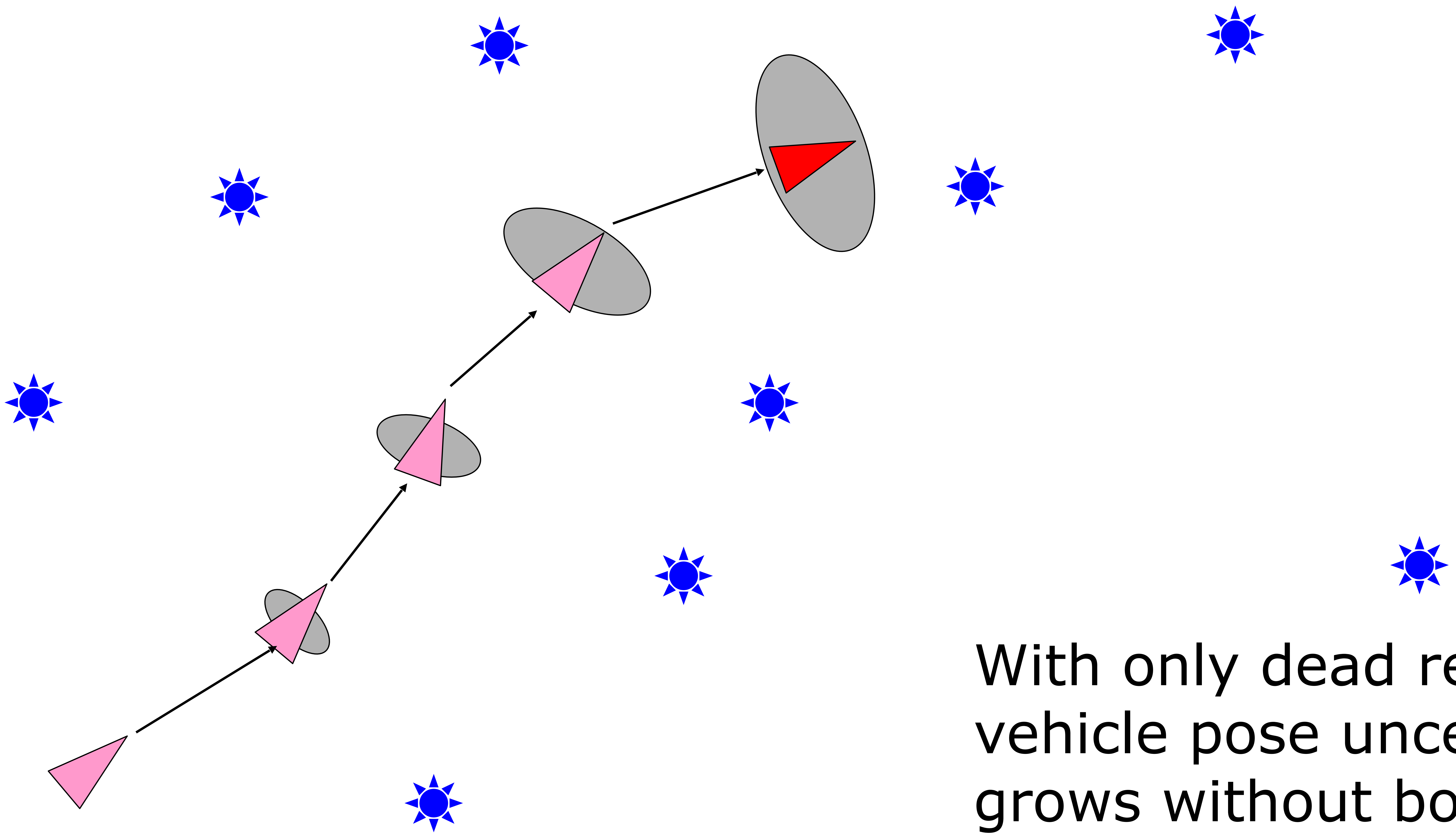


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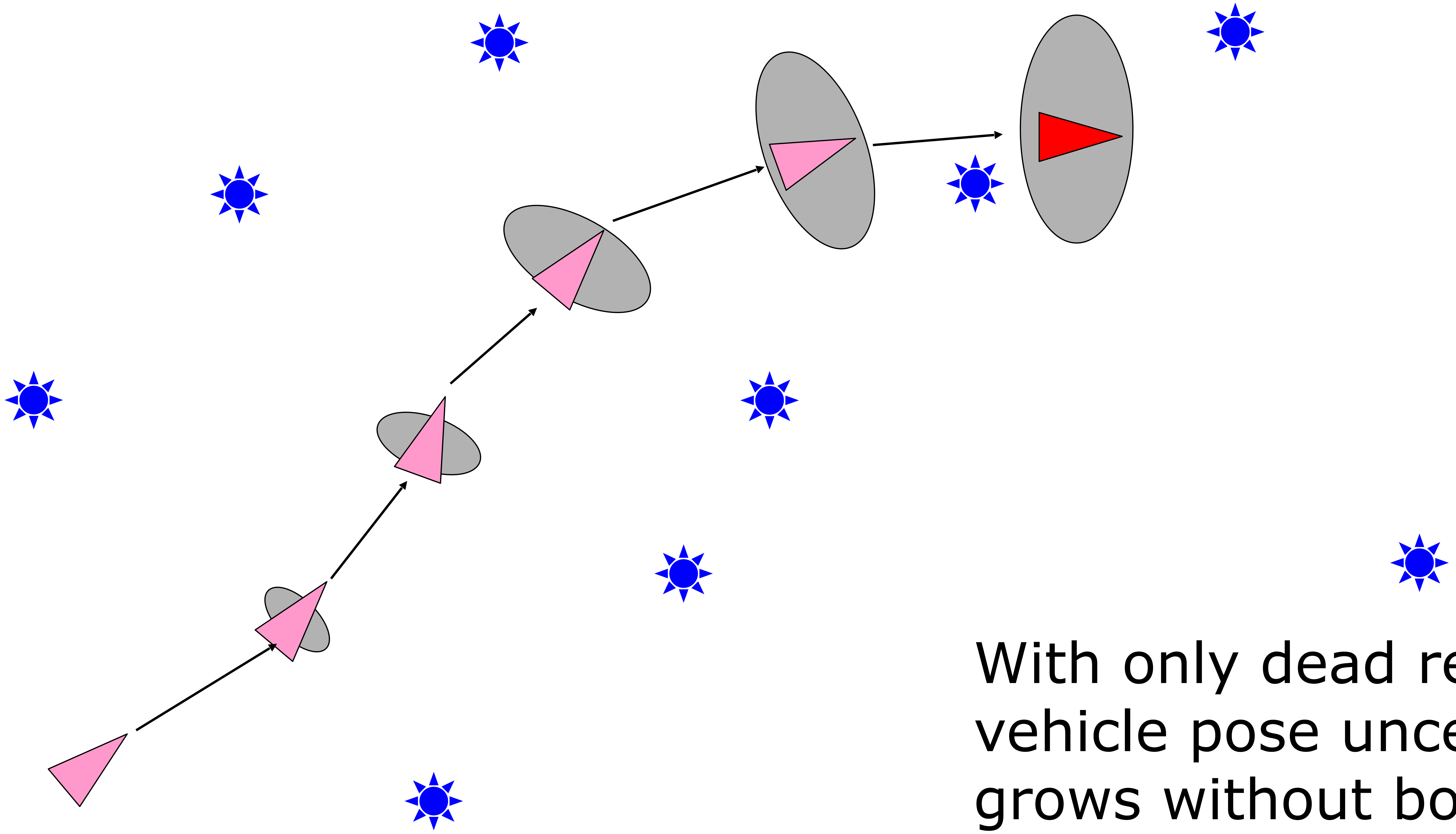
Illustration of SLAM without Landmarks

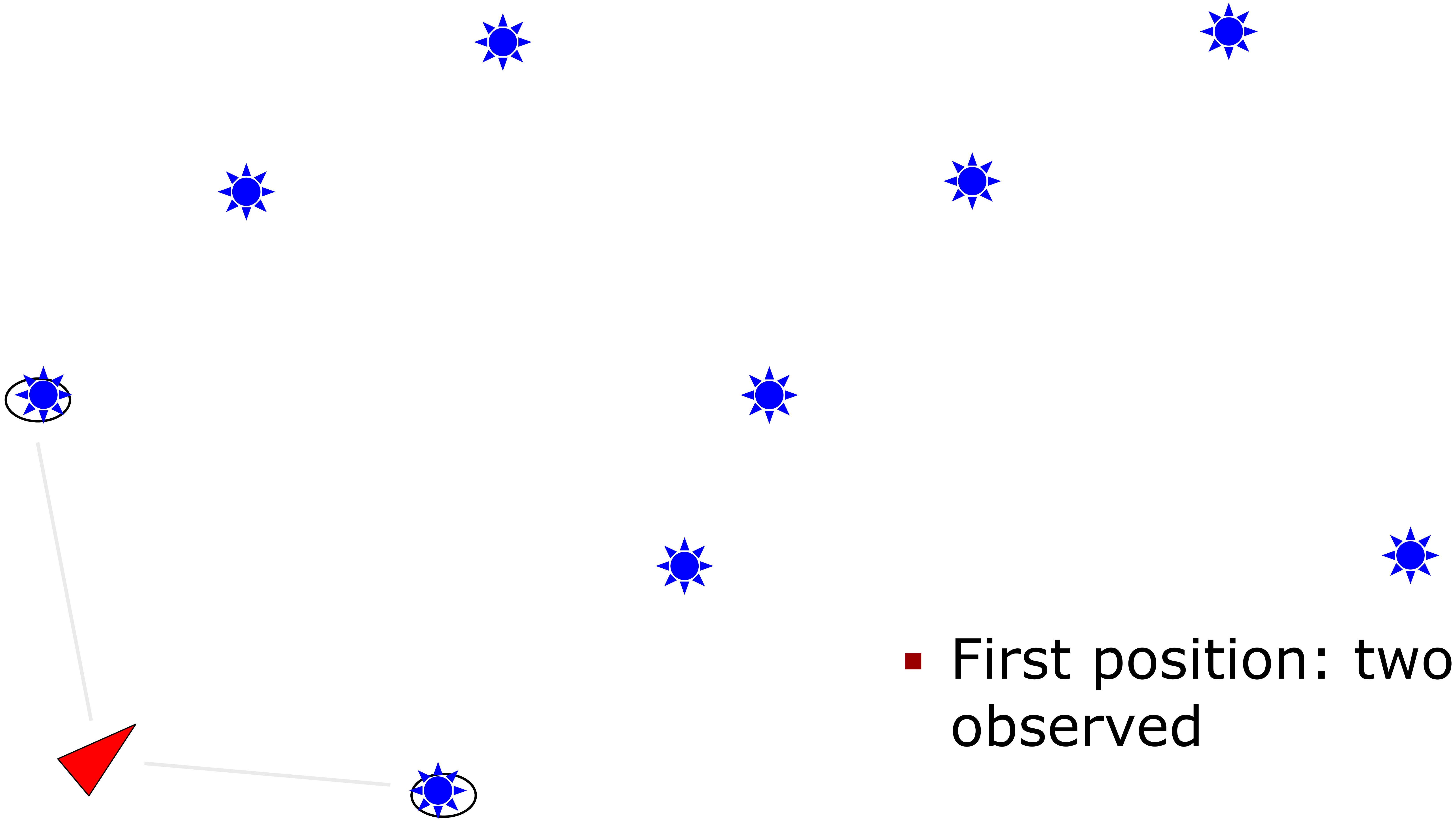


With only dead reckoning, vehicle pose uncertainty grows without bound

Courtesy J. Leonard

Slide borrowed from Dieter Fox

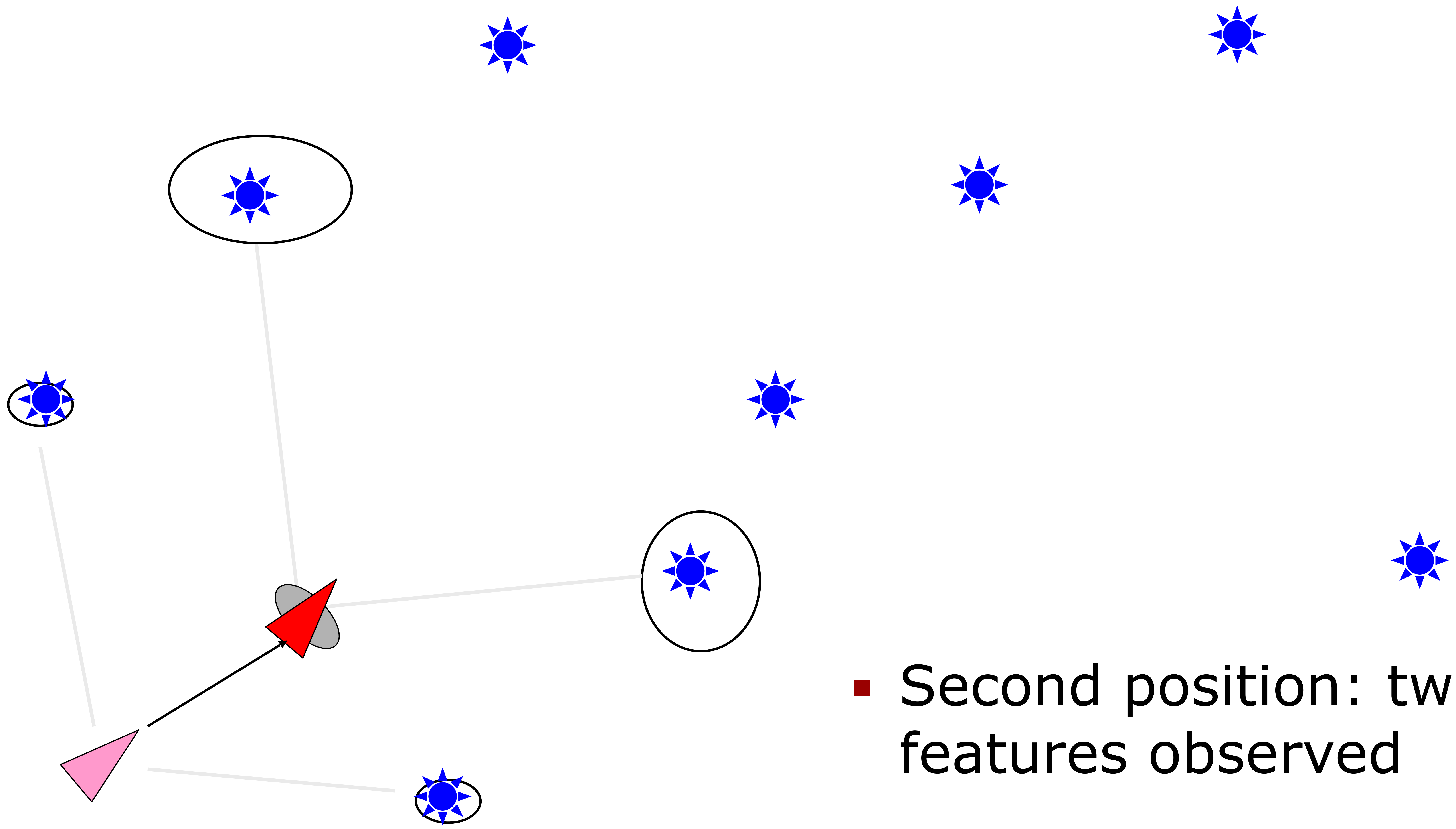
Repeat, with Measurements of Landmarks



- First position: two features observed

Courtesy J. Leonard

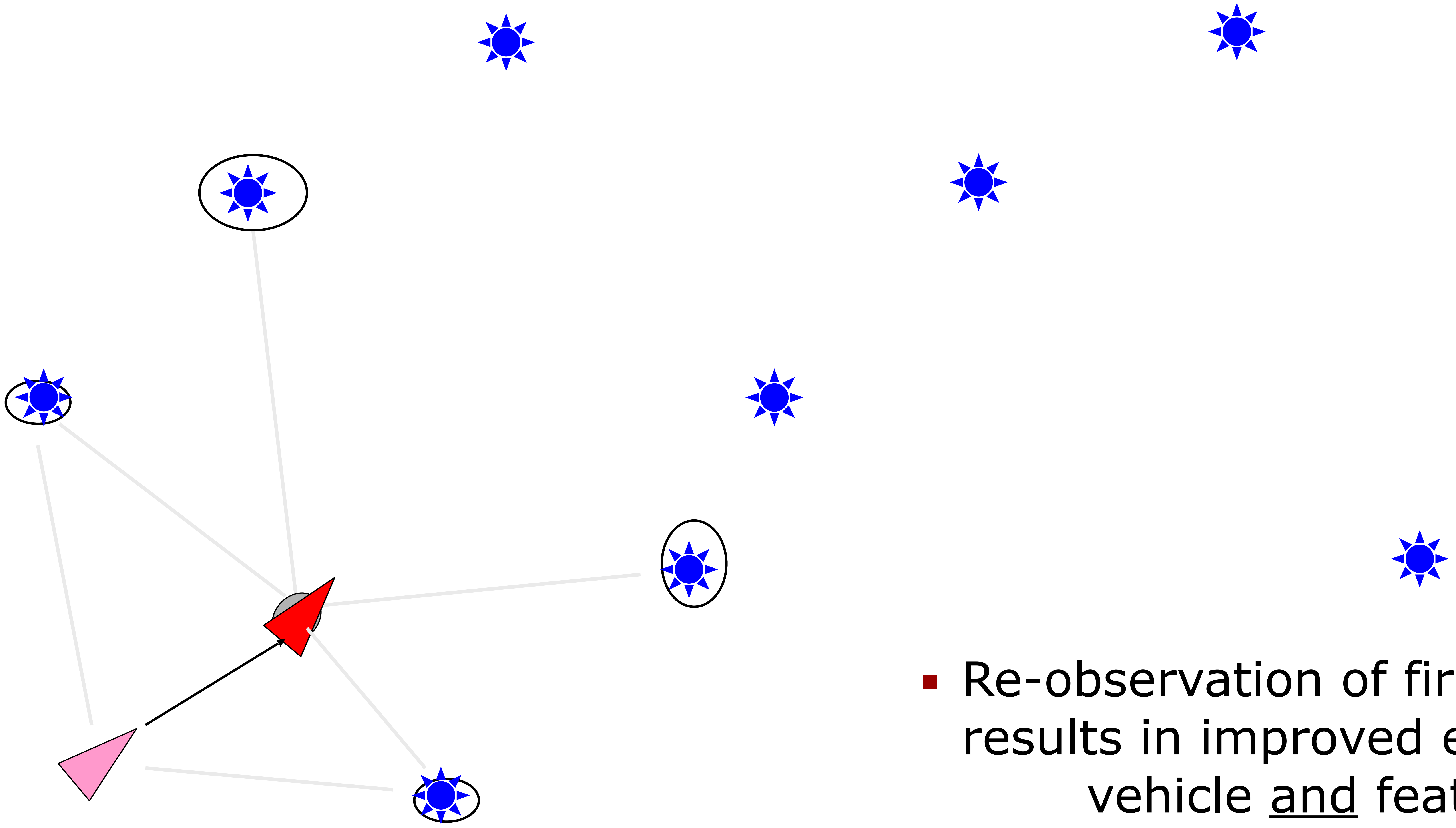
Illustration of SLAM with Landmarks



Courtesy J. Leonard

- Second position: two new features observed

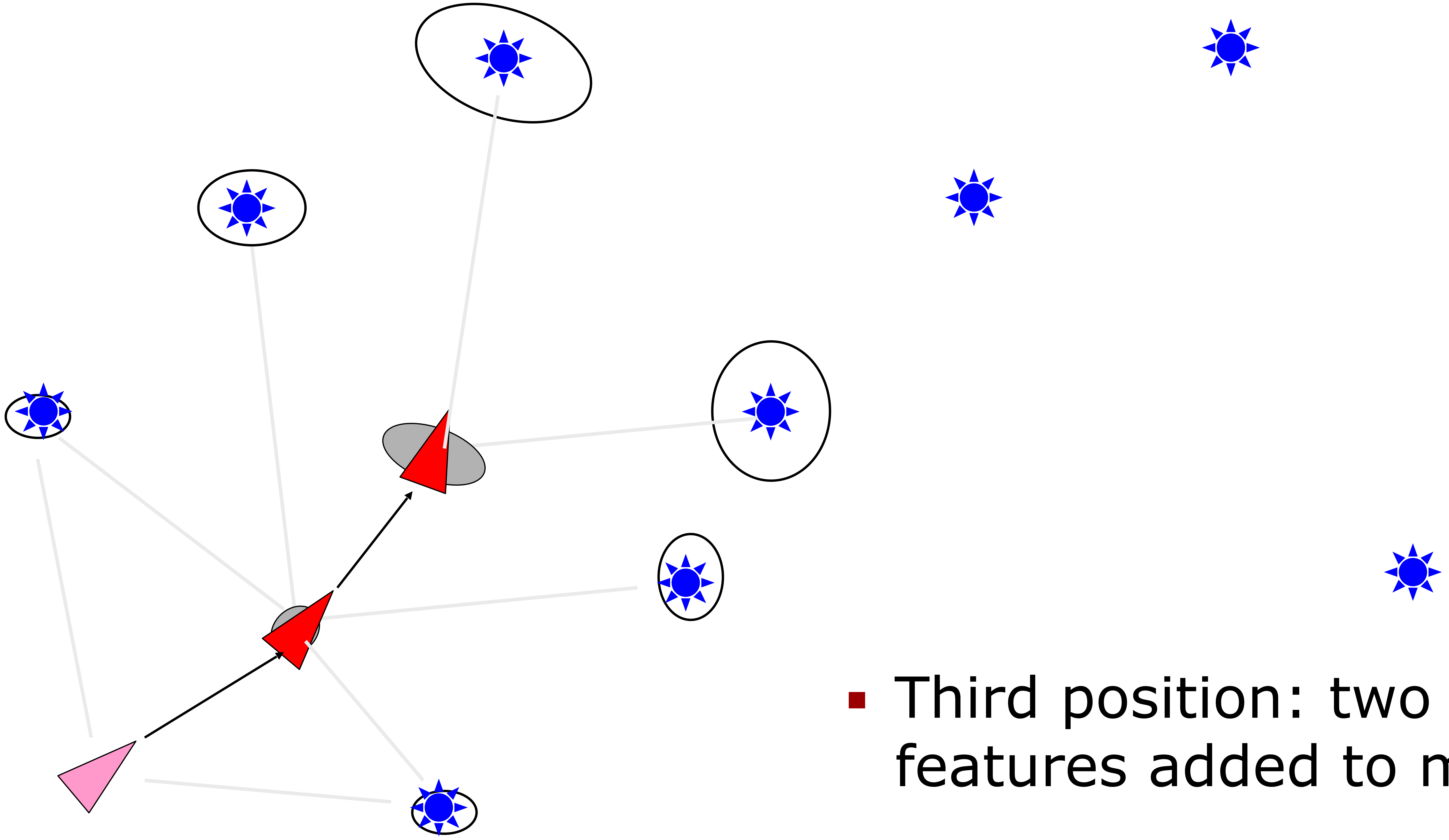
Illustration of SLAM with Landmarks



Courtesy J. Leonard

- Re-observation of first two features results in improved estimates for vehicle and feature

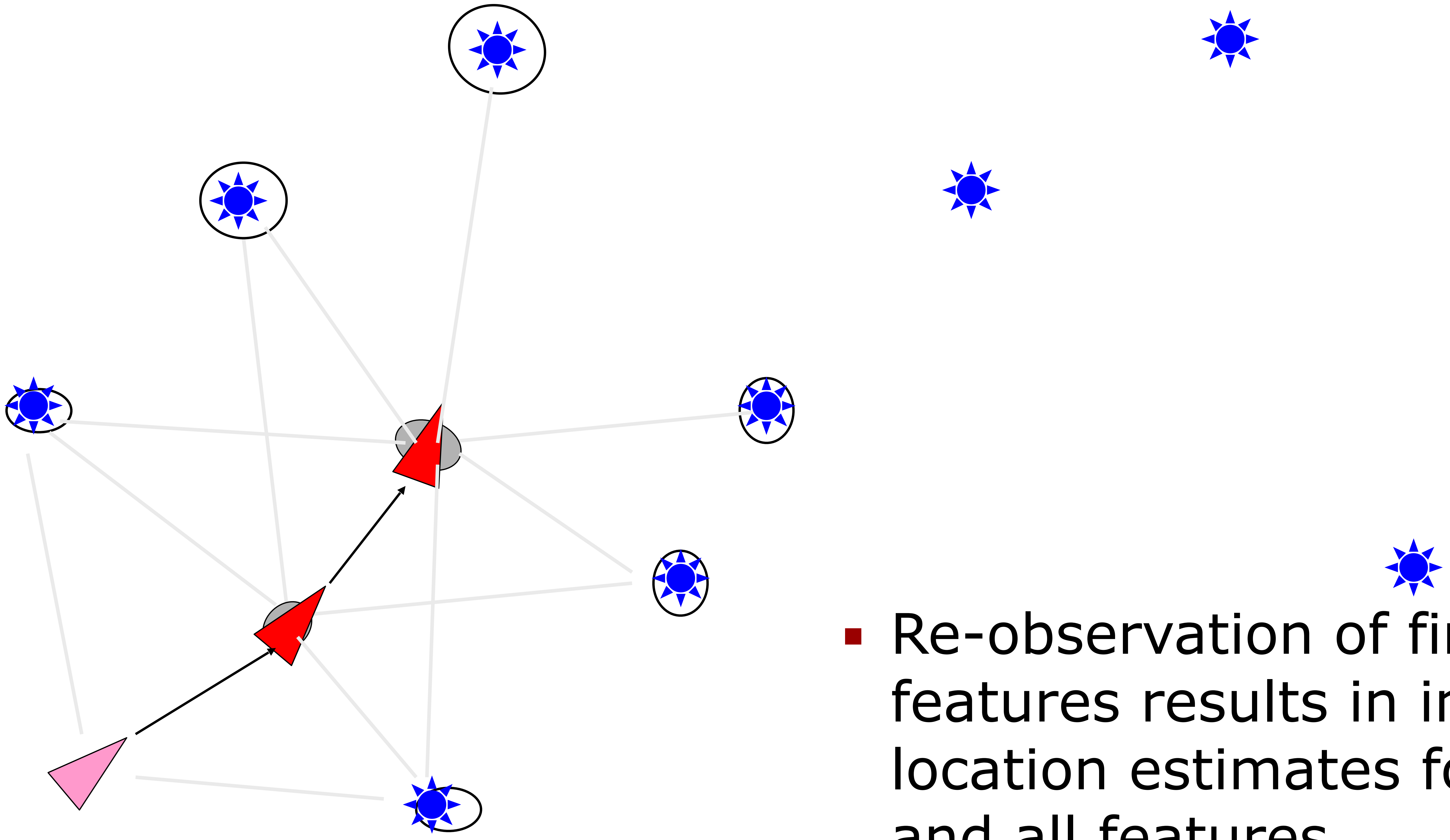
Illustration of SLAM with Landmarks



Courtesy J. Leonard

- Third position: two additional features added to map

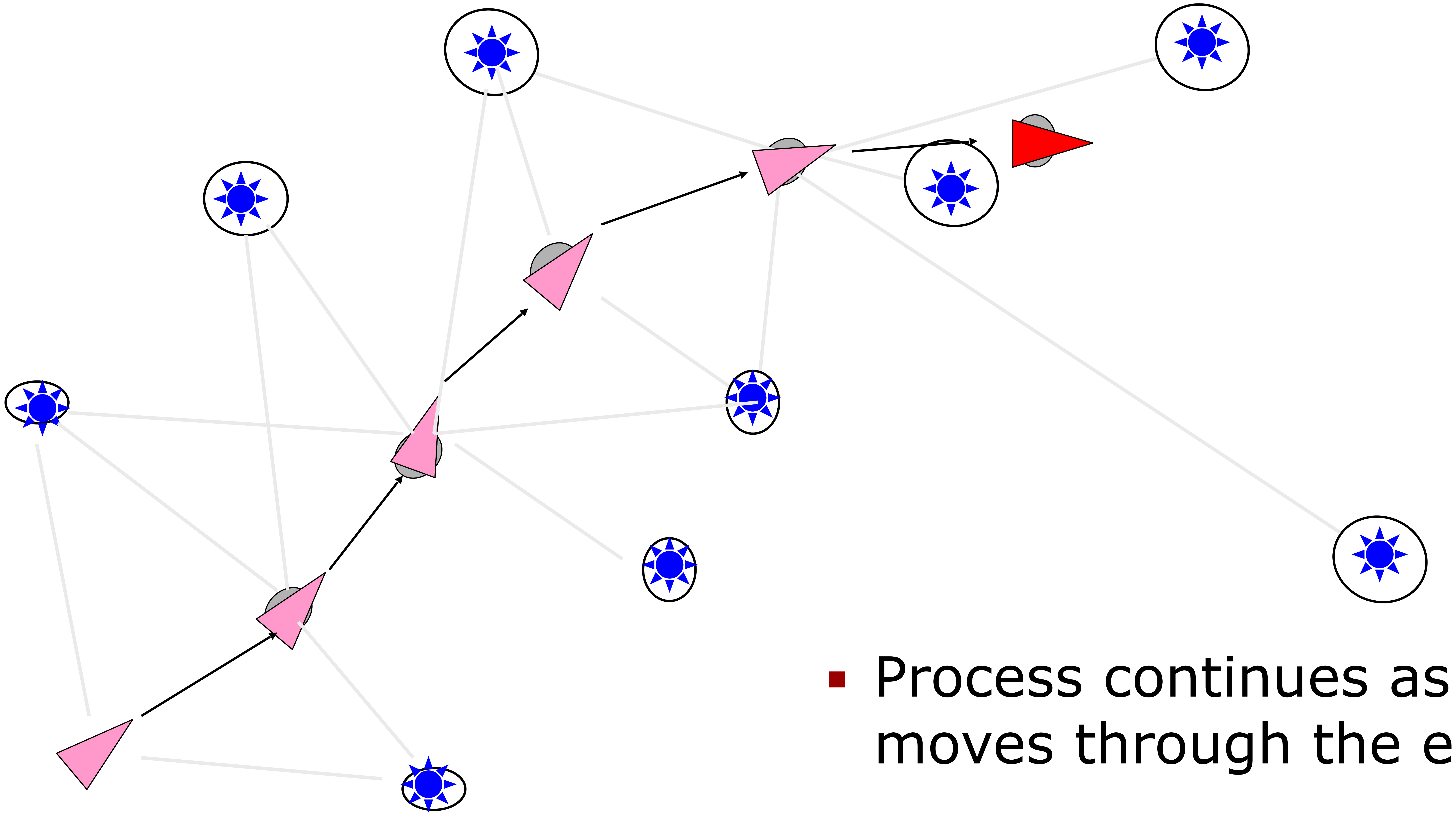
Illustration of SLAM with Landmarks



Courtesy J. Leonard

- Re-observation of first four features results in improved location estimates for vehicle and all features

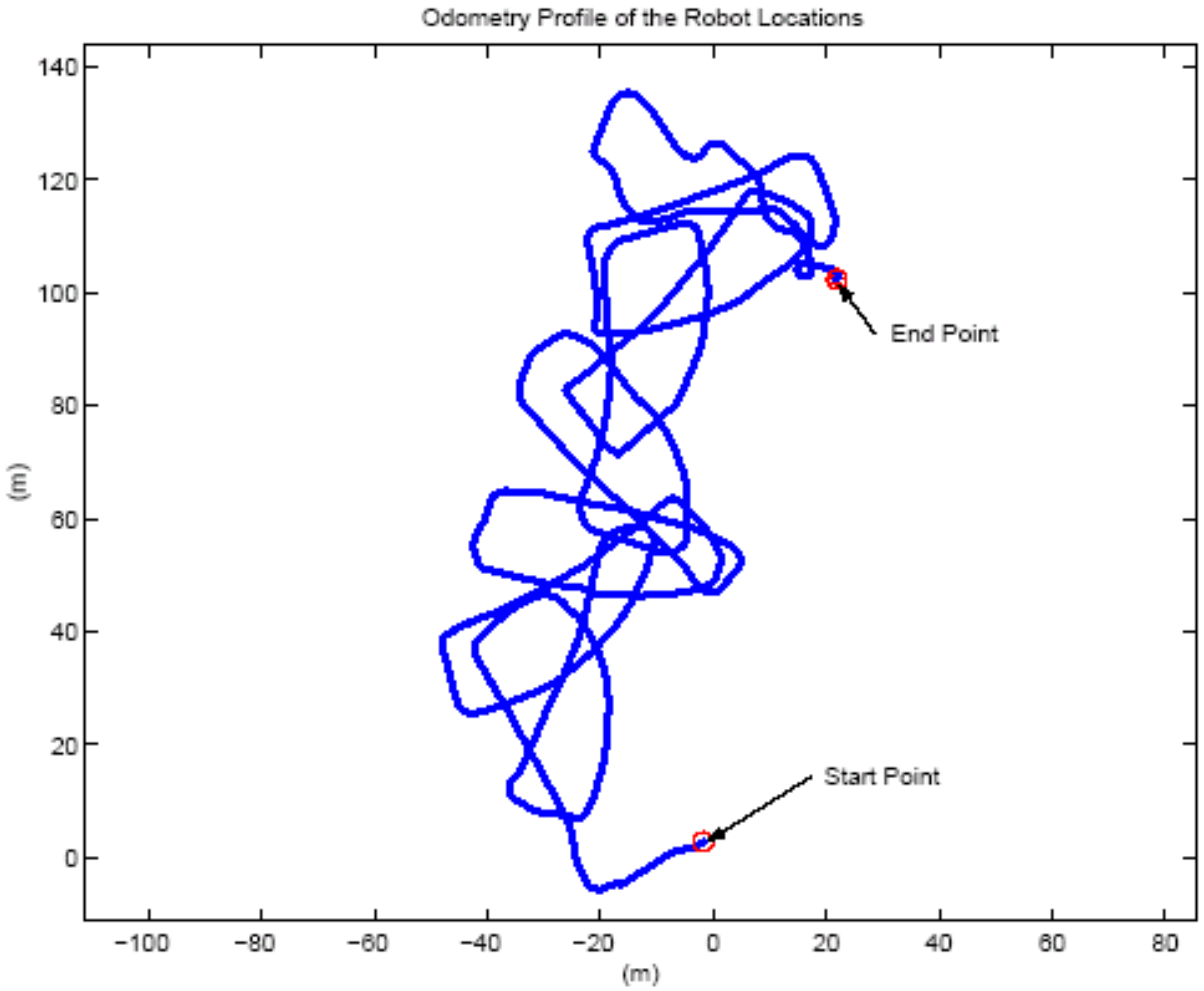
Illustration of SLAM with Landmarks



Courtesy J. Leonard

- Process continues as the vehicle moves through the environment

SLAM Using Landmarks



Courtesy J. Leonard

Test Environment (Point Landmarks)

Courtesy J. Leonard

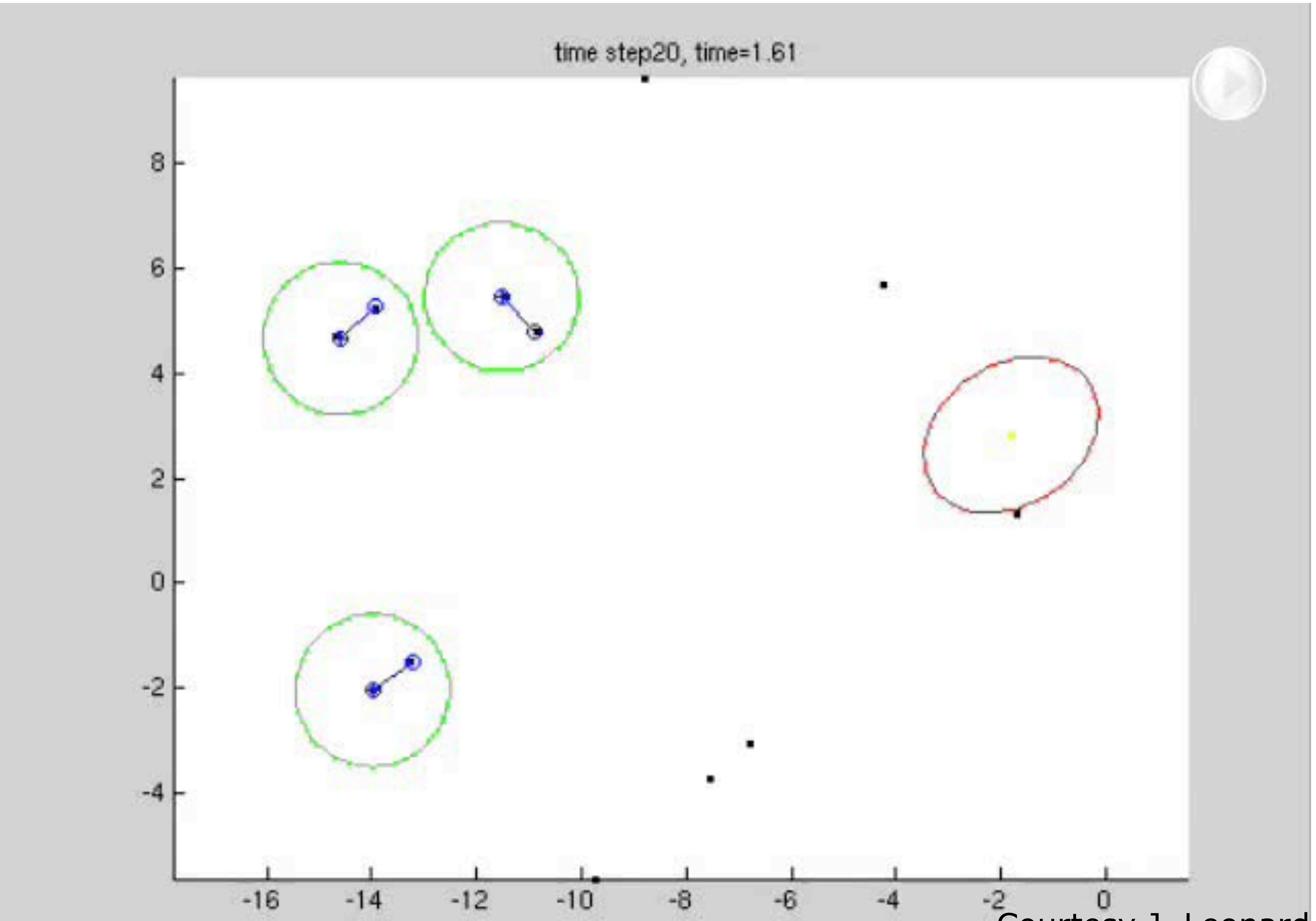
View from Vehicle

Courtesy J. Leonard

SLAM Using Landmarks

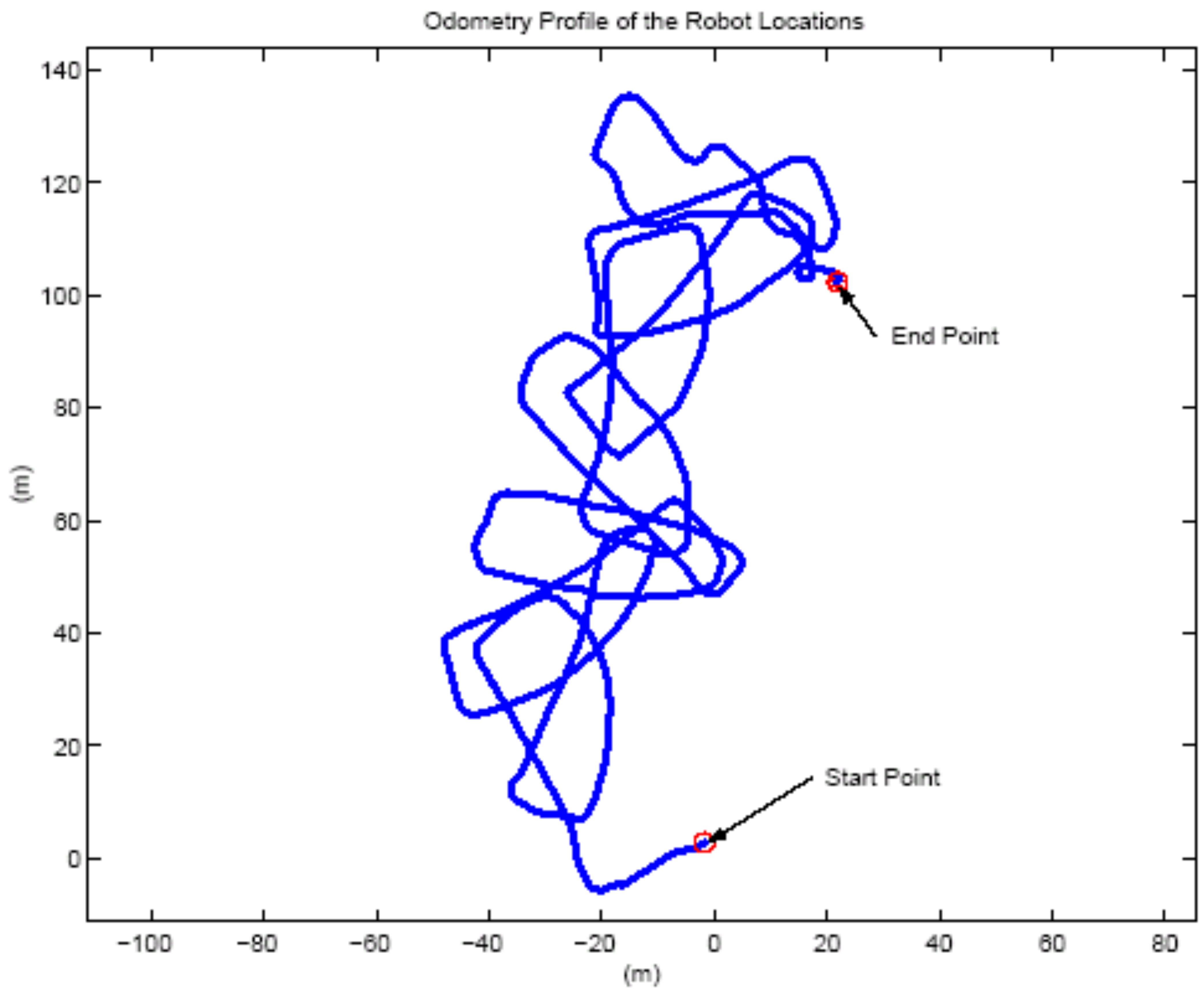
1. Move
2. Sense
3. Associate measurements with known features
4. Update state estimates for robot and previously mapped features
5. Find new features from unassociated measurements
6. Initialize new features
7. Repeat

MIT Indoor Track

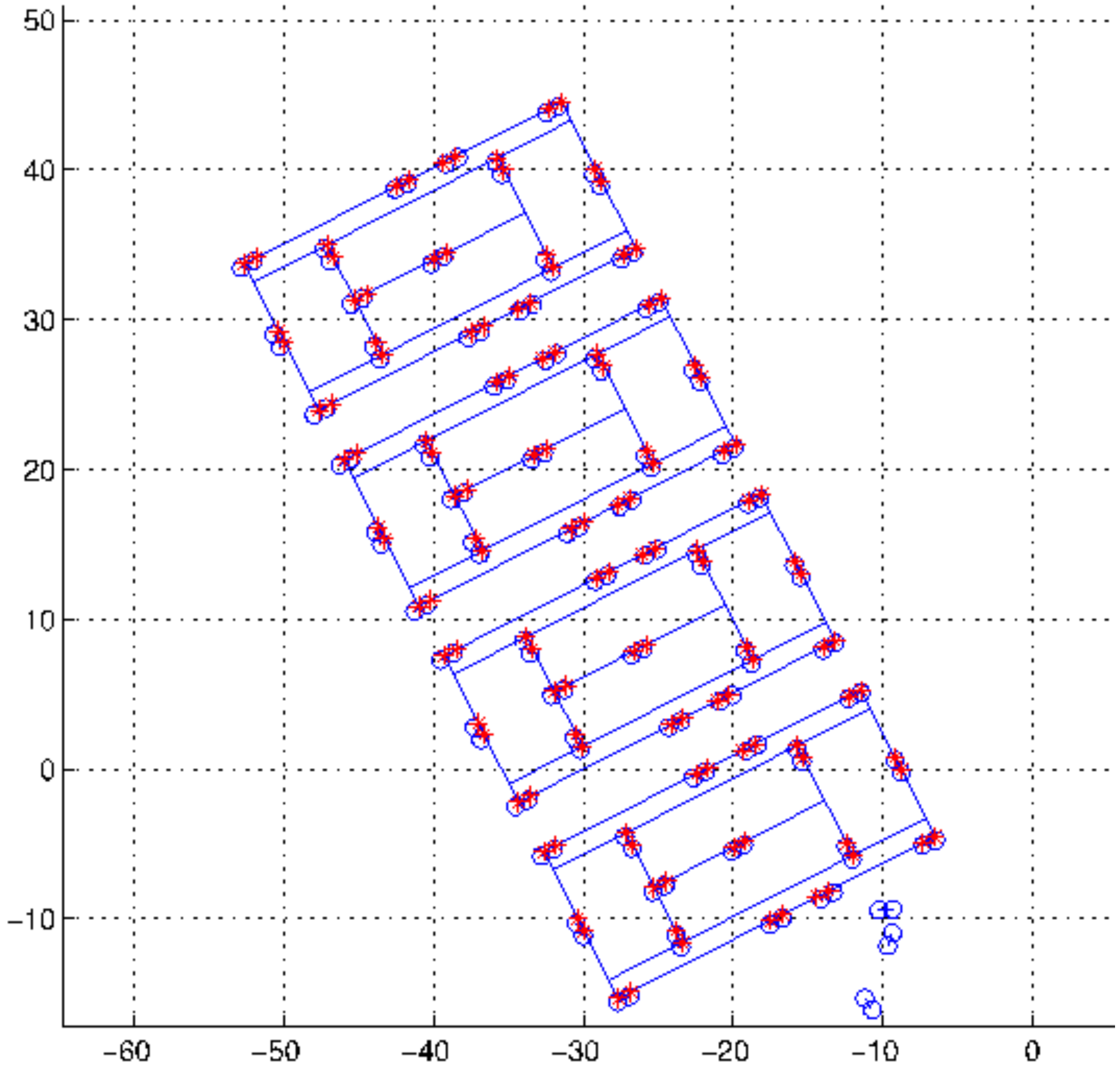


Courtesy J. Leonard

Comparison with Ground Truth



odometry

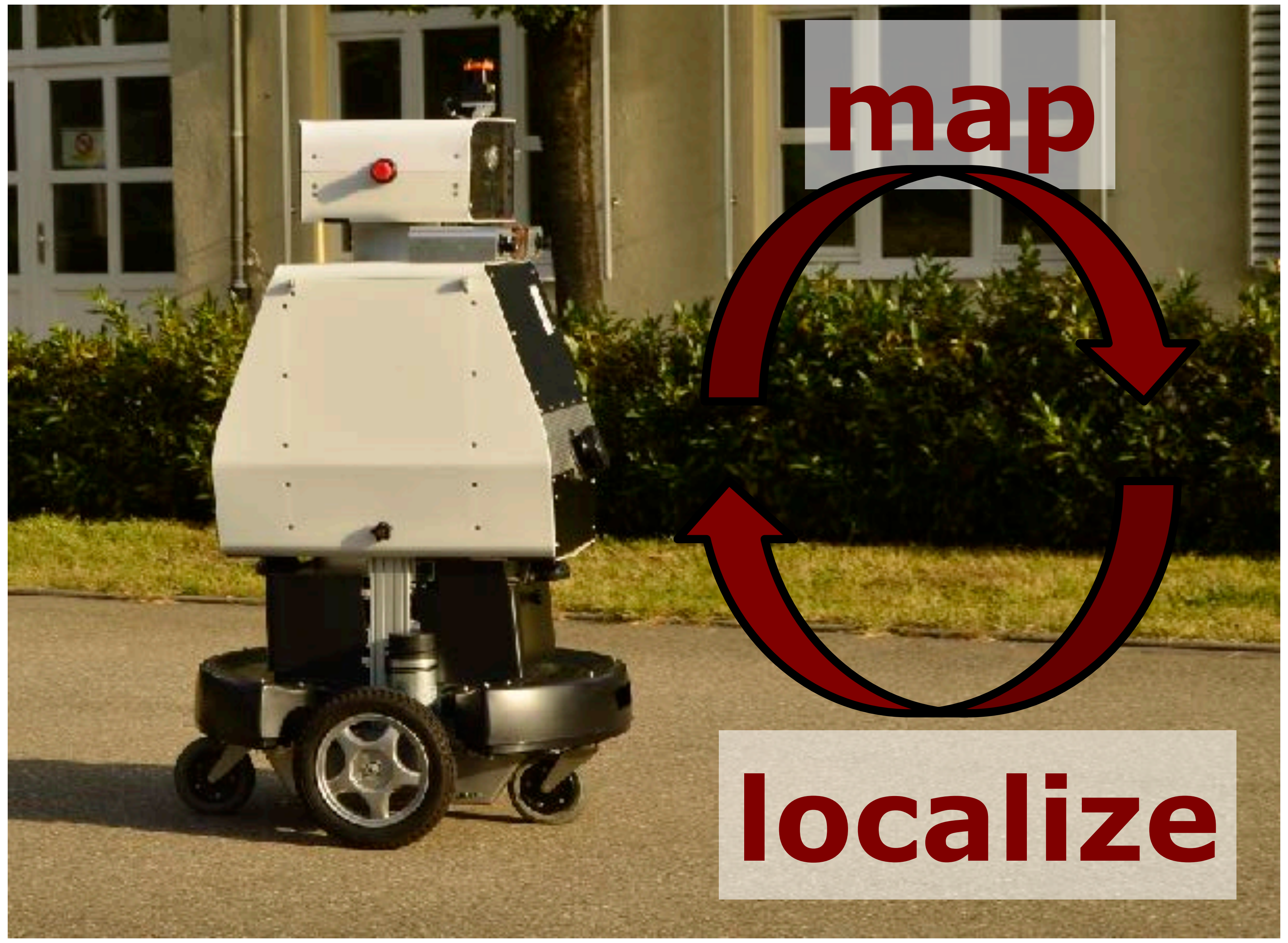


SLAM result

Courtesy J. Leonard

Simultaneous Localization and Mapping (SLAM)

- Building a map and locating the robot in the map at the same time
- Chicken-and-egg problem



Definition of the SLAM Problem

Given

- The robot's controls

$$u_{1:T} = \{u_1, u_2, u_3, \ldots, u_T\}$$

- Observations

$$z_{1:T} = \{z_1, z_2, z_3, \ldots, z_T\}$$

Wanted

- Map of the environment

$$m$$

- Path of the robot

$$x_{0:T} = \{x_0, x_1, x_2, \ldots, x_T\}$$

Three Main Paradigms

Kalman filter

Graph-based

Particle filter

EKF SLAM

- Application of the EKF to SLAM
- Estimate robot's pose and locations of landmarks in the environment
- Assumption: known correspondences **and ??**
- State space (for the 2D plane) is

$$x_t = (\underbrace{x, y, \theta}_{\text{robot's pose}}, \underbrace{m_{1,x}, m_{1,y}}_{\text{landmark 1}}, \ldots, \underbrace{m_{n,x}, m_{n,y}}_{\text{landmark n}})^T$$

EKF SLAM: State Representation

- Map with n landmarks: (3+2n)-dimensional Gaussian
- Belief is represented by

$$\underbrace{\begin{pmatrix} x \\ y \\ \theta \\ m_{1,x} \\ m_{1,y} \\ \vdots \\ m_{n,x} \\ m_{n,y} \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xm_{1,x}} & \sigma_{xm_{1,y}} & \cdots & \sigma_{xm_{n,x}} & \sigma_{xm_{n,y}} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} & \sigma_{ym_{1,x}} & \sigma_{ym_{1,y}} & \cdots & \sigma_{m_{n,x}} & \sigma_{m_{n,y}} \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} & \sigma_{\theta m_{1,x}} & \sigma_{\theta m_{1,y}} & \cdots & \sigma_{\theta m_{n,x}} & \sigma_{\theta m_{n,y}} \\ \sigma_{m_{1,x}x} & \sigma_{m_{1,x}y} & \sigma_{\theta} & \sigma_{m_{1,x}m_{1,x}} & \sigma_{m_{1,x}m_{1,y}} & \cdots & \sigma_{m_{1,x}m_{n,x}} & \sigma_{m_{1,x}m_{n,y}} \\ \sigma_{m_{1,y}x} & \sigma_{m_{1,y}y} & \sigma_{\theta} & \sigma_{m_{1,y}m_{1,x}} & \sigma_{m_{1,y}m_{1,y}} & \cdots & \sigma_{m_{1,y}m_{n,x}} & \sigma_{m_{1,y}m_{n,y}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{m_{n,x}x} & \sigma_{m_{n,x}y} & \sigma_{\theta} & \sigma_{m_{n,x}m_{1,x}} & \sigma_{m_{n,x}m_{1,y}} & \cdots & \sigma_{m_{n,x}m_{n,x}} & \sigma_{m_{n,x}m_{n,y}} \\ \sigma_{m_{n,y}x} & \sigma_{m_{n,y}y} & \sigma_{\theta} & \sigma_{m_{n,y}m_{1,x}} & \sigma_{m_{n,y}m_{1,y}} & \cdots & \sigma_{m_{n,y}m_{n,x}} & \sigma_{m_{n,y}m_{n,y}} \end{pmatrix}}_{\Sigma}$$

EKF SLAM: State Representation

- More compactly

$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

EKF SLAM: State Representation

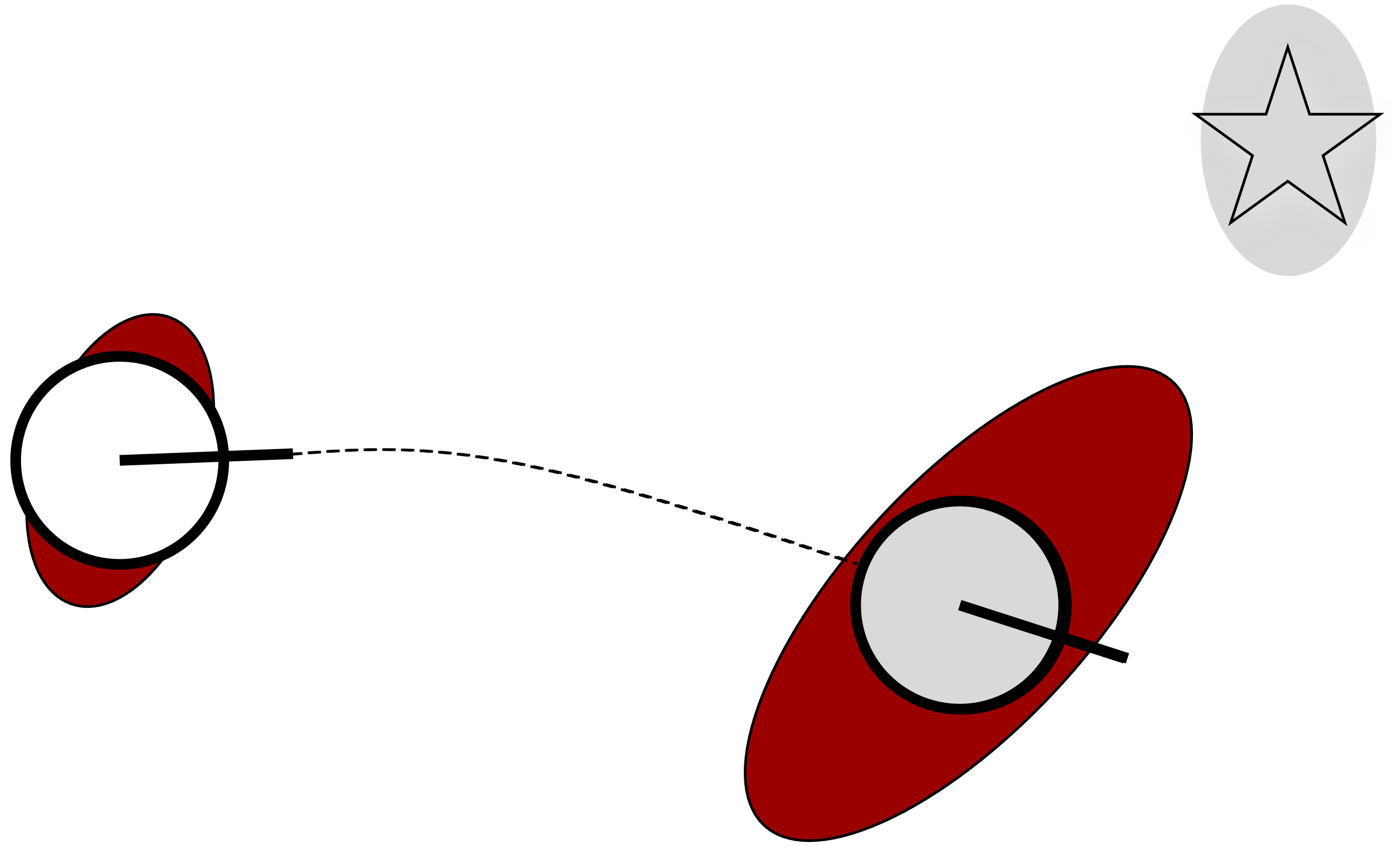
- Even more compactly (note: $x_R \rightarrow x$)

$$\underbrace{\begin{pmatrix} x \\ m \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{xx} & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_{mm} \end{pmatrix}}_{\Sigma}$$

EKF SLAM: Filter Cycle

1. State prediction
2. Measurement prediction
3. Measurement
4. Data association
5. Update

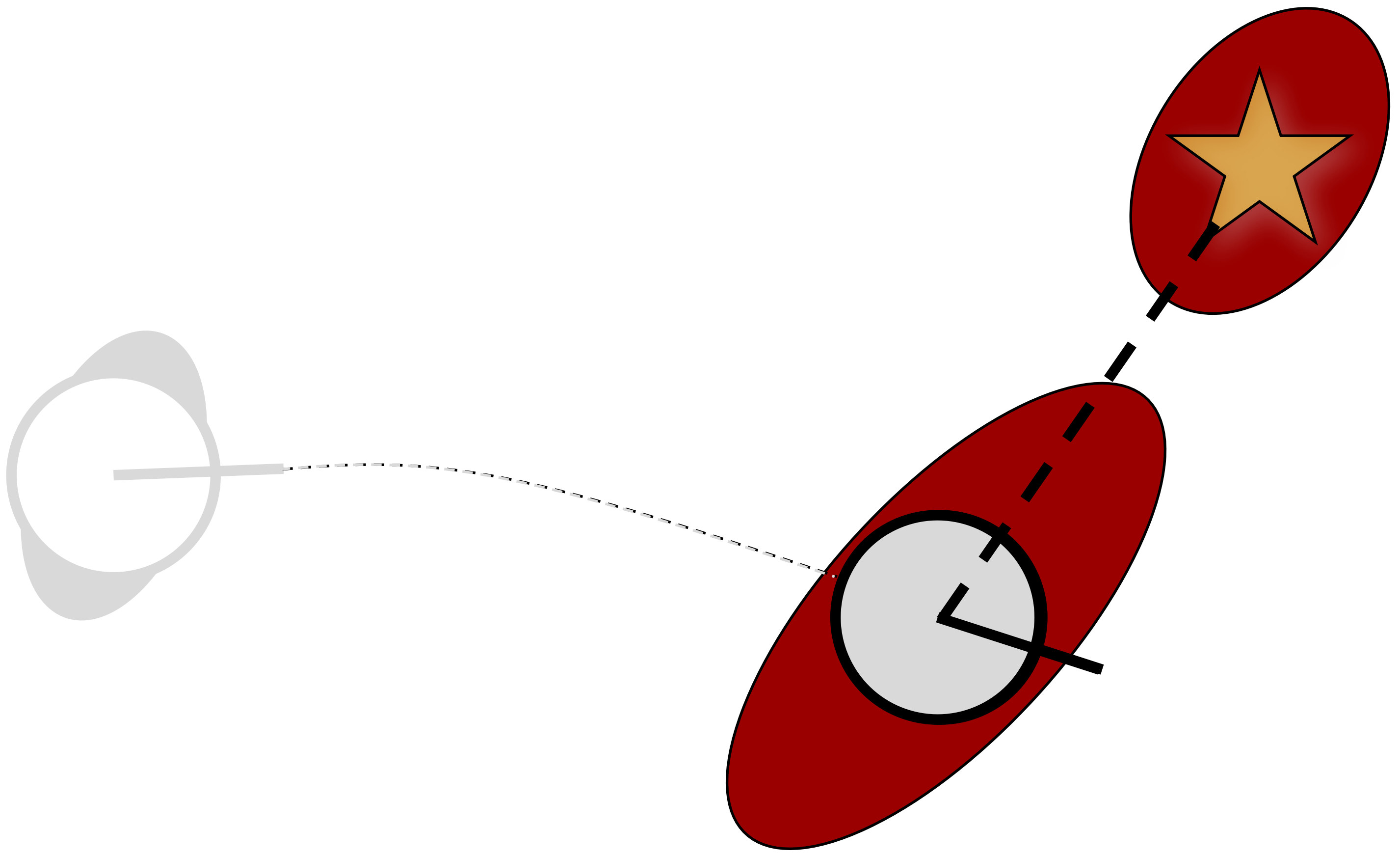
EKF SLAM: State Prediction



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

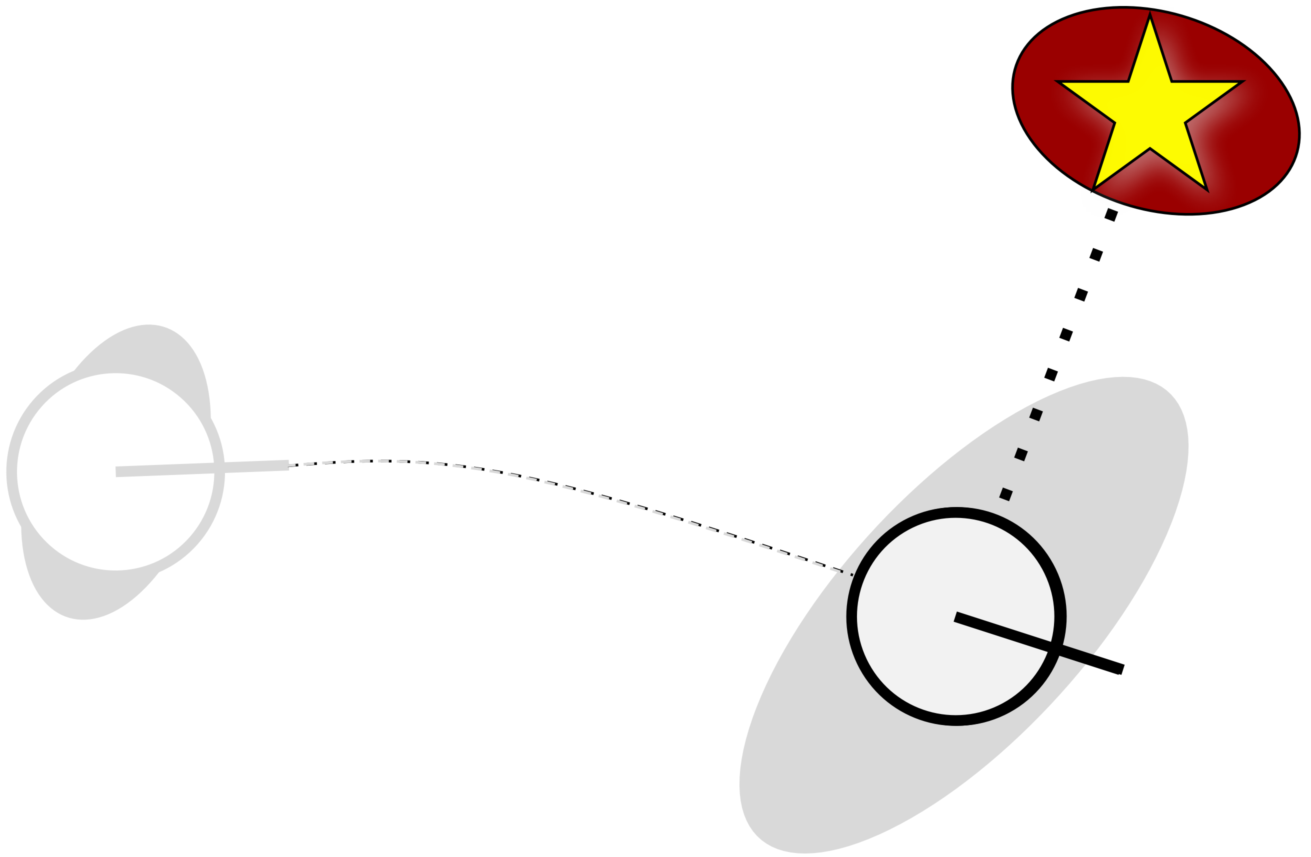
EKF SLAM: Measurement Prediction



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

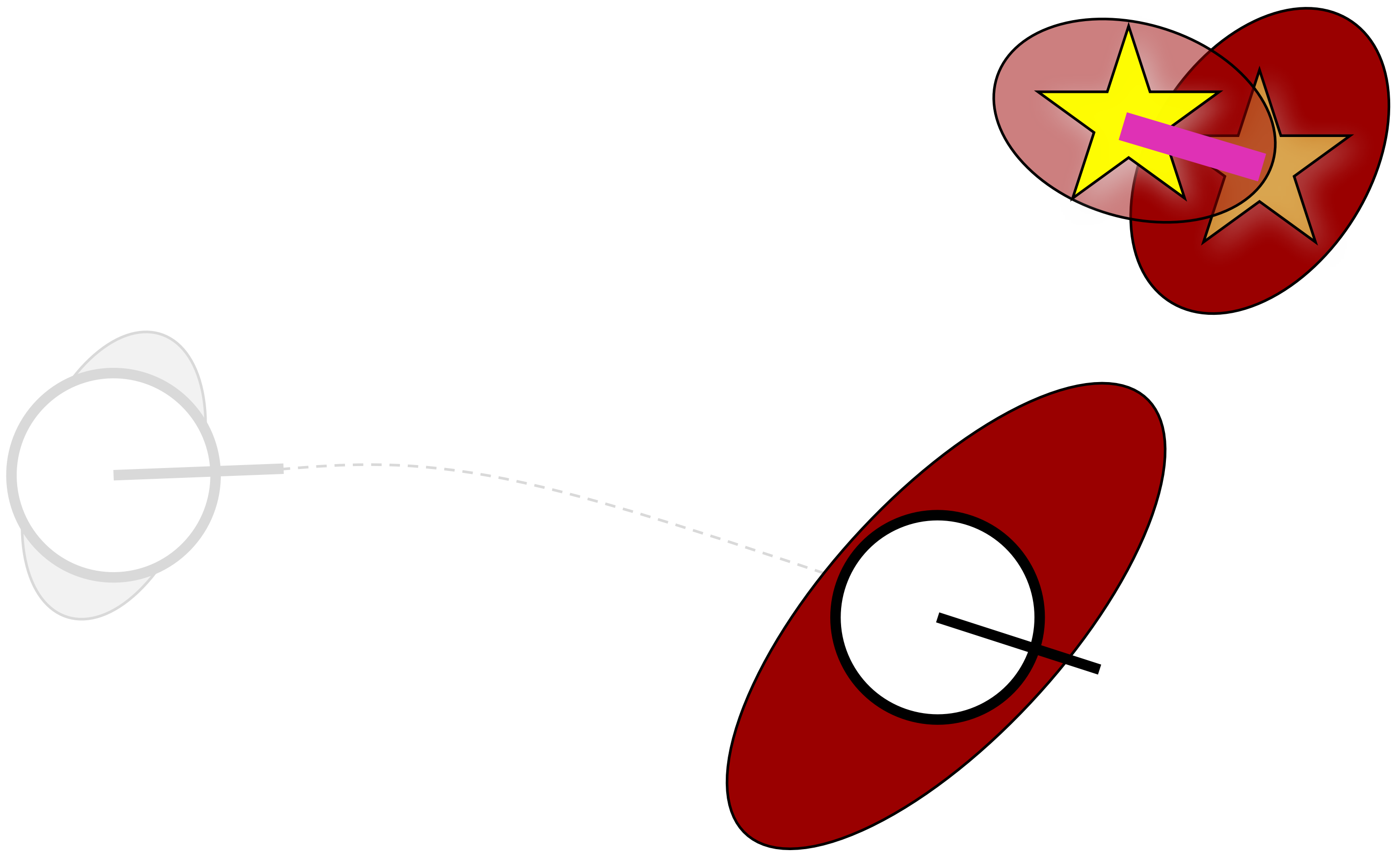
EKF SLAM: Obtained Measurement



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

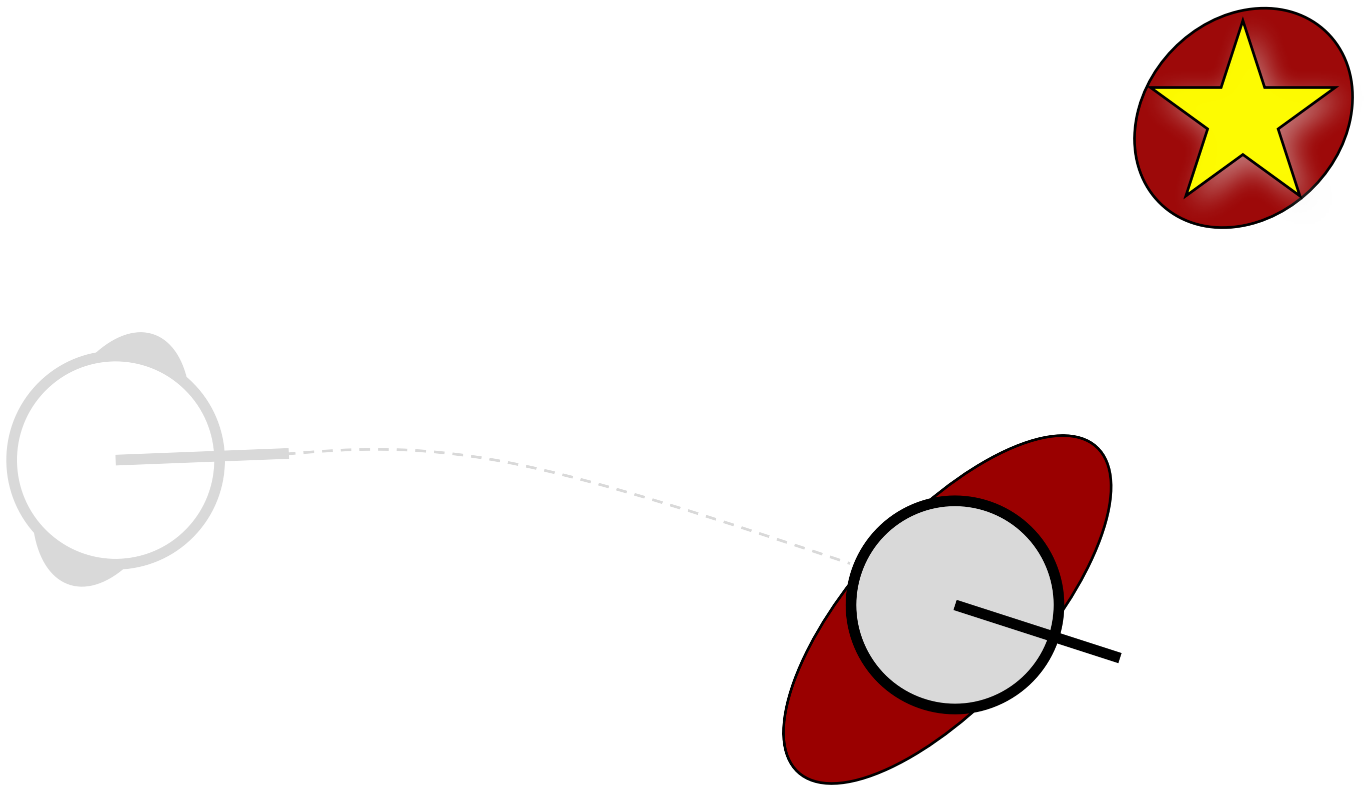
EKF SLAM: Data Association and Difference Between h(x) and z



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

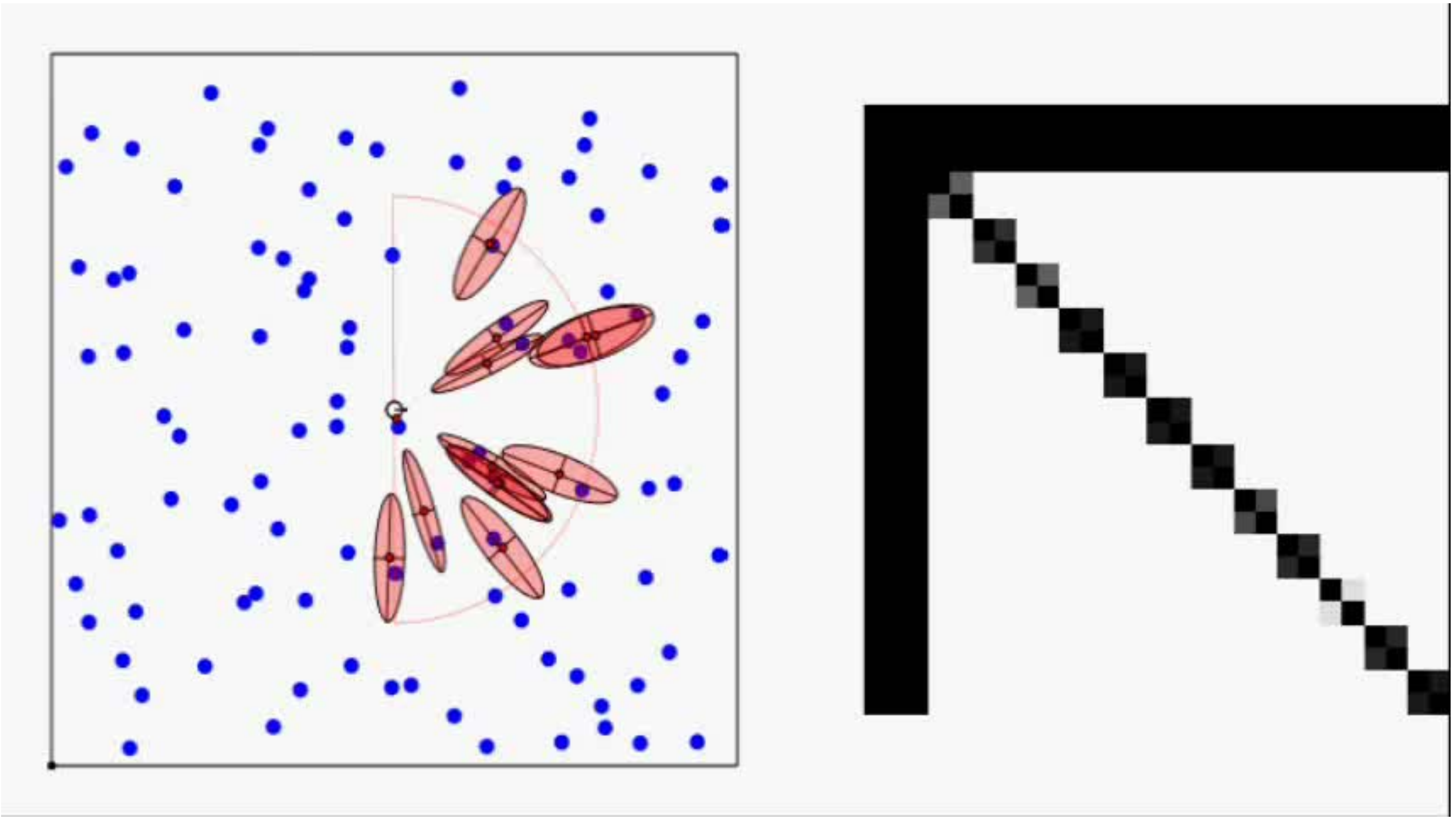
EKF SLAM: Update Step



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

EKF SLAM Correlations

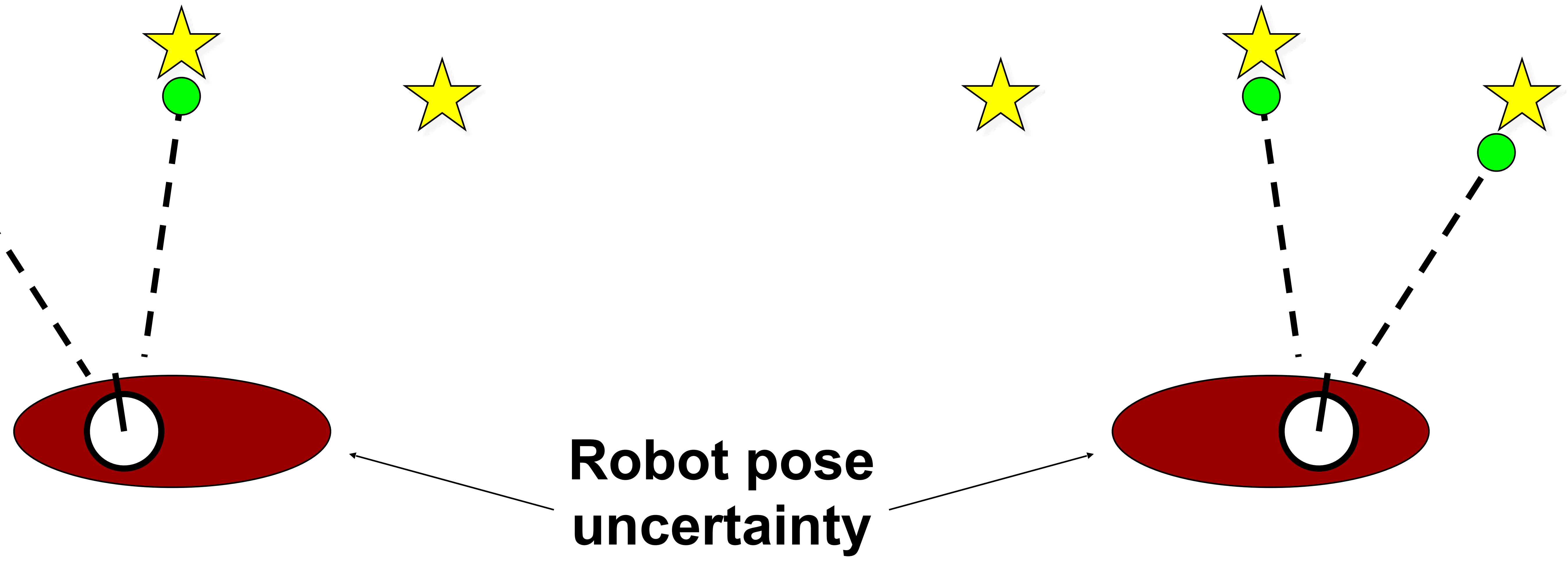


Blue path = true path **Red path** = estimated path **Black path** = odometry

- Approximate the SLAM posterior with a high-dimensional Gaussian [Smith & Cheesman, 1986] …
- Single hypothesis data association

Courtesy: M. Montemerlo

Data Association in SLAM



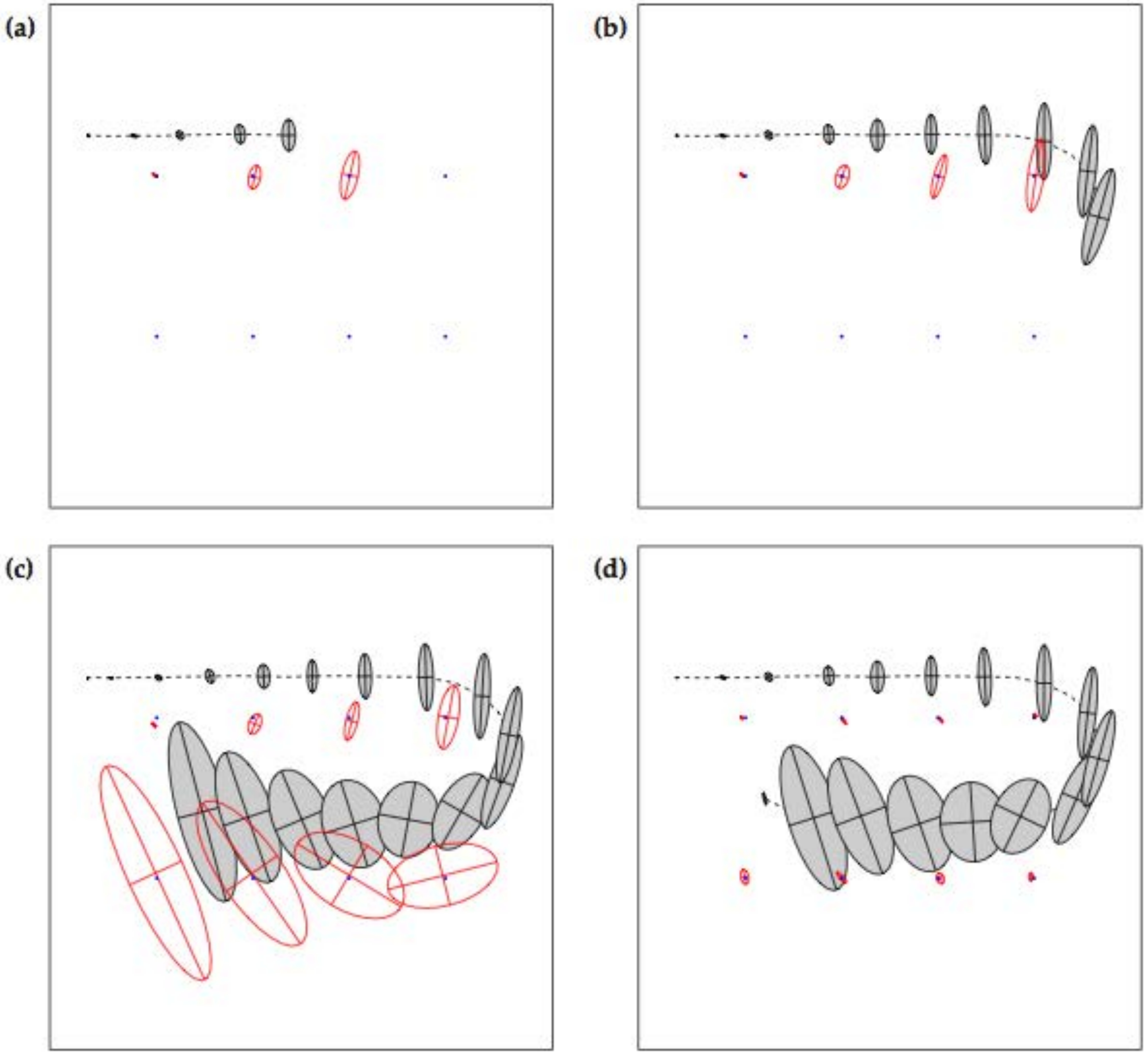
- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
  - EKF SLAM is brittle in this regard
- Pose error correlates data associations

Slide borrowed from Dieter Fox

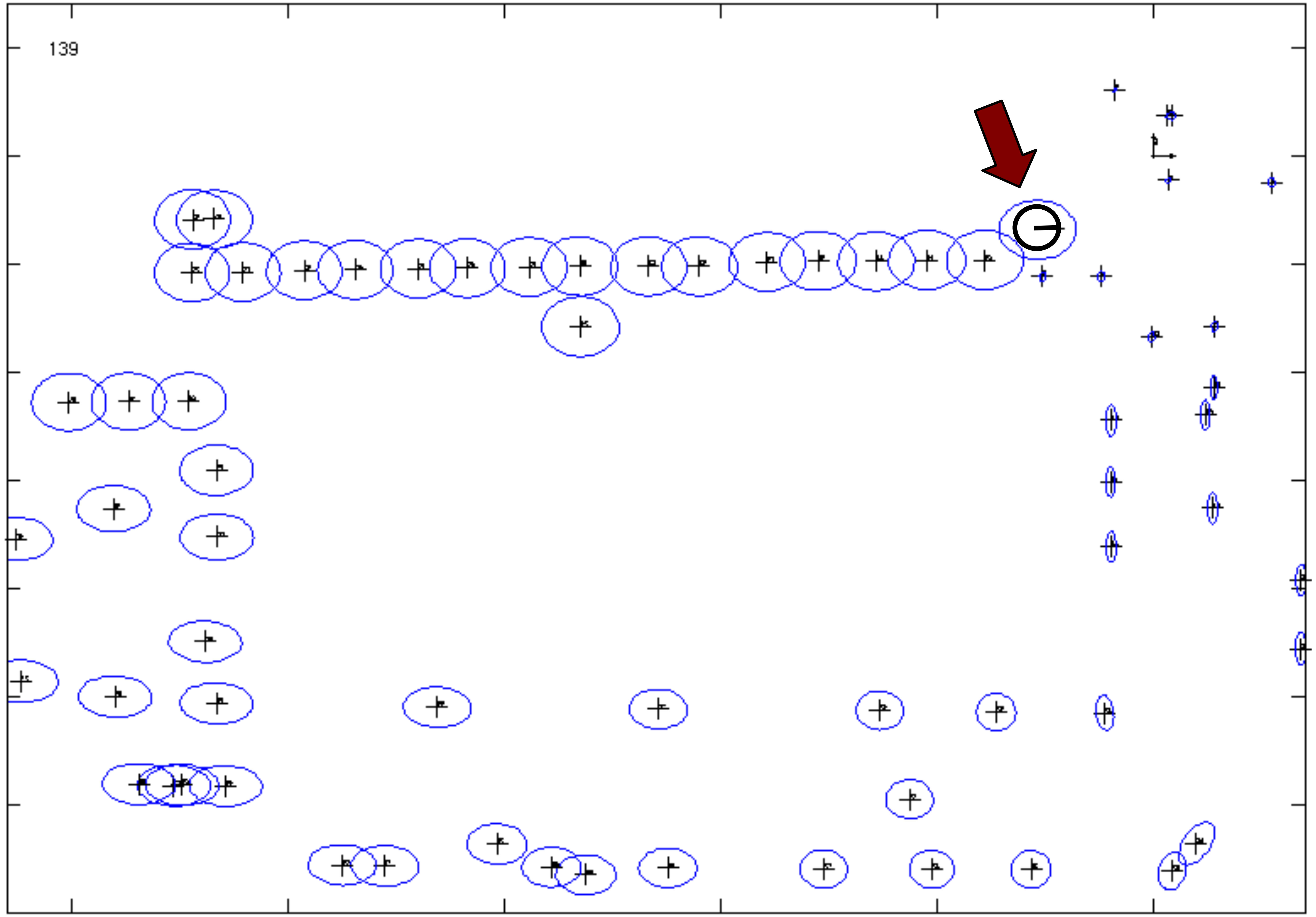
Loop-Closing

- Loop-closing means recognizing an already mapped area
- Data association under
  - high ambiguity
  - possible environment symmetries
- Uncertainties **collapse** after a loop-closure (whether the closure was correct or not)

Online SLAM Example

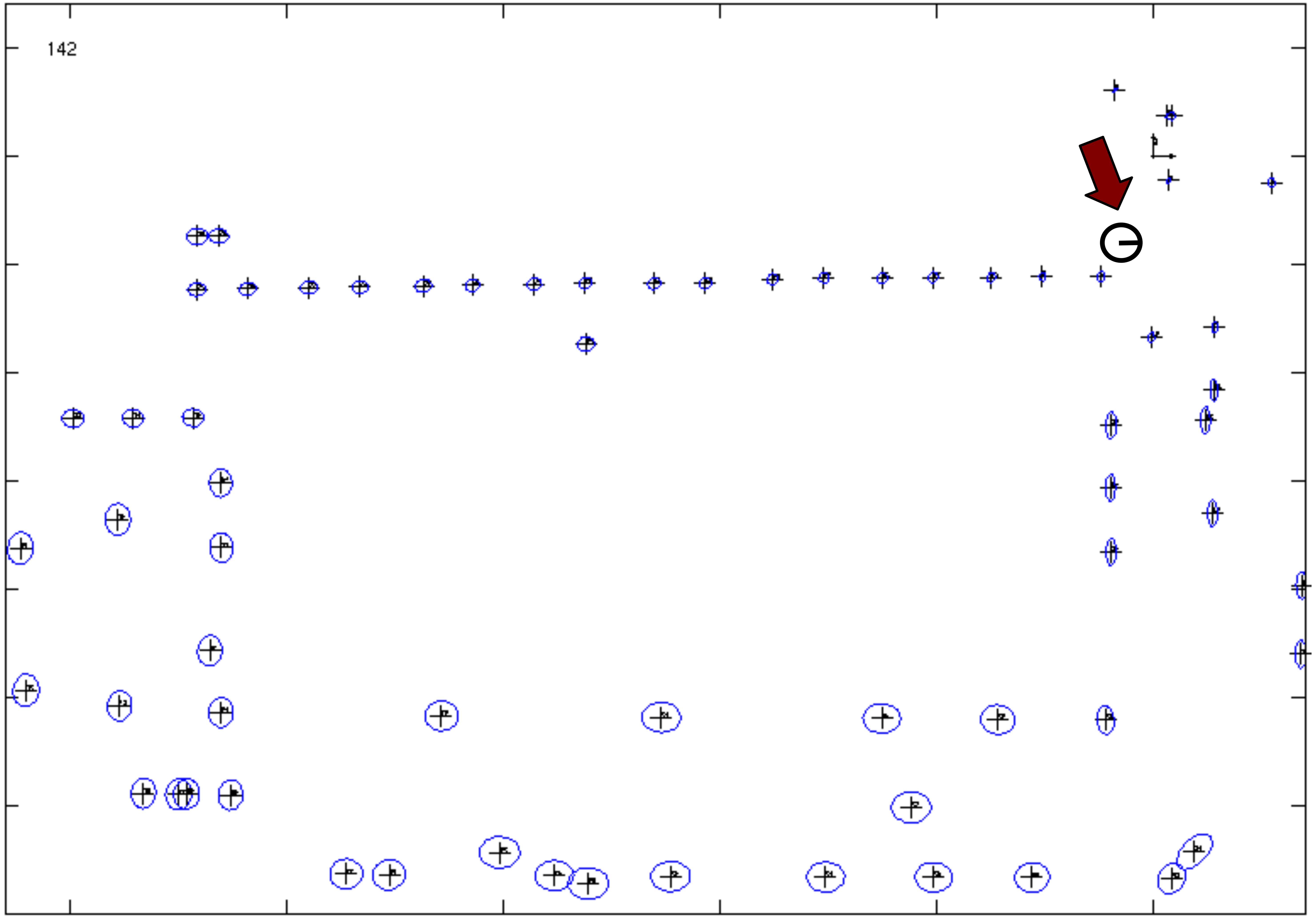


Before the Loop-Closure



Slide borrowed from Dieter Fox

After the Loop-Closure



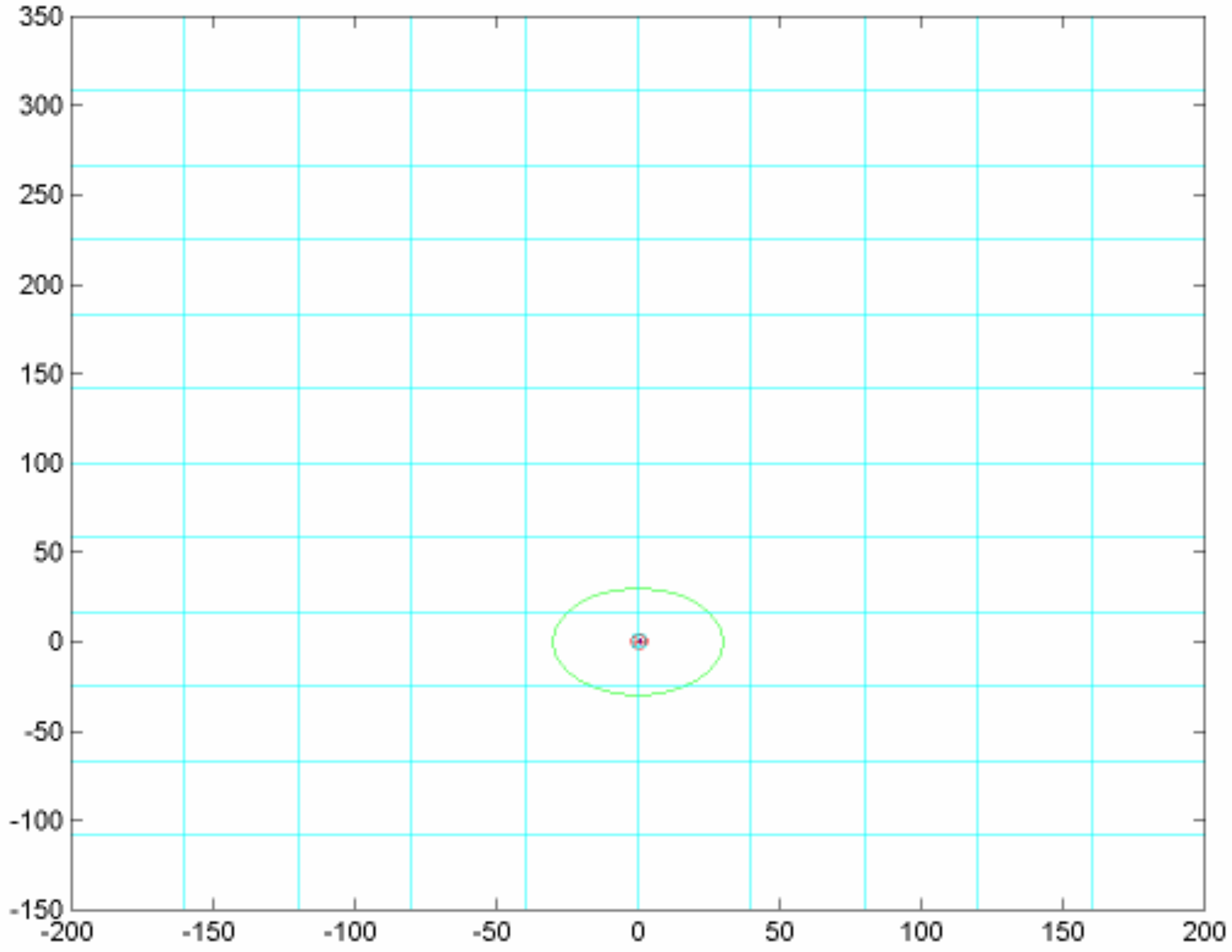
Example: Victoria Park Dataset

Courtesy: E. Nebot

Victoria Park: Data Acquisition

Courtesy: E. Nebot

Victoria Park: EKF Estimate



Courtesy: E. Nebot

Victoria Park: Landmarks

Courtesy: E. Nebot

Andrew Davison: MonoSLAM

EKF SLAM Summary

- Quadratic in the number of landmarks: $O(n^2)$
- Convergence results for the linear case.
- Can diverge if nonlinearities are large!
- Have been applied successfully in large-scale environments.
- Approximations reduce the computational complexity.

EKF Algorithm

1. **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

2. Prediction:
3. $\bar{\mu}_t = g(u_t, \mu_{t-1})$ $\longleftarrow$ $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$ $\longleftarrow$ $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

5. Correction:
6. $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$ $\longleftarrow$ $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$ $\longleftarrow$ $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8. $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$ $\longleftarrow$ $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

$$H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t} \qquad G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}$$

Literature

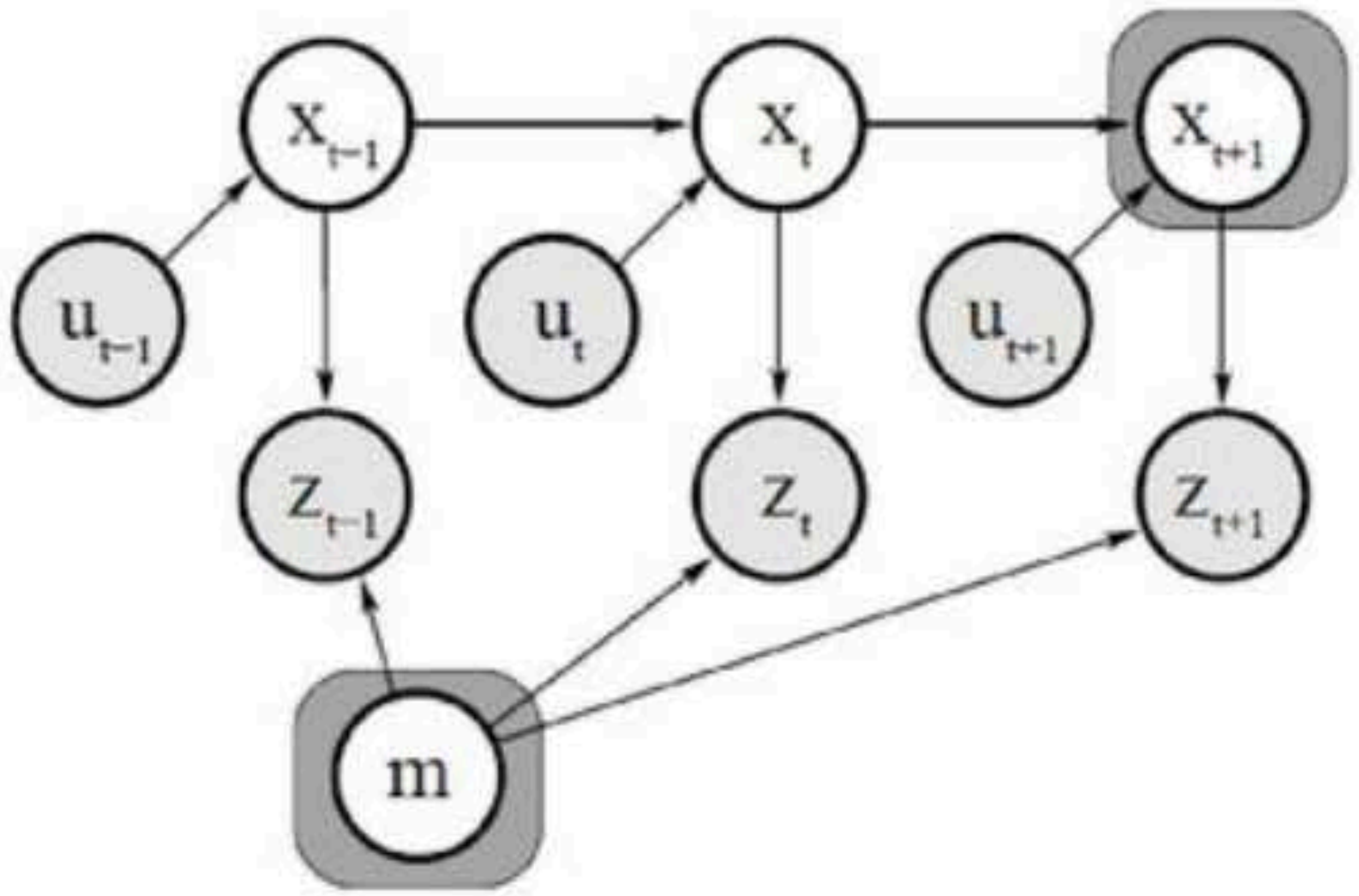
EKF SLAM

- “Probabilistic Robotics”, Chapter 10
- Smith, Self, & Cheeseman: “Estimating Uncertain Spatial Relationships in Robotics”
- Dissanayake et al.: “A Solution to the Simultaneous Localization and Map Building (SLAM) Problem”
- Durrant-Whyte & Bailey: “SLAM Part 1” and “SLAM Part 2” tutorials

Online vs Full SLAM

Online SLAM problem

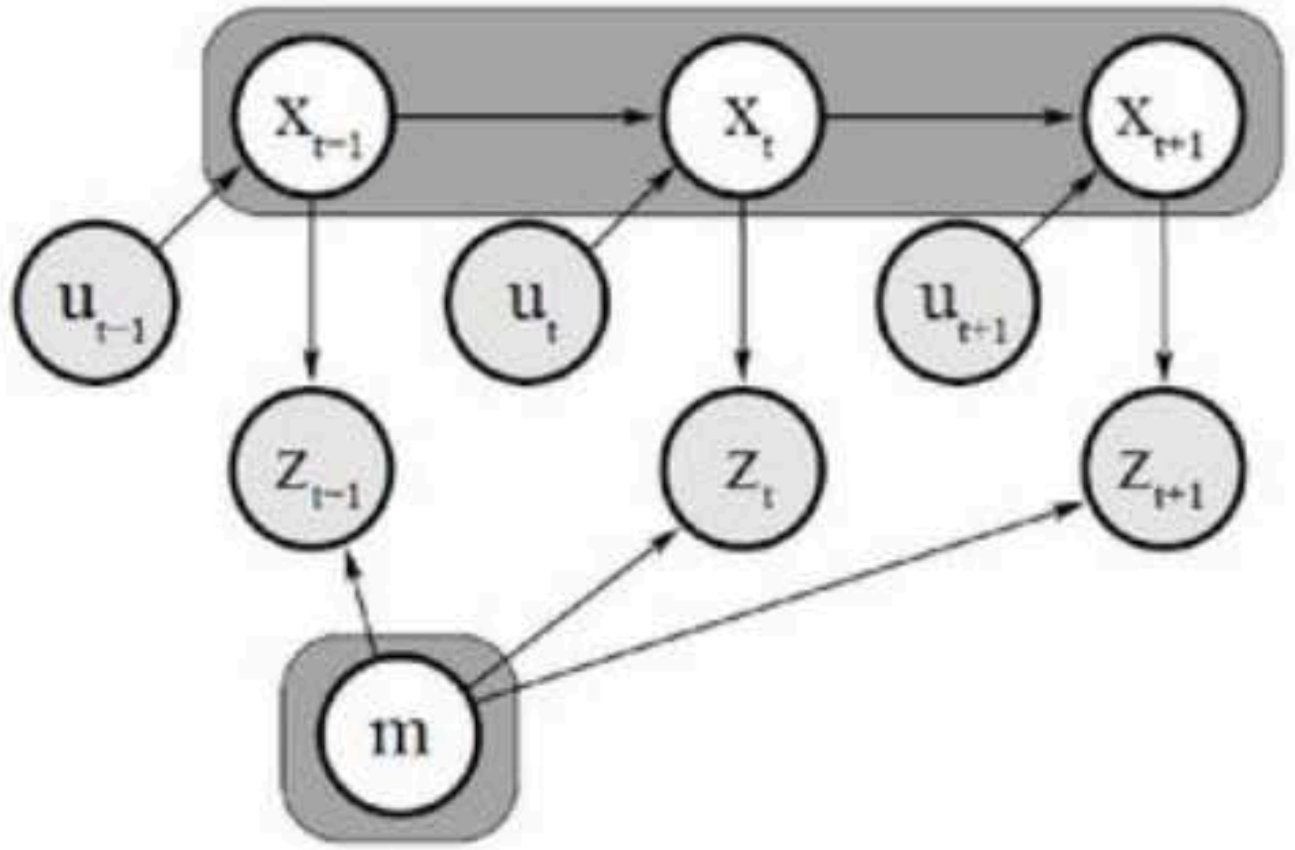
$$p(x_t, m \mid z_{1:t}, u_{1:t})$$



Estimate map *m* and current position x_t

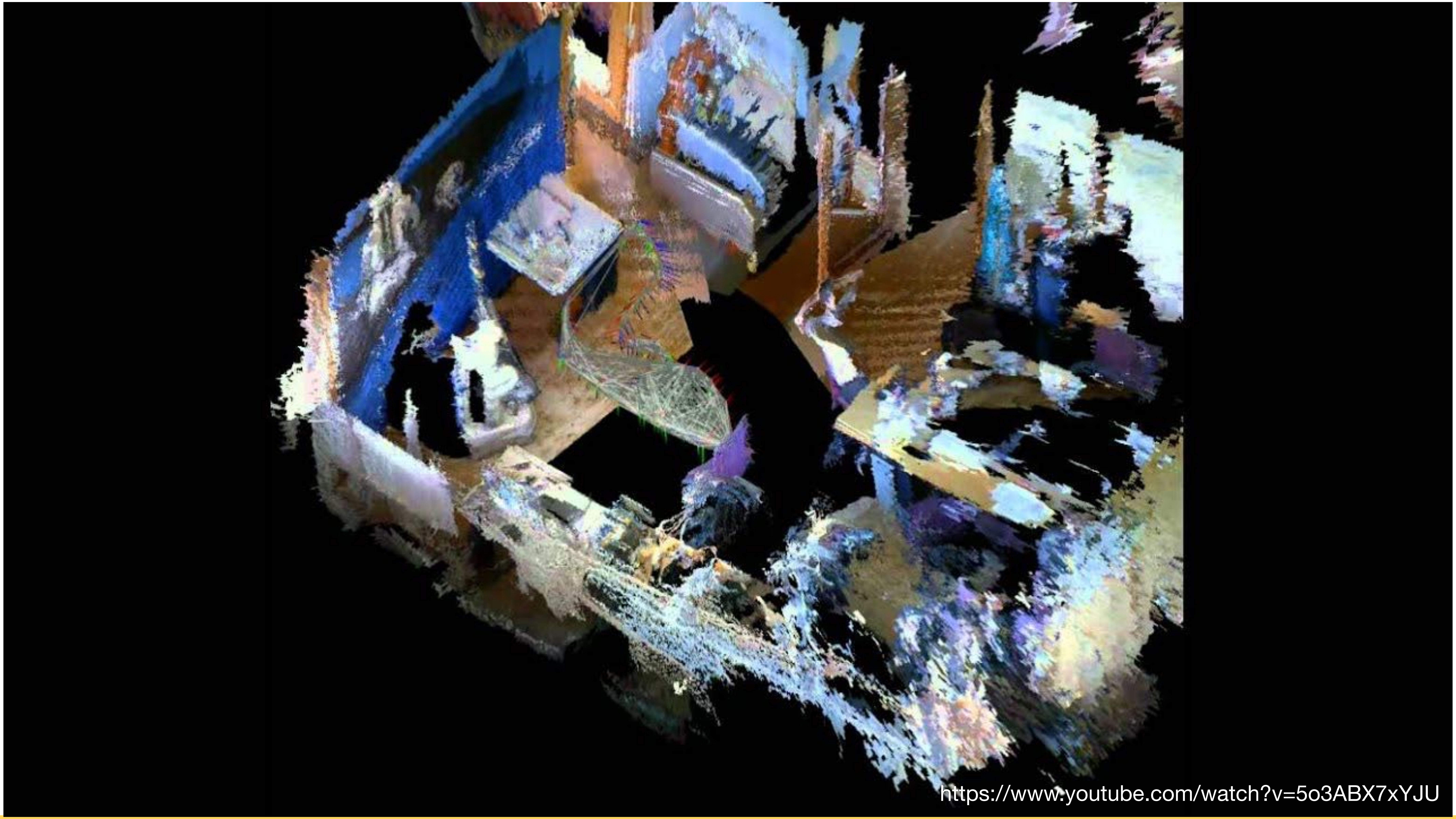
Full SLAM problem

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

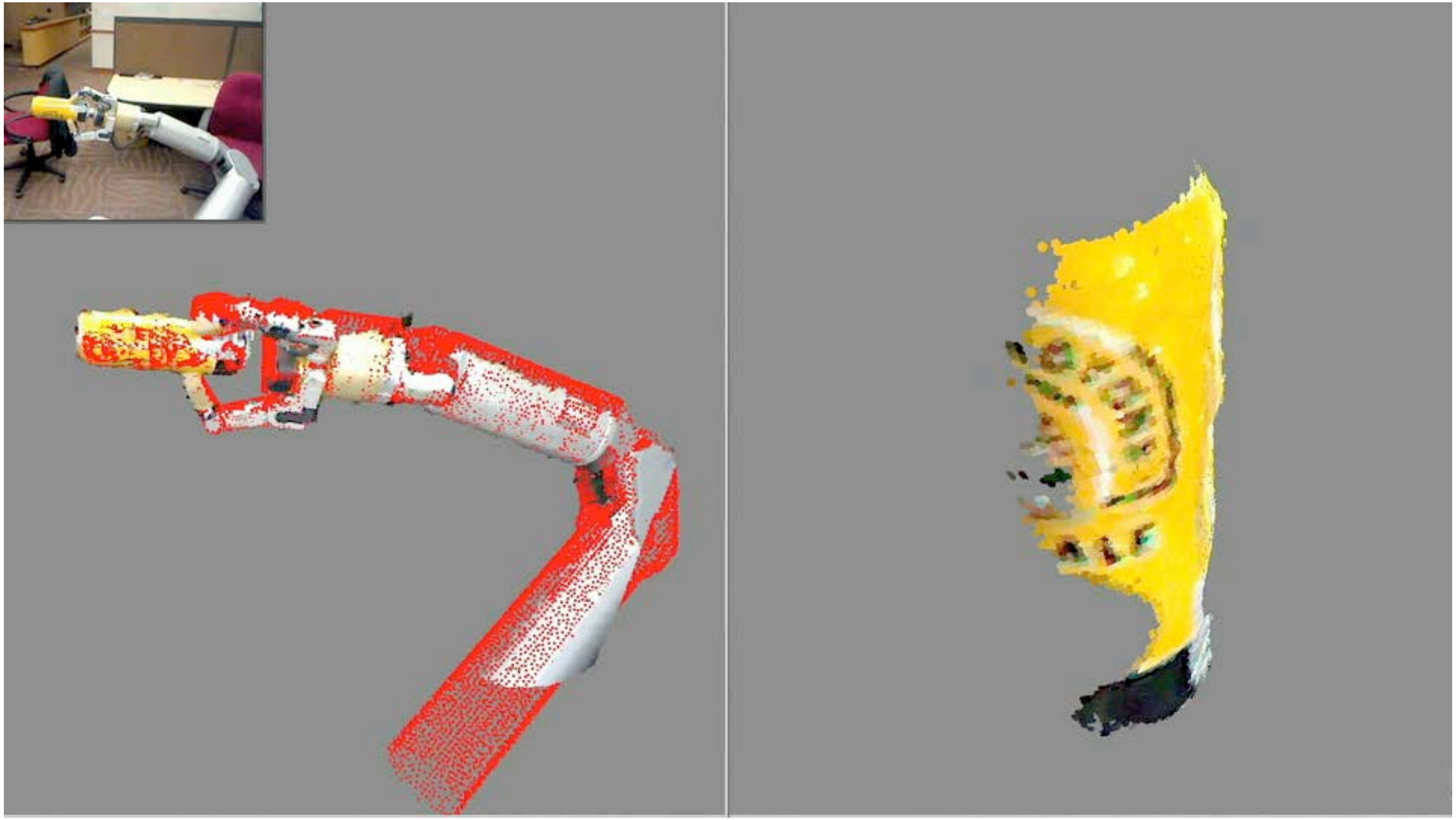


Estimate map *m* and driven path $x_{1:t}$

RGBD Mapping



Active Object Modeling: Joint Tracking and Modeling

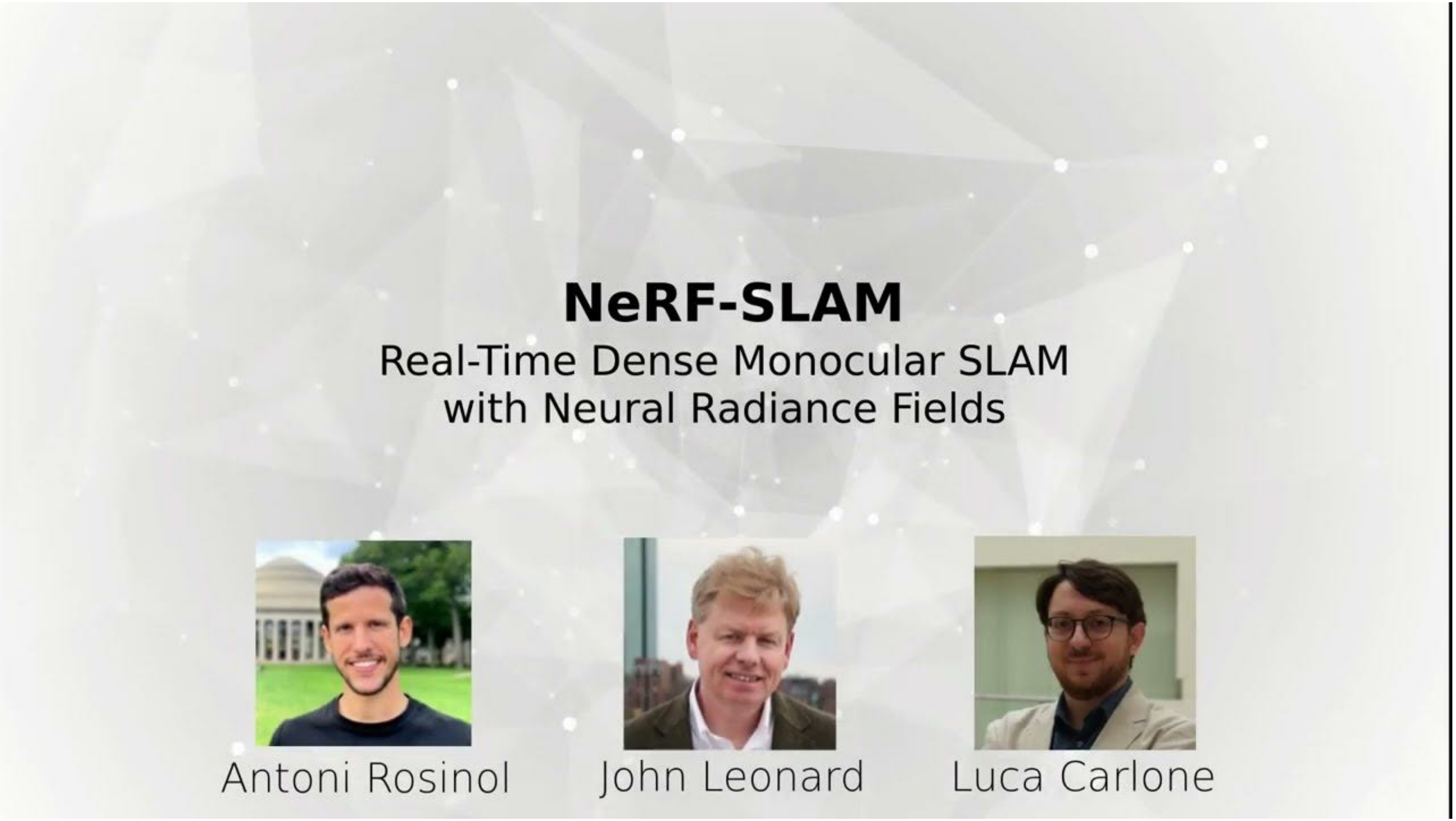


Robotic In-Hand 3D Object Modeling, UW Robotics and State Estimation Lab Michael Krainin, Peter Henry, Xiaofeng Ren, Dieter Fox, and Brian Curless

NeRF: Neural Radiance Fields

https://www.youtube.com/watch?v=JuH79E8rdKc

NeRF-SLAM



https://www.youtube.com/watch?v=-6ufRJugcEU

Thats the end of the course lectures!

Representations

1. Transformations
2. Rotations & Quaternions

Manipulation

1. Forward Kinematics
2. Inverse Kinematics

Planning

1. Path Planning
2. Bugs
3. Configuration space
4. Sampling based planners
5. Potential Fields
6. Collision Detection

Motion Control

Mobile Robotics

1. Probabilistic Robotics
2. Sensor and Motion models
3. Kalman Filter, Particle Filters
4. Localization
5. Mapping
6. SLAM

Upcoming weeks

04/10	Open Ended Final Project Pitches
04/15	Open Ended Final Project Pitches
04/17	Open Ended Final Project Pitches
04/22	Thomas Cohn - MIT - Guest Lecture
04/24	Guest Lectures / Extra office hours
04/29	Extra office hours
05/01	Alphonsus Antwi Adu - Boston Dynamics - Guest Lecture

- Groups 1-4: 04/10
- Groups 5-8: 04/15
- Groups 9-13: 04/17