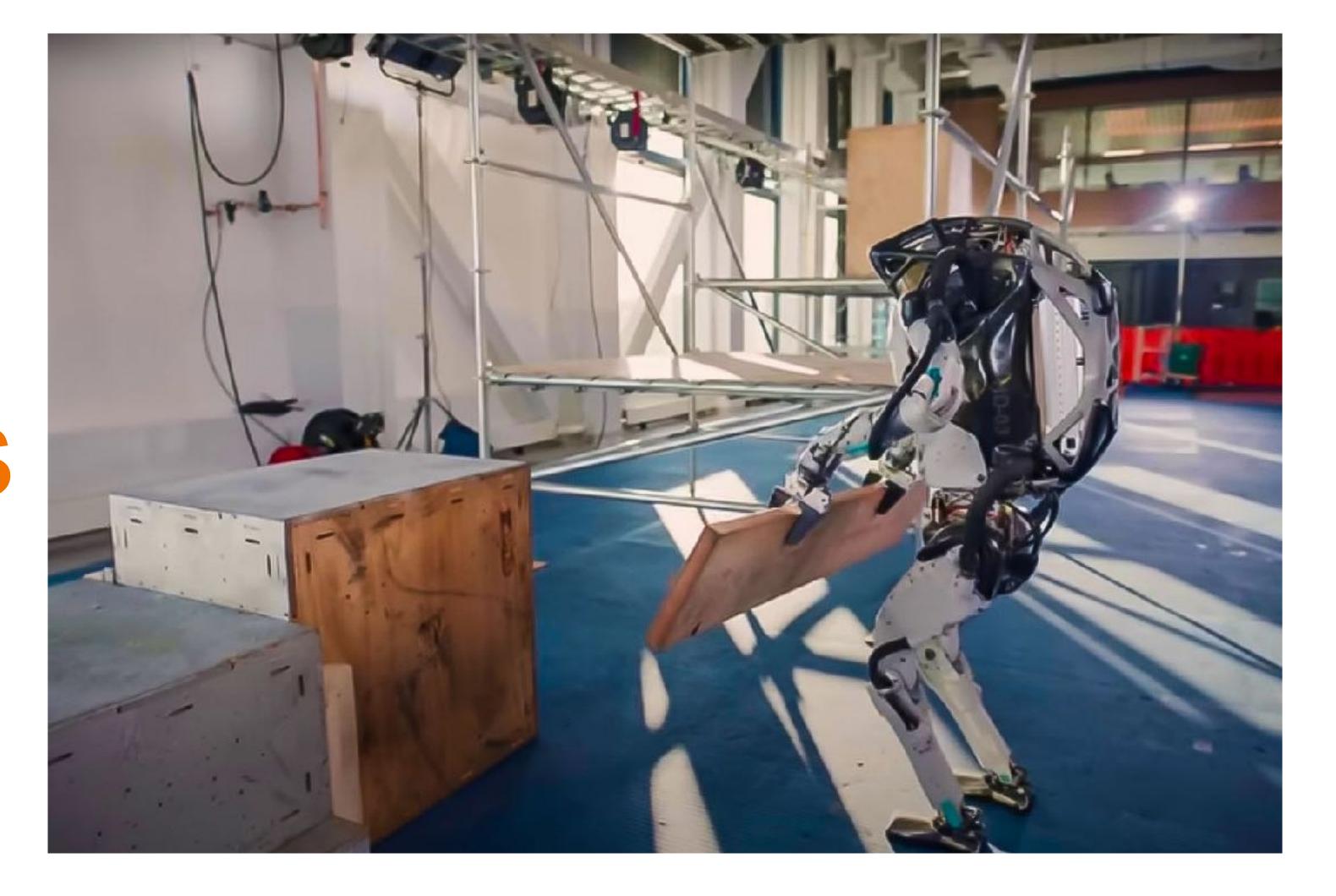
# Lecture 09 IK cont ... & Manipulation New Frontiers







# Course Logistics

- Quiz 4 was posted yesterday and was due at noon today.
- Project 3 was posted on 02/07 and will be due 02/15 (tomorrow).
- Project 4 will be posted 02/14 (today) and will be due on 02/28.

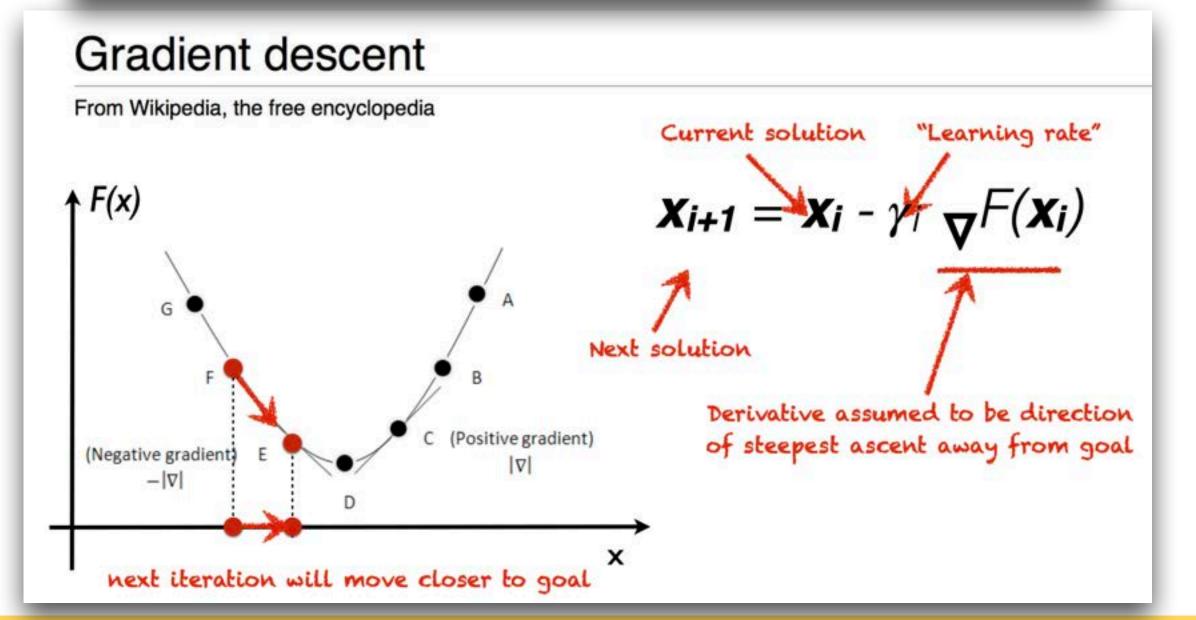


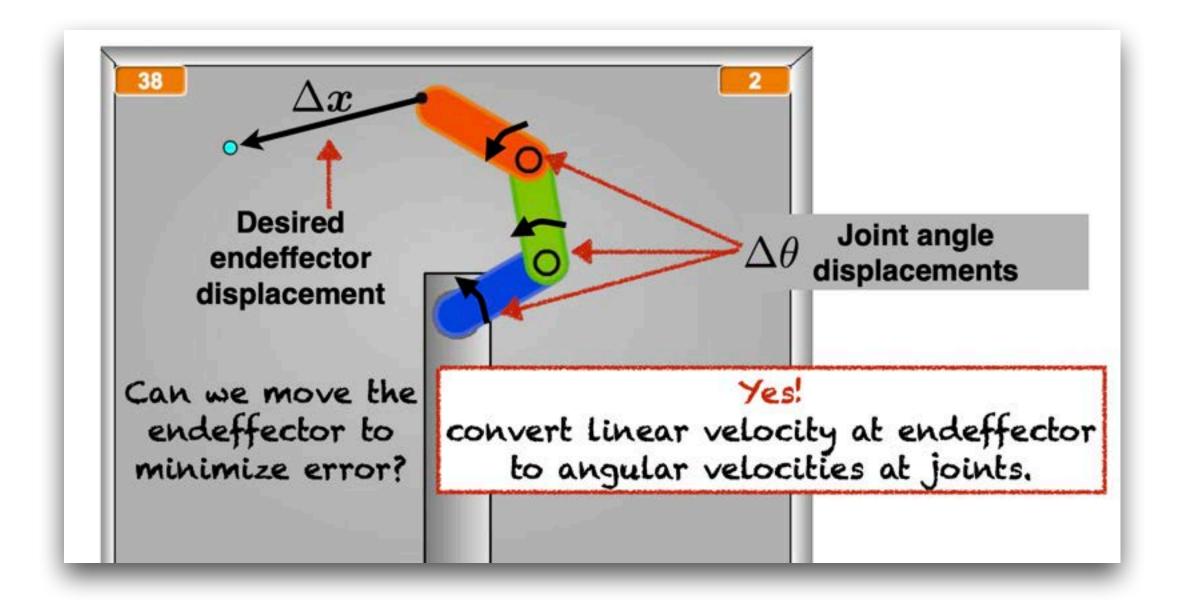
# Previously

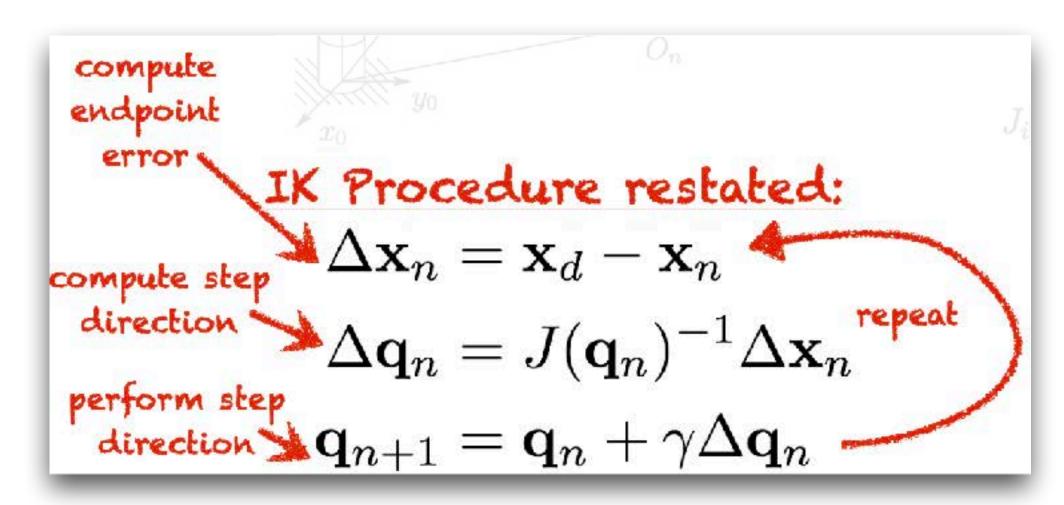
**Inverse kinematics**: how to solve for  $q = \{\theta_1, ..., \theta_N\}$  from  $T^0_N$ ?

#### Inverse Kinematics: 2 possibilites

- Closed-form solution: geometrically infer satisfying configuration
  - Speed: solution often computed in constant time
  - Predictability: solution is selected in a consistent manner
- **Solve by optimization**: minimize error of endeffector to desired pose
- often some form of Gradient Descent (a la Jacobian Transpose)
- Generality: same solver can be used for many different robots



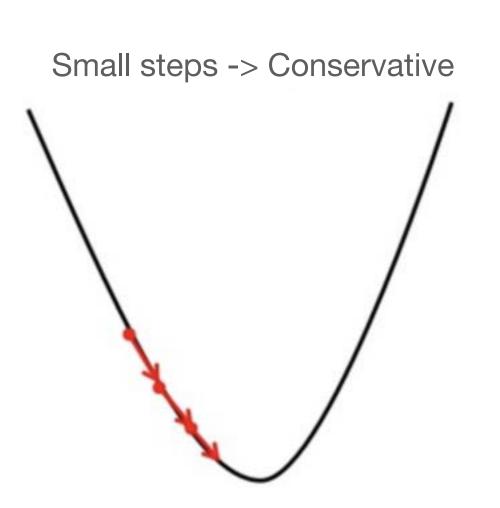


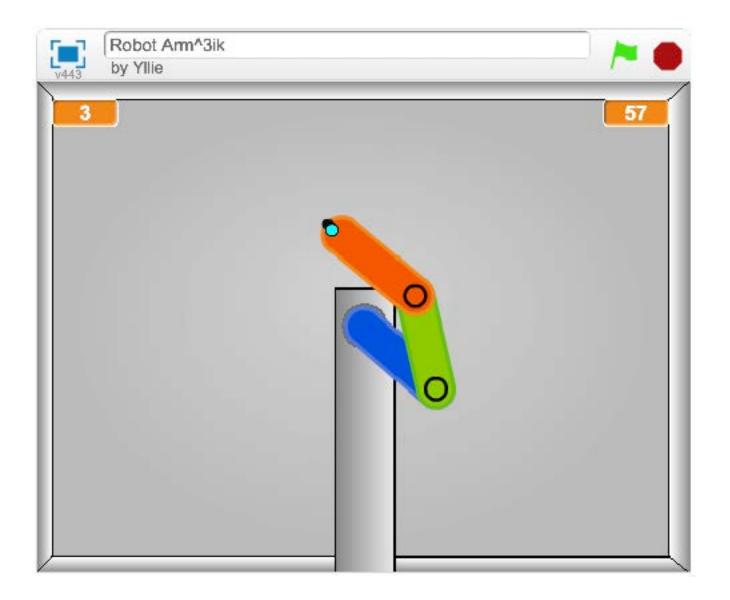


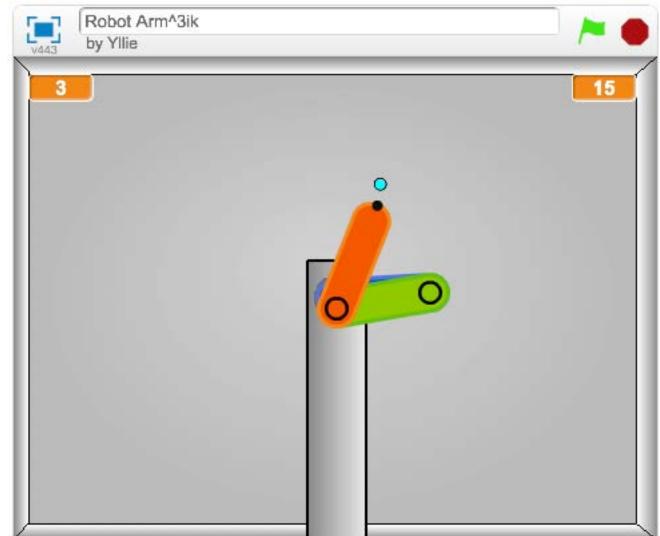


# IK by optimization

Big steps -> Aggressive







**Inverse kinematics**: how to solve for  $q = \{\theta_1, ..., \theta_N\}$  from  $T^0_N$ ?

Wait IK should give only the final robot configuration, isn't it?

In these videos, we see the entire path from the initial configuration.

What's going on?

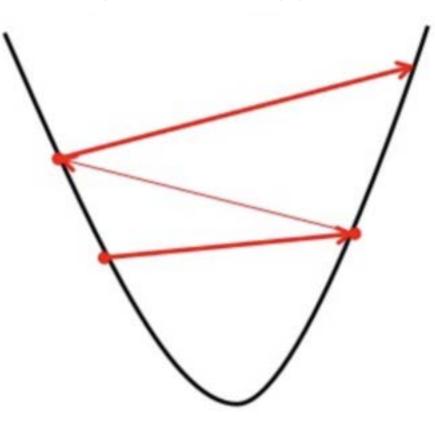
These videos are *illustrating* the optimization steps

In practice, you will use the solution (q\_desired) from the IK solver and invoke a motion planner that will plan a collision free trajectory/path 4 to your solution

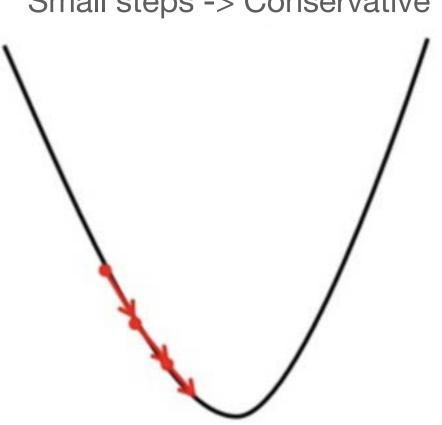


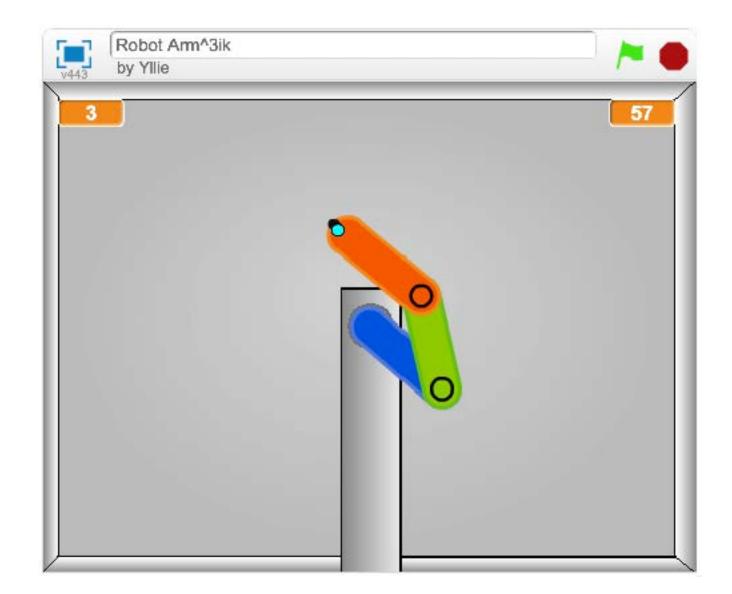
# IK by optimization

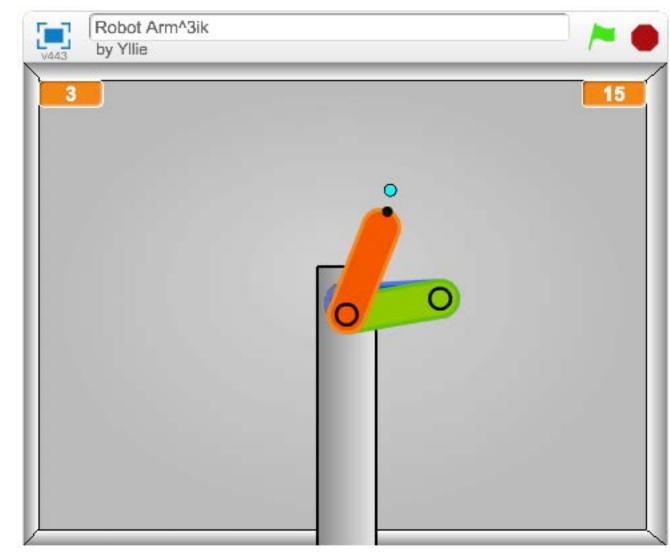
Big steps -> Aggressive



Small steps -> Conservative







**Inverse kinematics**: how to solve for  $q = \{\theta_1, ..., \theta_N\}$  from  $T^0_N$ ?

Wait IK should give only the final robot configuration, isn't it?

In these videos, we see the entire path from the initial configuration. What's going on?

> These videos are *illustrating* the optimization steps

 $q_{\text{desired}} = \text{IKSolver}(x_{\text{desired}})$ 

Trajectory = MotionPlanner( $q_{current}$ ,  $q_{desired}$ )

We will talk about this in the future classes



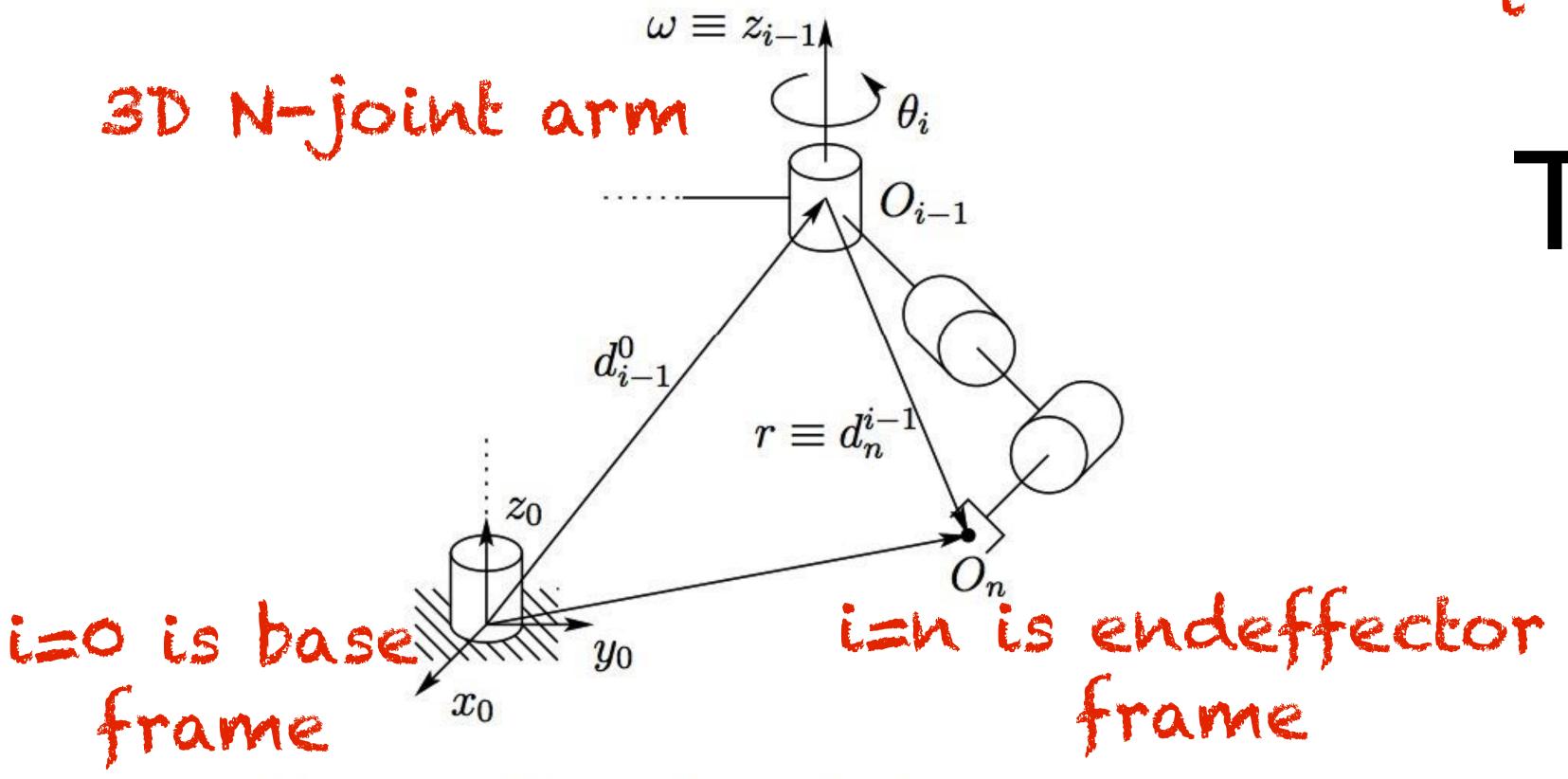


Figure 5.1: Motion of the end-effector due to link i.

i-1th frame maps to ith column in

# The Jacobian

A 6xN matrix

$$J = [J_1 J_2 \cdots J_n]$$



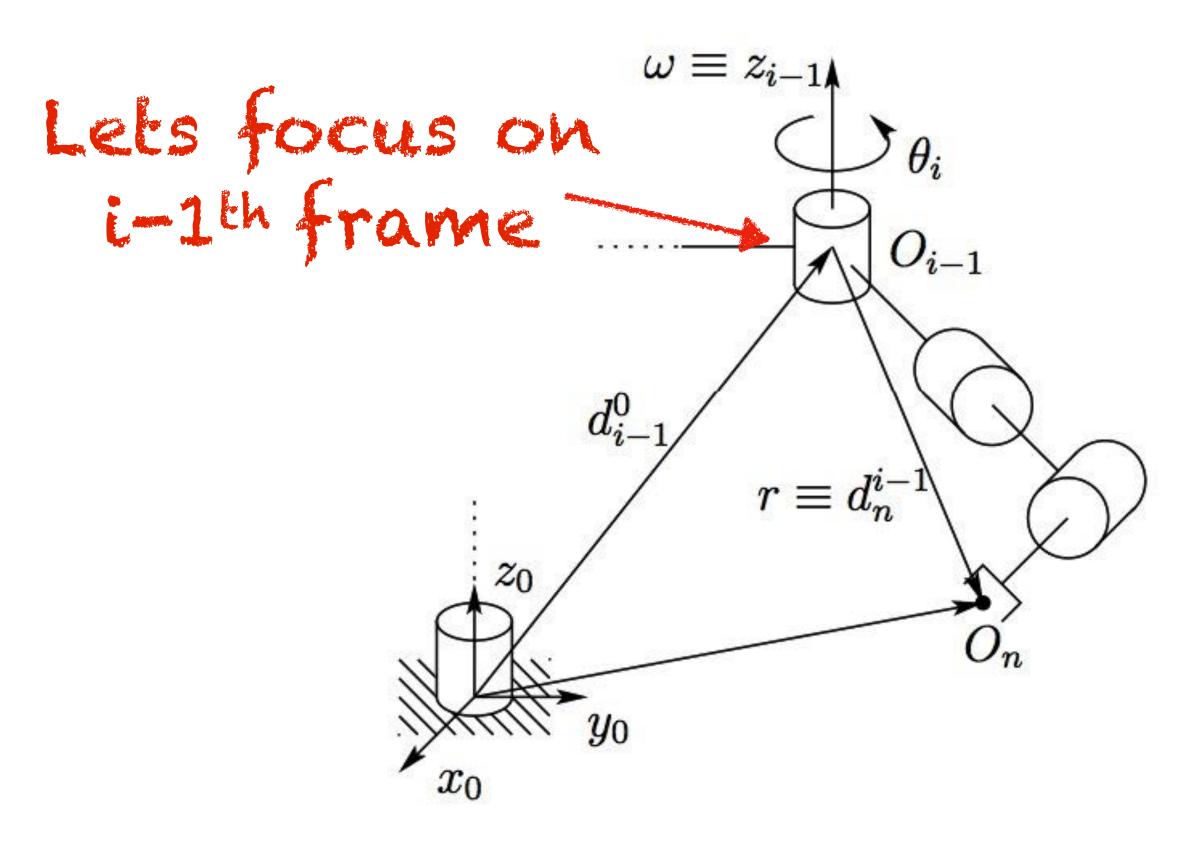


Figure 5.1: Motion of the end-effector due to link i.

i-1th frame maps to ith column in

# The Jacobian

A 6xN matrix

$$J=[J_1J_2\cdots J_n]$$
 This will correspond to ith column



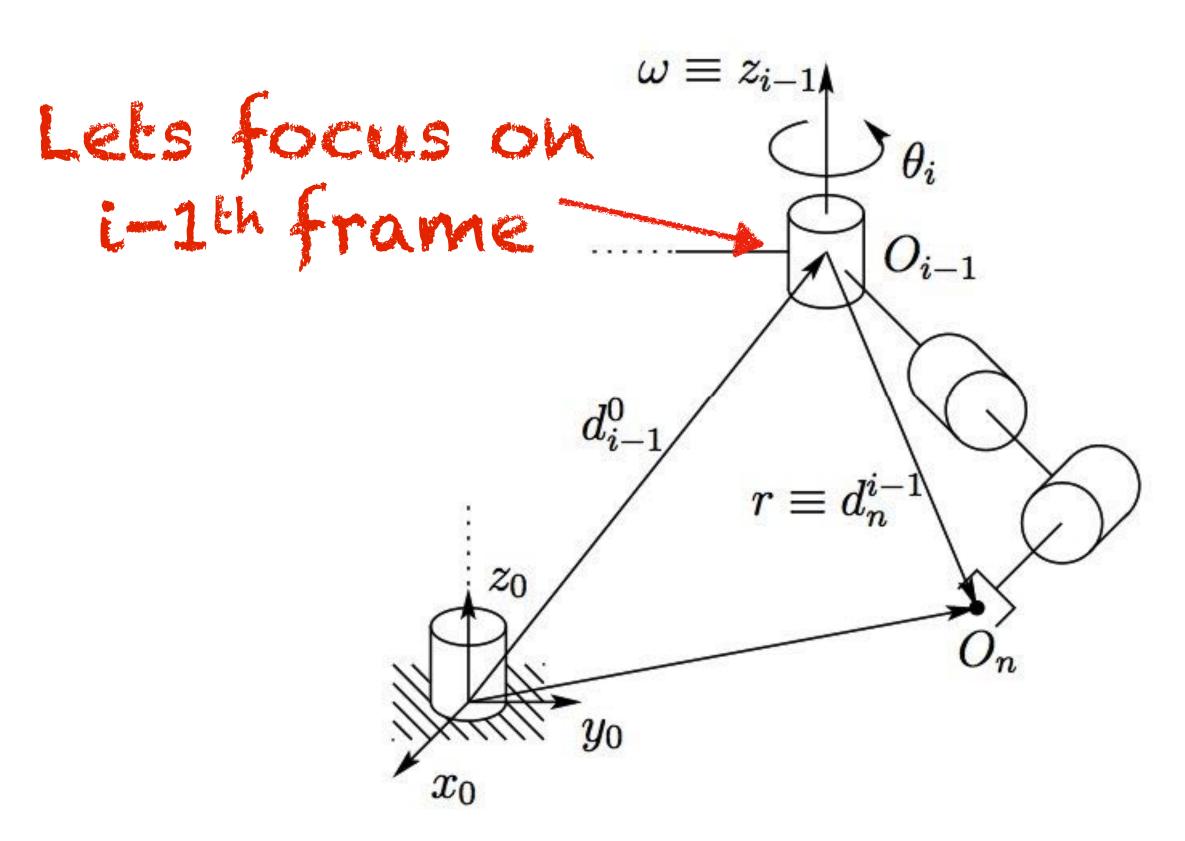


Figure 5.1: Motion of the end-effector due to link i.

Ji for a prismatic joint

$$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$$

 $J_i = \left[ egin{array}{c} z_{i-1} imes (o_n - o_{i-1}) \ z_{i-1} \end{array} 
ight]$ 

i-1th frame maps to ith column in

# The Jacobian

A 6xN matrix

$$J = [J_1 J_2 \cdots J_n]$$

consisting of two 3xN matrices

$$J = \left[ rac{J oldsymbol{v}}{J_{\omega}} 
ight]$$

Ji for a rotational joint

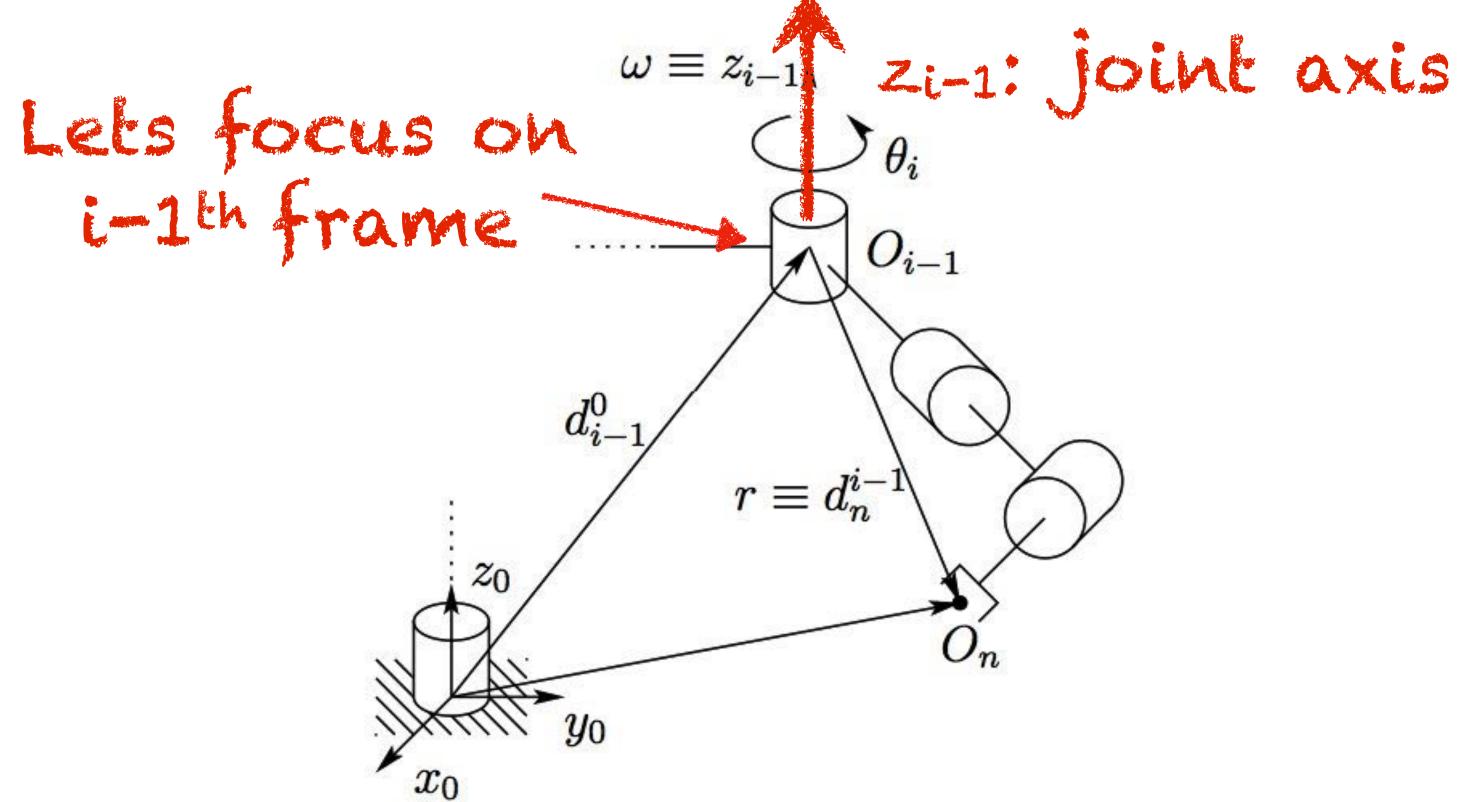


Figure 5.1: Motion of the end-effector due to link i.

If the i-1th joint is prismatic

Ji for a prismatic joint

$$J_i = \left[egin{array}{c} z_{i-1} \ 0 \end{array}
ight]$$

# What is zi-1 capturing?

z<sub>i-1</sub> is a 3x1 vector capturing the influence of this joint on the end-effector pose.

Only influences the translational (linear) component



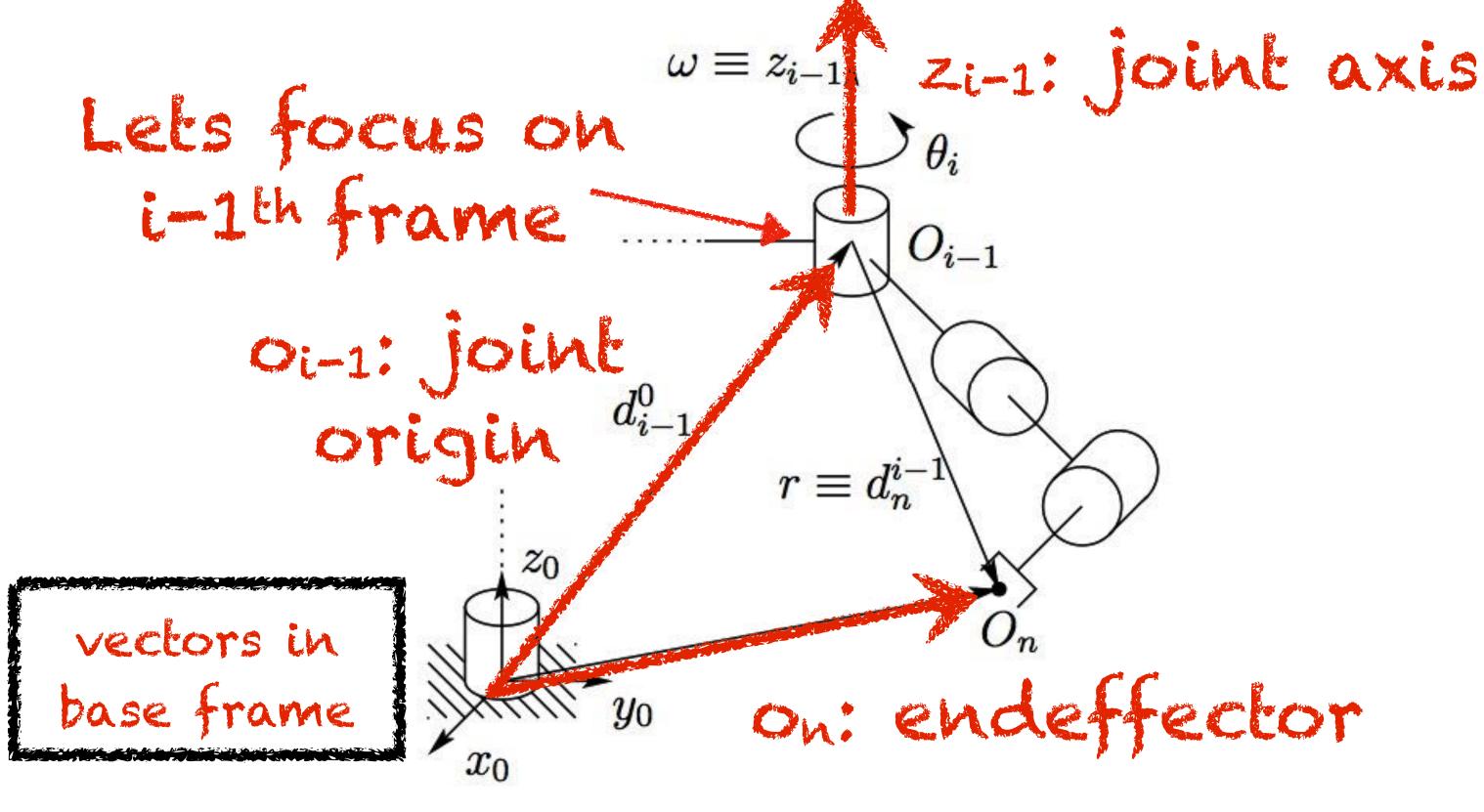


Figure 5.1: Motion of the end-effector due to link i.

If the i-1th joint is revolute

Ji for a rotational joint

$$J_i = \left[egin{array}{c} z_{i-1} imes (o_n - o_{i-1}) \ z_{i-1} \end{array}
ight]$$

What is zi-1 x (On - Oi-1) capturing?



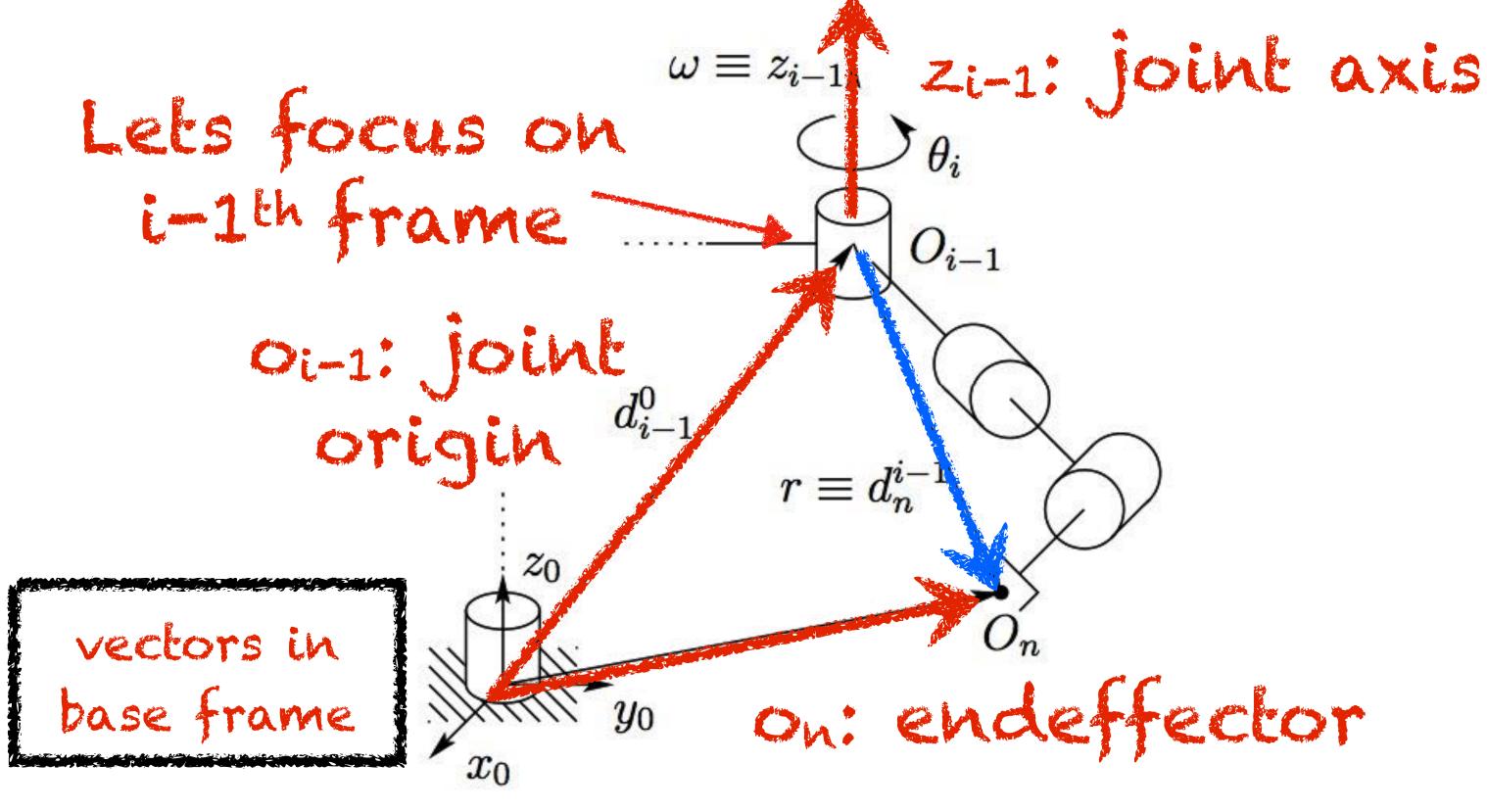


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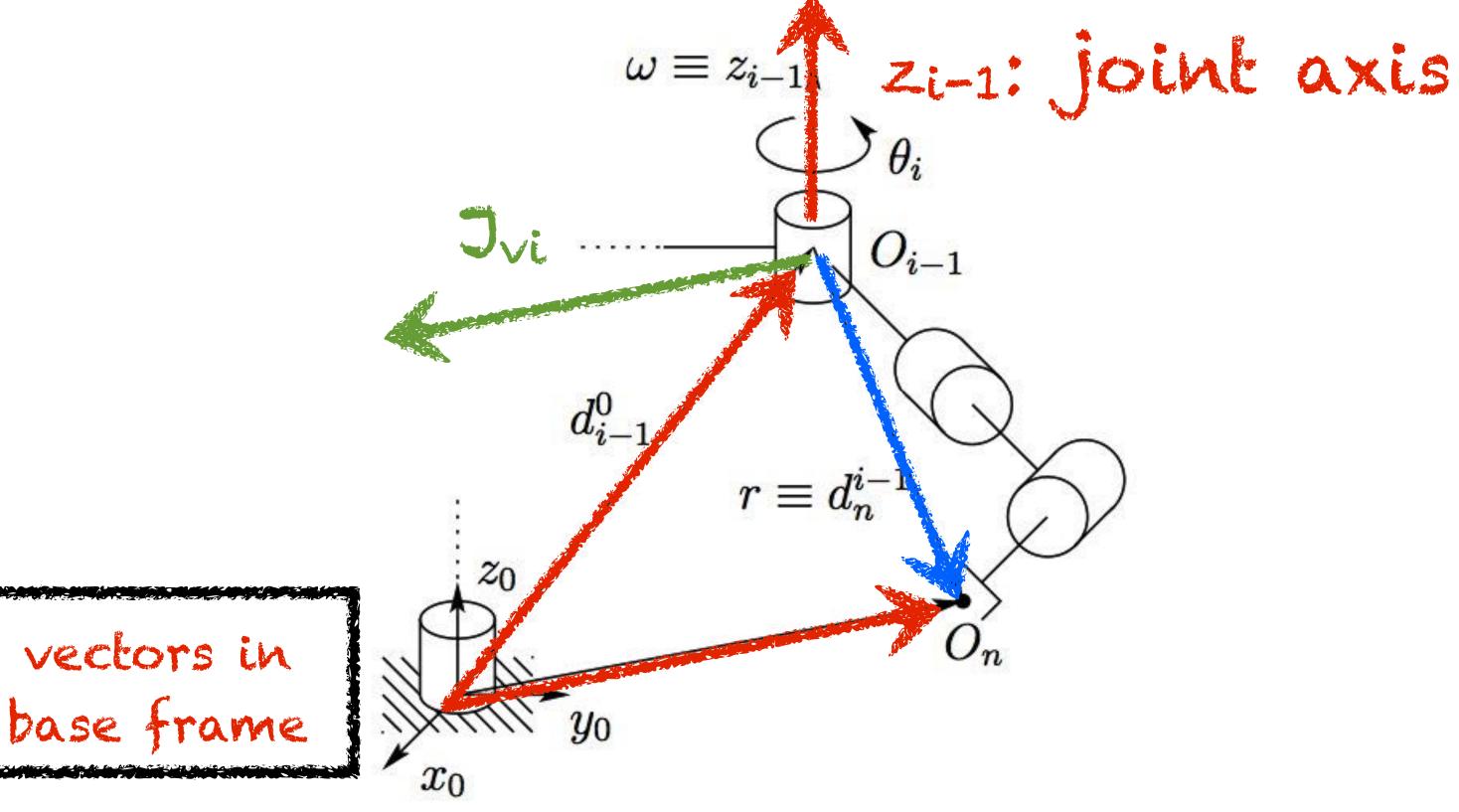


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$$J_i = \left[egin{array}{c} z_{i-1} imes (o_n - o_{i-1}) \ z_{i-1} \end{array}
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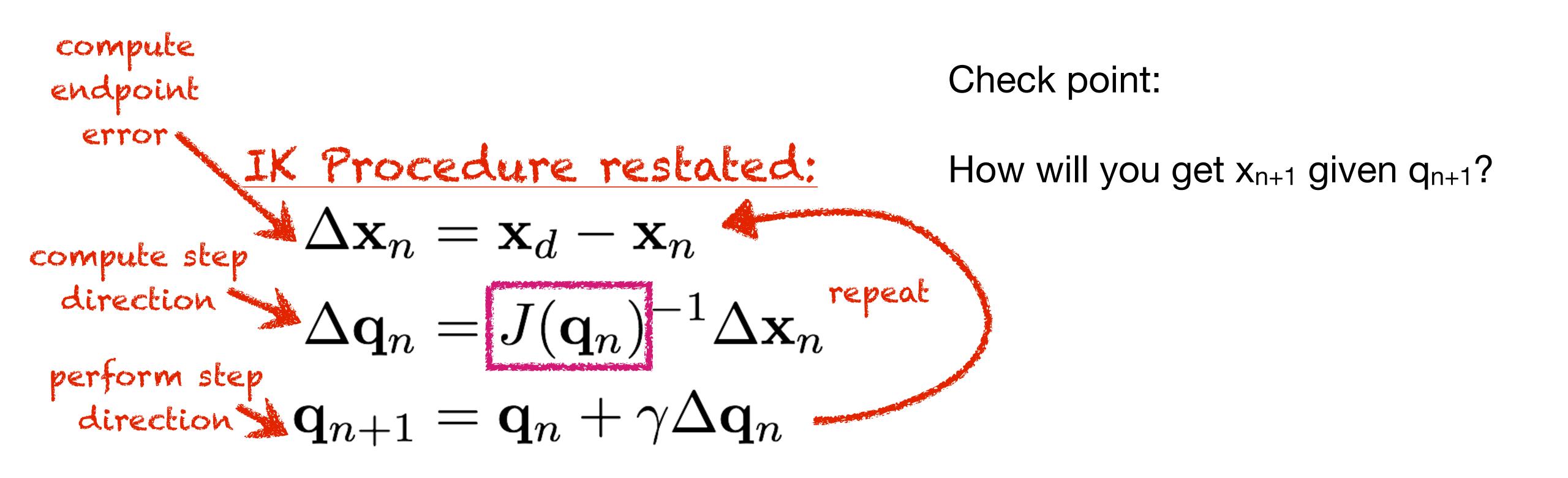
The influence of this joint on the endeffector's translational component.

## What is zi-1 capturing?

The influence of this joint on the endeffector's rotational component.

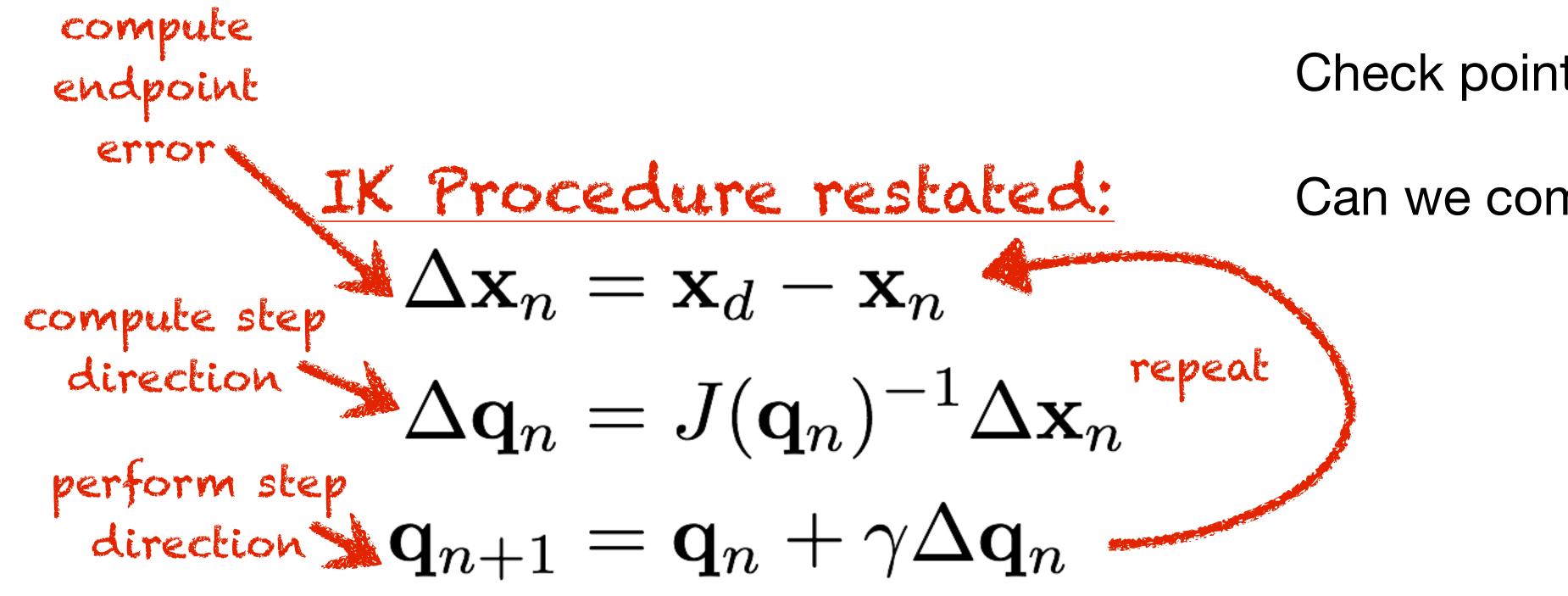


# How to use this Jacobian for IK as optimization?





# How to use this Jacobian for IK as optimization?



Check point:

Can we compute the J-1 all the time?



# How to use this Jacobian for IK as optimization?

compute endpoint  $\Delta \mathbf{x}_n = \mathbf{x}_d - \mathbf{x}_n$  compute step  $\Delta \mathbf{q}_n = J(\mathbf{q}_n)^{-1}\Delta \mathbf{x}_n$  repeat perform step direction  $\mathbf{q}_{n+1} = \mathbf{q}_n + \gamma \Delta \mathbf{q}_n$ 

Check point:

Can we compute the J-1 all the time?

No

We can use pseudoinverse!

- For matrix A with dimensions N x M with full rank
- Left pseudoinverse, for when N > M, (i.e., "tall", less than than 6 DoFs)

$$A_{\text{left}}^{-1} = \left(A^T A\right)^{-1} A^T$$
 s.t.  $A_{\text{left}}^{-1} A = I_n$ 

Right pseudoinverse, for when N < M, (i.e., "wide", more than 6 DoFs)</li>

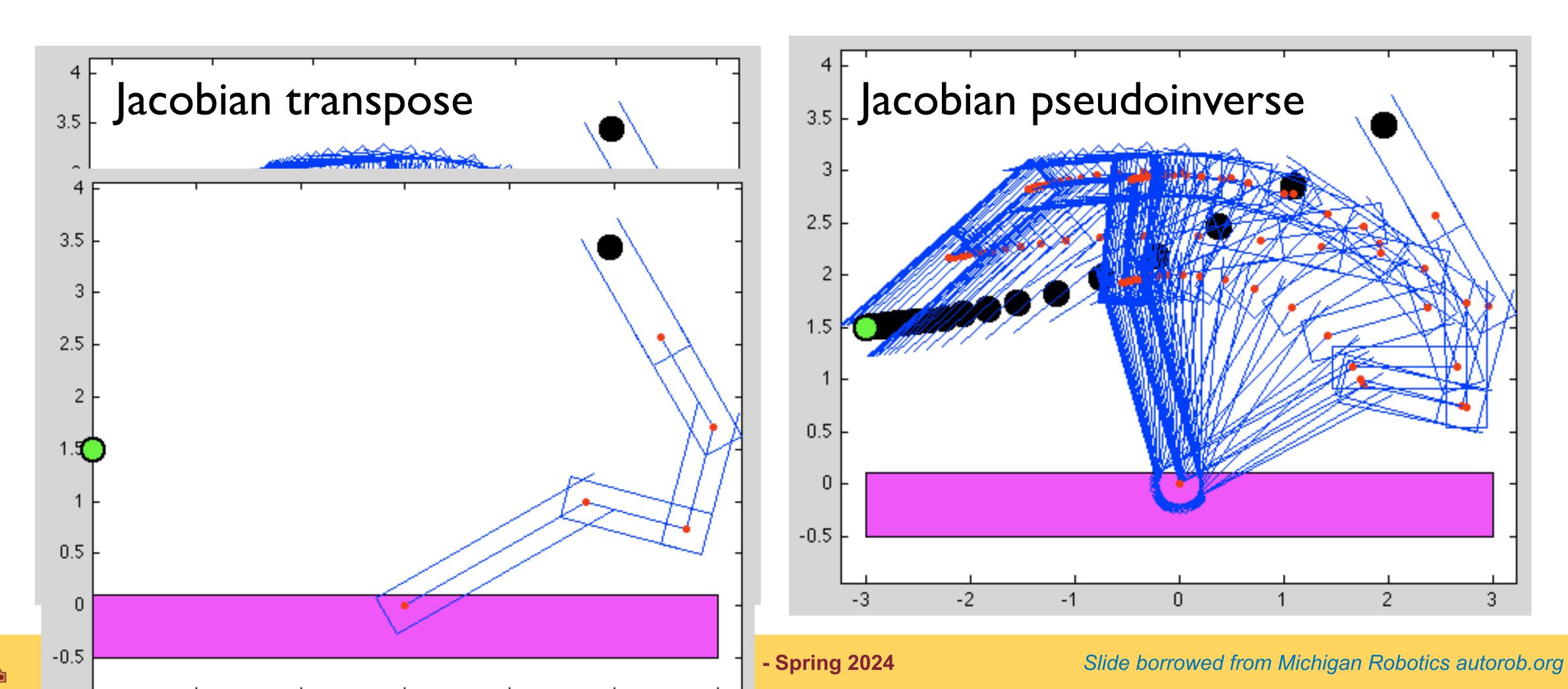
$$A_{\text{right}}^{-1} = A^T \left( A A^T \right)^{-1}$$
 s.t.  $A A_{\text{right}}^{-1} = I_m$ 



# Jacobian Pseudoinverse



# Matlab 5-link arm example: Jacobian Pseudoinverse



#### Error Minimization by Jacobian Pseudoinverse

$$J(\mathbf{q})\Delta\mathbf{q} = \Delta\mathbf{x}$$

$$\Delta\mathbf{q} = J(\mathbf{q})^{-1}\Delta\mathbf{x}$$

Jacobian gives mapping from configuration displacement to endeffector displacement

Inverse of Jacobian maps endeffector displacement to configuration displacement

 $\underset{\Delta \mathbf{q}}{\arg\min} ||J(\mathbf{q})\Delta \mathbf{q} - \Delta \mathbf{x}||^2$ 

But, inverse of Jacobian is rarely an option. Why?

Instead, find configuration displacement that minimizes endeffector error squared

#### Error Minimization by Jacobian Pseudoinverse

$$\underset{\Delta \mathbf{q}}{\operatorname{arg\,min}} ||J(\mathbf{q})\Delta \mathbf{q} - \Delta \mathbf{x}||^2$$

Instead, find configuration displacement that minimizes endeffector error squared

$$\begin{split} C &= (J(\mathbf{q})\Delta\mathbf{q} - \Delta\mathbf{x})^2 & \text{ Define cost function} \\ &= (J(\mathbf{q})\Delta\mathbf{q} - \Delta\mathbf{x})^T (J(\mathbf{q})\Delta\mathbf{q} - \Delta\mathbf{x}) \\ &= (J(\mathbf{q})\Delta\mathbf{q} - \Delta\mathbf{x})^T (J(\mathbf{q})\Delta\mathbf{q} - \Delta\mathbf{x}) \\ &= \Delta\mathbf{q}^T J(\mathbf{q})^T J(\mathbf{q})\Delta\mathbf{q} - \Delta\mathbf{q}^T J(\mathbf{q})^T \Delta\mathbf{x} - \Delta\mathbf{x}^T J(\mathbf{q})\Delta\mathbf{q} + \Delta\mathbf{x}^T \Delta\mathbf{x} \\ &= \Delta\mathbf{q}^T J(\mathbf{q})^T J(\mathbf{q})\Delta\mathbf{q} - 2\Delta\mathbf{q}^T J(\mathbf{q})^T \Delta\mathbf{x} + \Delta\mathbf{x}^T \Delta\mathbf{x} \end{split}$$



#### Error Minimization by Jacobian Pseudoinverse

Define cost function expressing squared error

$$C = \Delta \mathbf{q}^T J(\mathbf{q})^T J(\mathbf{q}) \Delta \mathbf{q} - 2\Delta \mathbf{q}^T J(\mathbf{q})^T \Delta \mathbf{x} + \Delta \mathbf{x}^T \Delta \mathbf{x}$$

Take cost derivative

$$\frac{dC}{d\Delta \mathbf{q}} = 2J(\mathbf{q})^T J(\mathbf{q}) \Delta \mathbf{q} - 2J(\mathbf{q})^T \Delta \mathbf{x} + 0$$

Set to zero and solve for configuration displacement

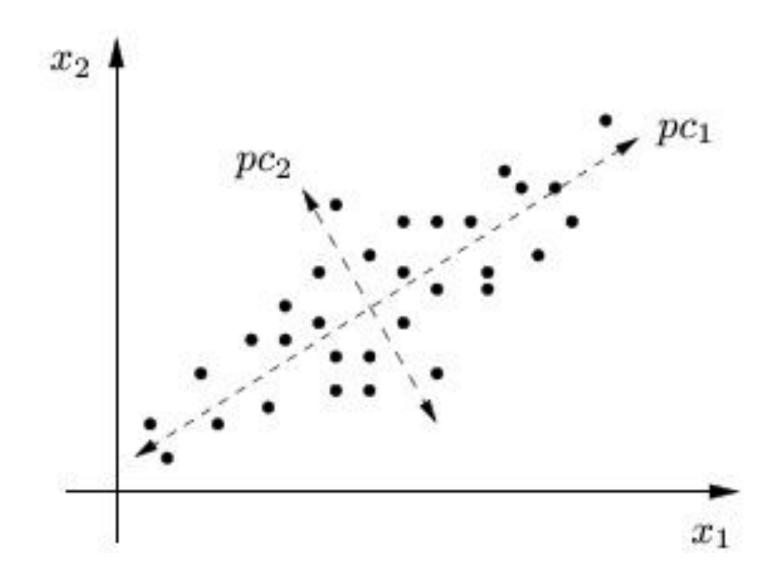
$$0 = 2J(\mathbf{q})^T J(\mathbf{q}) \Delta \mathbf{q} - 2J(\mathbf{q})^T \Delta \mathbf{x}$$

$$J(\mathbf{q})^T J(\mathbf{q}) \Delta \mathbf{q} = J(\mathbf{q})^T \Delta \mathbf{x}$$
 Normal form

$$\Delta \mathbf{q} = (J(\mathbf{q})^T J(\mathbf{q}))^{-1} J(\mathbf{q})^T \Delta \mathbf{x}$$

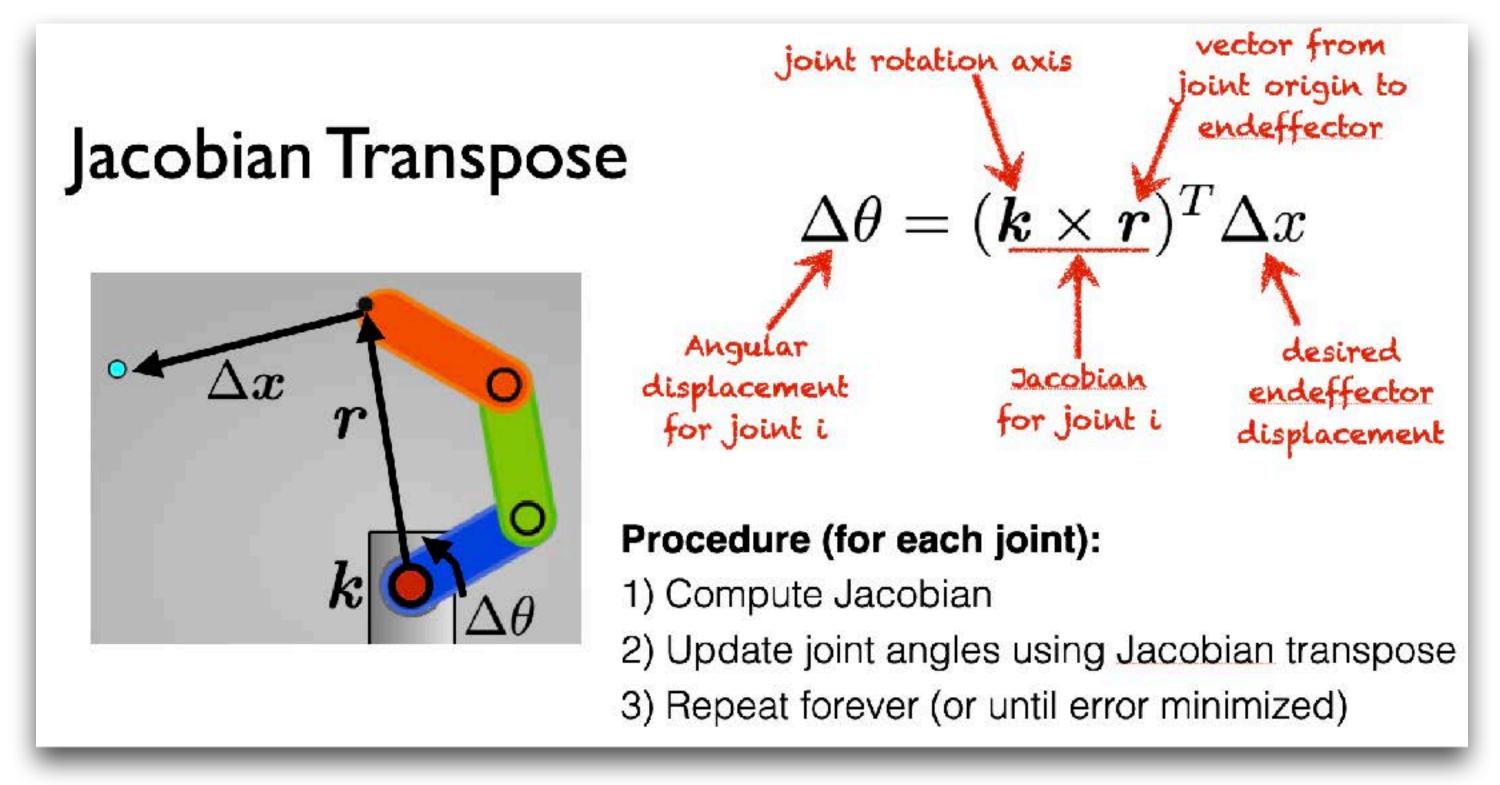


# Pseudoinverse, More Generally



- Pseudoinverse of matrix A:  $A^{+}=(A^{T}A)^{-1}A^{T}$  approximates solution to linear system Ax=b
- The pseudoinverse A+ is a least squares "best fit" approximate solution of an **overdetermined** system Ax=b, where there are more equations (m) than unknowns (n), or vice versa
- Often used for data fitting, as a singular value decomposition

# Didn't we say **Jacobian Transpose** in the earlier lecture and not inverse?



We can also use Jacobian Transponse

# Jacobian Transpose revisited



#### Error Minimization by Jacobian Transpose

Define cost function expressing squared error

$$C = \Delta \mathbf{q}^T J(\mathbf{q})^T J(\mathbf{q}) \Delta \mathbf{q} - 2\Delta \mathbf{q}^T J(\mathbf{q})^T \Delta \mathbf{x} + \Delta \mathbf{x}^T \Delta \mathbf{x}$$

 $\frac{dC}{d\Delta\mathbf{q}} = 2J(\mathbf{q})^TJ(\mathbf{q})\Delta\mathbf{q} - 2J(\mathbf{q})^T\Delta\mathbf{x} + 0$  Take cost derivative wrt, change in configuration

Evaluate at convergence point, where change in configuration is zero

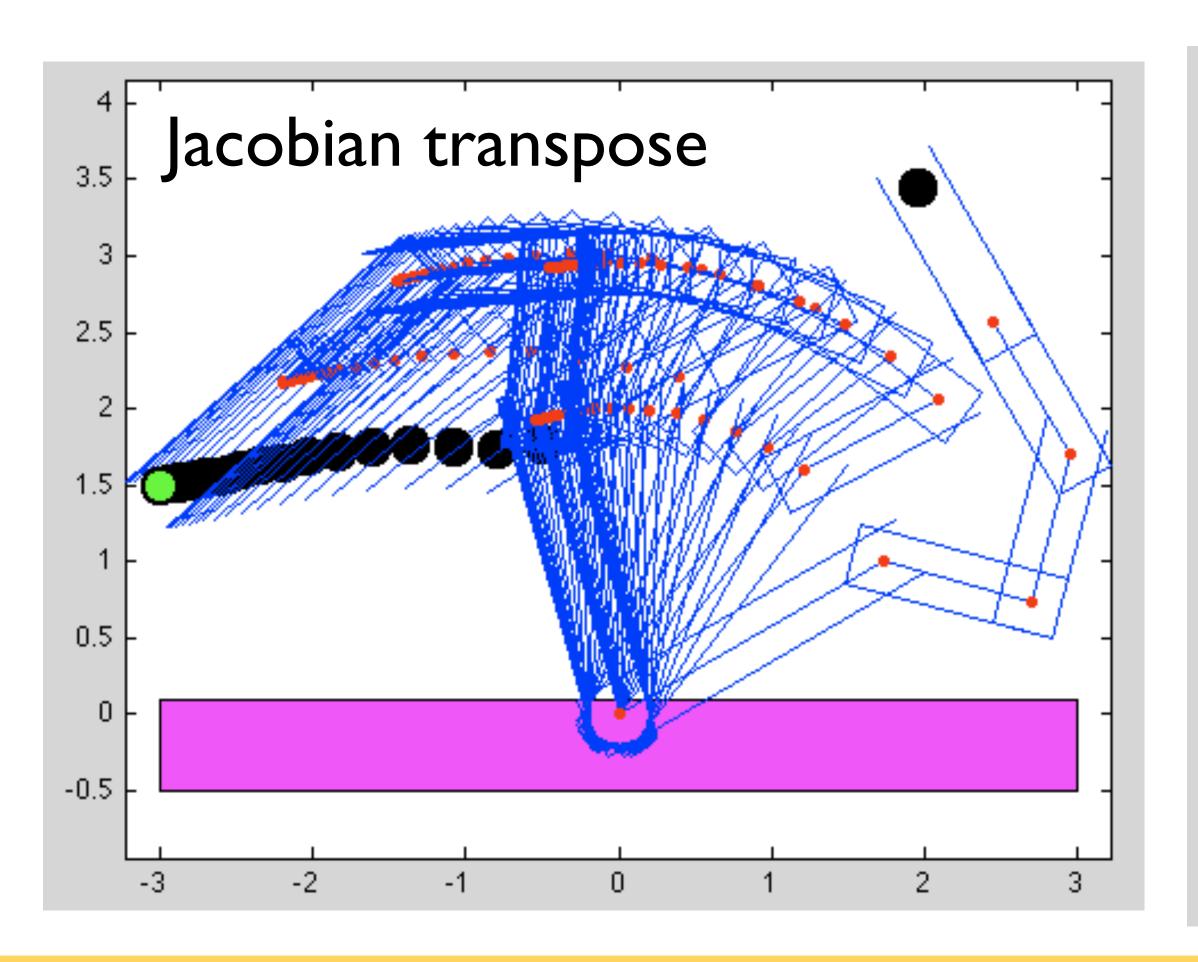
$$\frac{dC}{d\Delta \mathbf{q}}\Big|_{\Delta \mathbf{q}=0} = 2J(\mathbf{q})^T J(\mathbf{q}) \Delta \mathbf{q} - 2J(\mathbf{q})^T \Delta \mathbf{x}|_{\Delta \mathbf{q}=0}$$

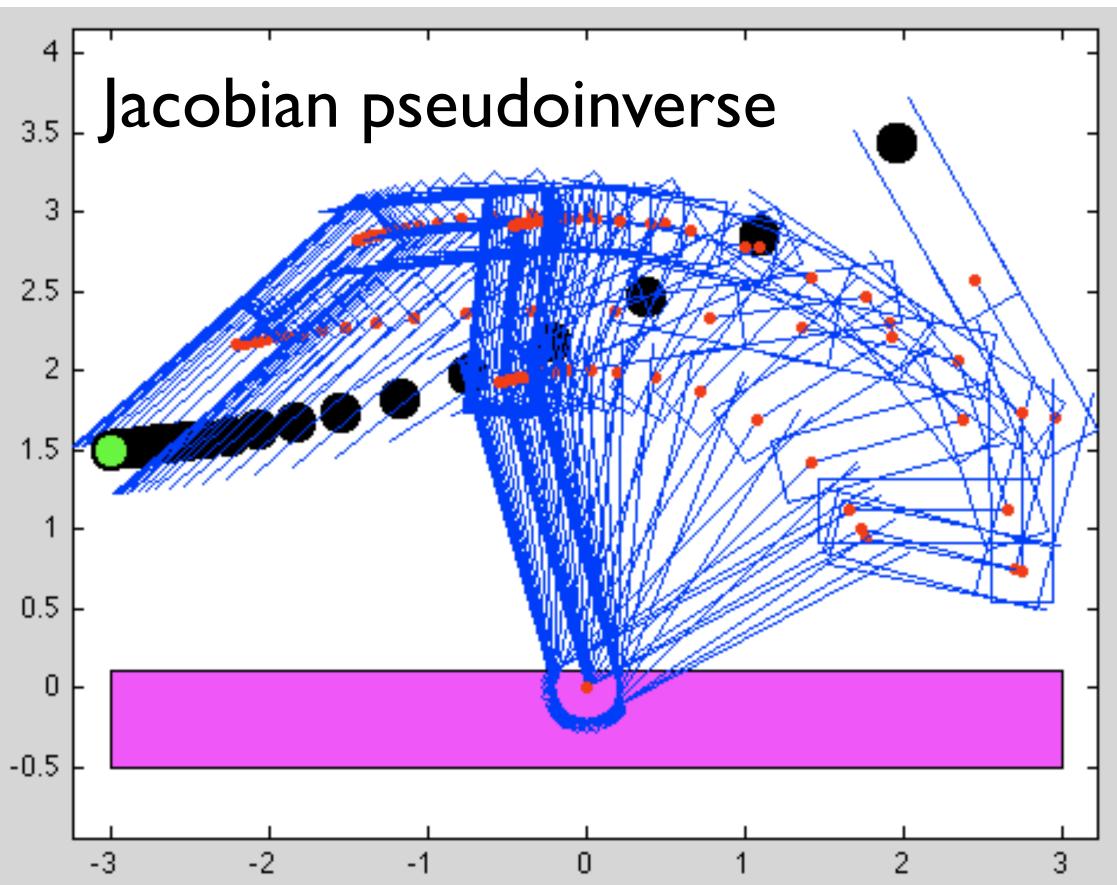
$$= 2J(\mathbf{q})^T \Delta \mathbf{x}$$
$$= \gamma J(\mathbf{q})^T \Delta \mathbf{x}$$

step length (gamma) chosen as update step scale



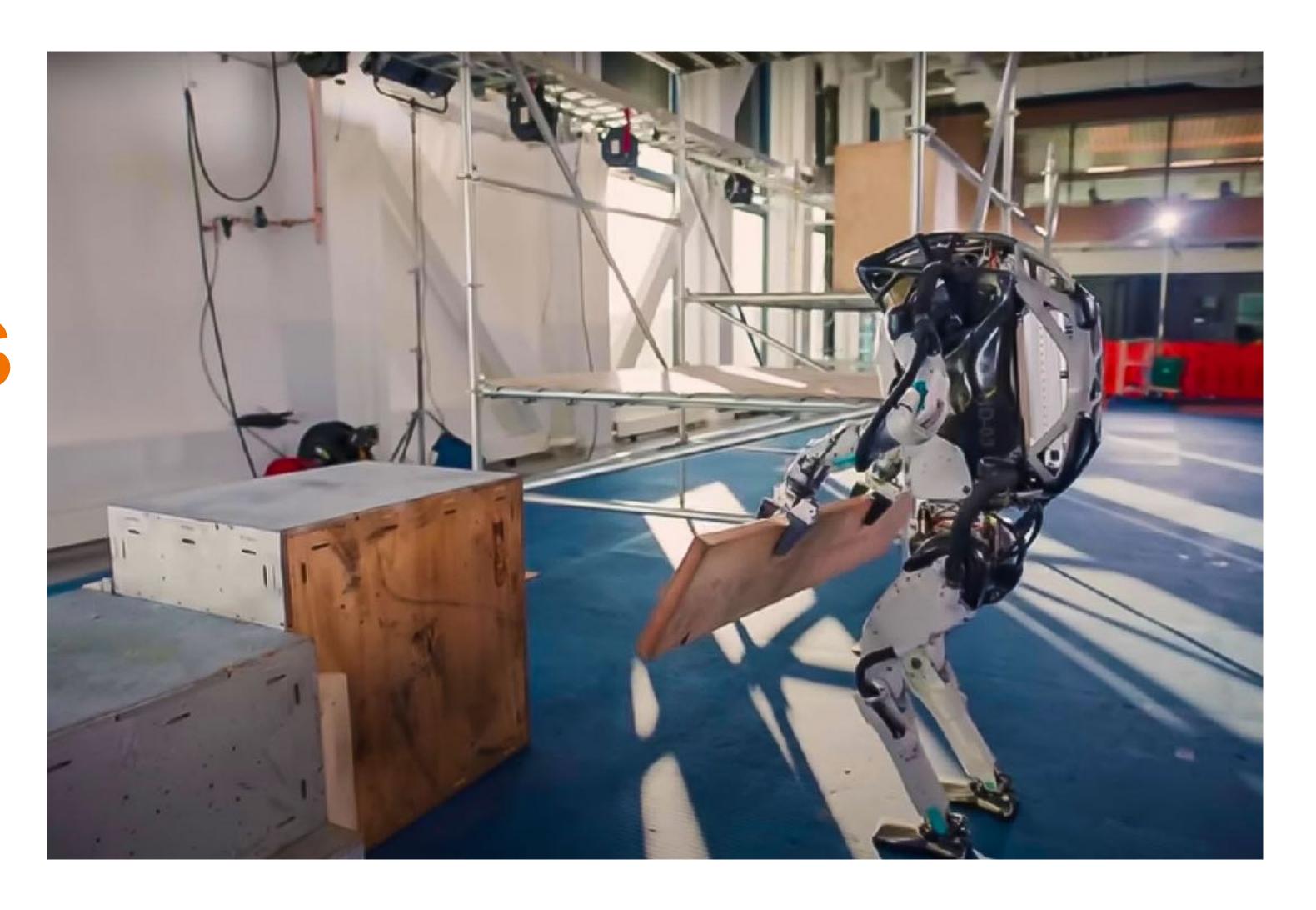
# Matlab 5-link arm example: Jacobian transpose







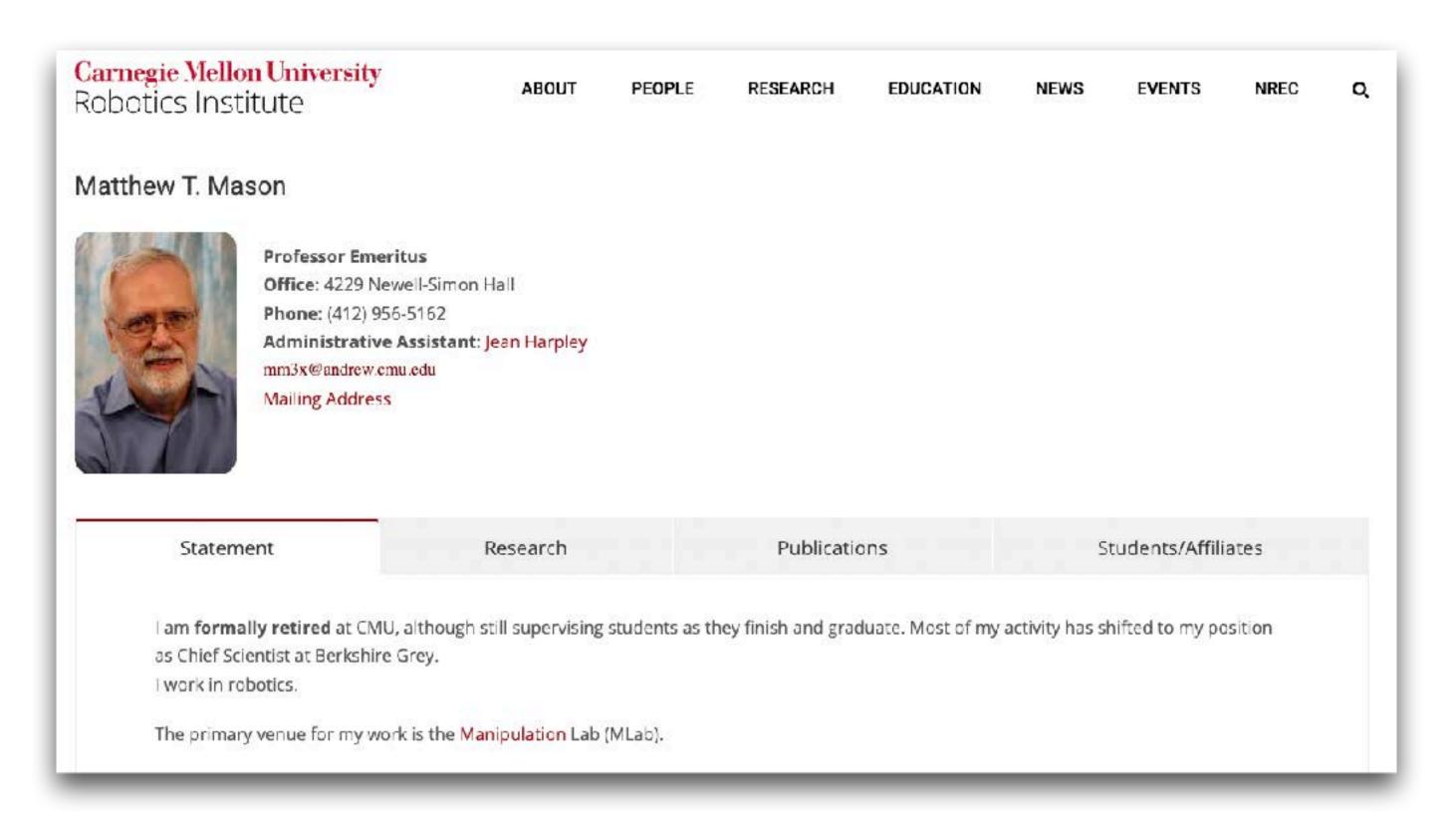
# Manipulation New Frontiers







Mason, Matthew T. "Toward robotic manipulation." *Annual Review of Control, Robotics, and Autonomous Systems* 1 (2018): 1-28.



This lecture uses the structure and material from this review paper!





Annual Review of Control, Robotics, and Autonomous Systems

#### Toward Robotic Manipulation

#### Matthew T. Mason

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Annu. Rev. Control Robot, Auton, Syst. 2018, 1:19.1-19.28

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https://doi.org/10.1146/annurev-control-060117-104848

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#### Keyword

robot, manipulation, evolution, engineering

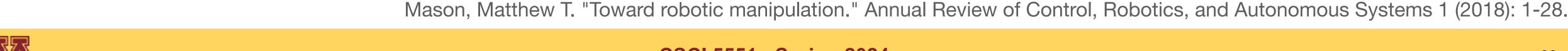
#### Abstract

This article surveys manipulation, including both biological and robotic manipulation. Biology inspires robotics and demonstrates aspects of manipulation that are far in the future of robotics. Robotics develops concepts and principles that become evident only in the creative process. Robotics also provides a test of our understanding. As Richard Feynman put it: "What I cannot create, I do not understand."

19.1



Very few definitions of manipulation appear in the robotics literature. A European research road map defined manipulation as "the function of utilising the characteristics of a grasped object to achieve a task" (1, p. 38). A NASA road-mapping effort yields the following: "Manipulation pertains to making an intentional change in the environment or to objects that are being manipulated" (2, p. 13). My own earlier attempt at defining manipulation was "using one's hands to rearrange one's environment" (3, p. 1). Rather than sorting the pros and cons of those definitions, let us apply the shotgun method and identify every approach that we can.



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Definition 1 (etymological). Manipulation refers to the activities performed by hands.

**Definition 2 (genus/differentia, ends only).** Manipulation is when an agent moves things other than itself.

**Definition 3 (genus/differentia, ends and means).** Manipulation is when an agent moves things other than itself through selective contact.

**Definition 4 (bottom up).** Manipulation is pick-and-place manipulation plus in-hand manipulation plus mechanical assembly plus....

**Definition 5.** Manipulation refers to an agent's control of its environment through selective contact.

Mason, Matthew T. "Toward robotic manipulation." Annual Review of Control, Robotics, and Autonomous Systems 1 (2018): 1-28.







Smaller-scale manipulation exhibited by flagella and cilia starting billion years ago

https://makeagif.com/gif/flagella-cilia-VjpqAa



"The brain of an ant is one of the most marvellous atoms of matter in the world, perhaps more marvellous than the brain of man." - Darvin



Intermediate-scale Manipulation Weaver ants ~20 million years ago

https://www.youtube.com/watch?v=1pkjpC4O\_TM



Intermediate-scale Manipulation Dung Beetle Mobile Manipulation???

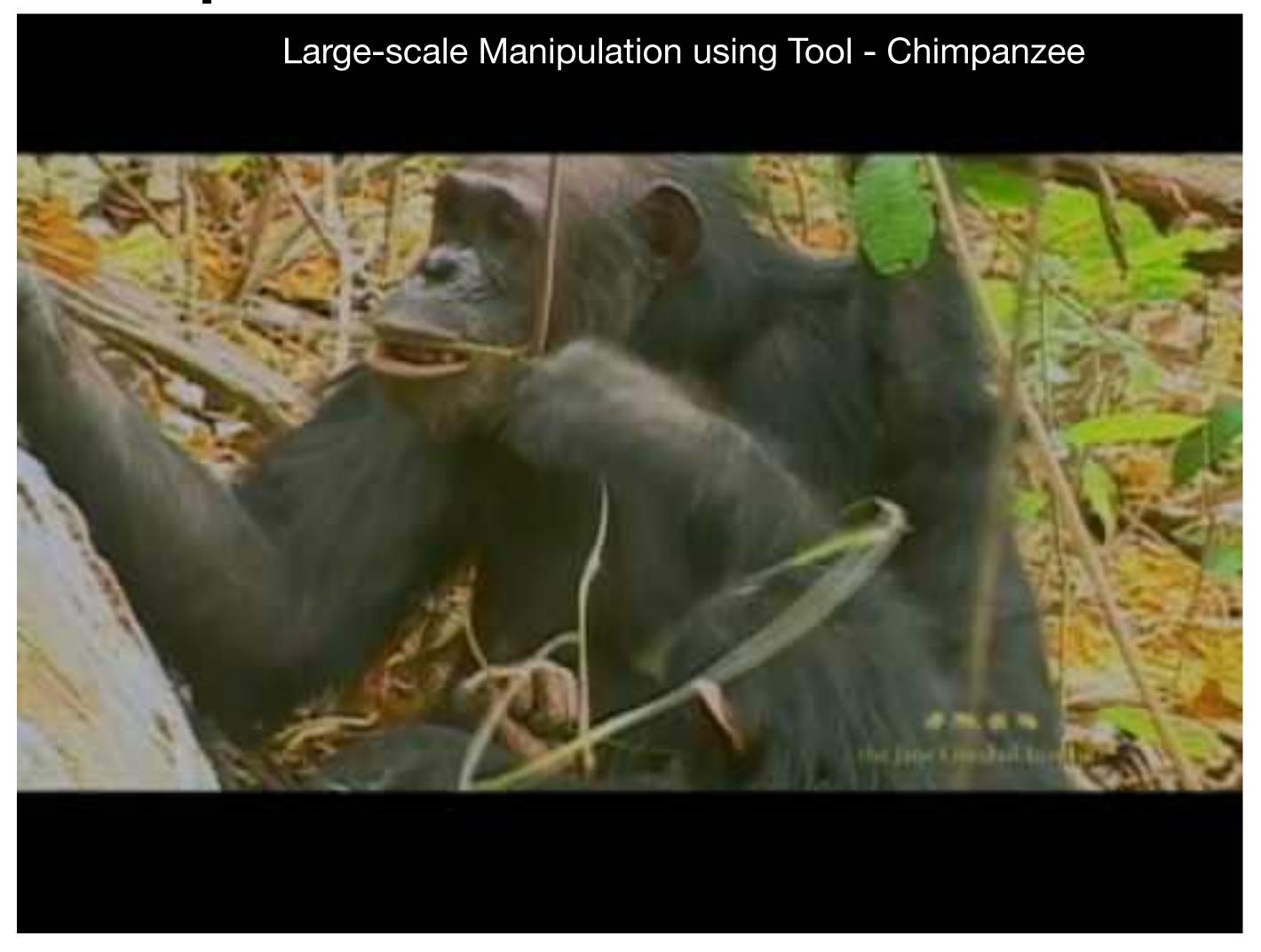


Locomotion is a form of manipulation??
Duality Principle

https://youtu.be/xNjymt6oCcQ

https://cdn2.vectorstock.com/i/1000x1000/53/51/big-dung-beetle-that-pushes-dirty-ball-vector-19965351.jpg https://t3.ftcdn.net/jpg/01/62/59/04/360 F 162590489 5lcesYmlOK0RC4T4r5lydft8aQmpCwl7.jpg





https://www.youtube.com/watch?v=inFkERO30oM



#### Animal Manipulation



https://www.youtube.com/watch?v=YePKbjODrto



### Animal Manipulation



https://www.youtube.com/watch?v=BXi3xJriGZY



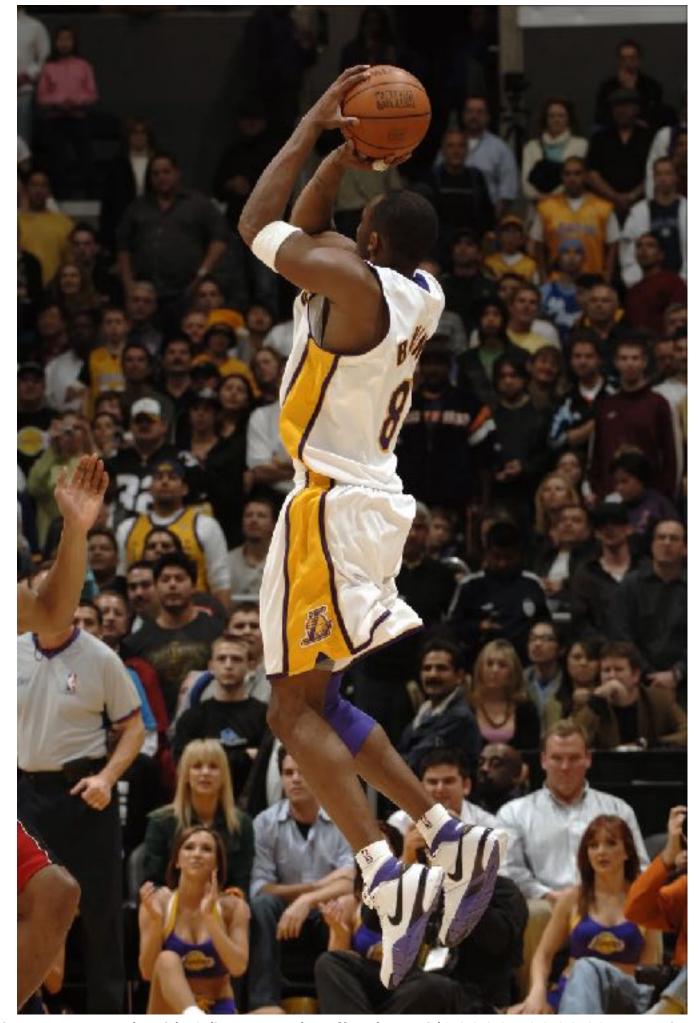
#### Animal Manipulation



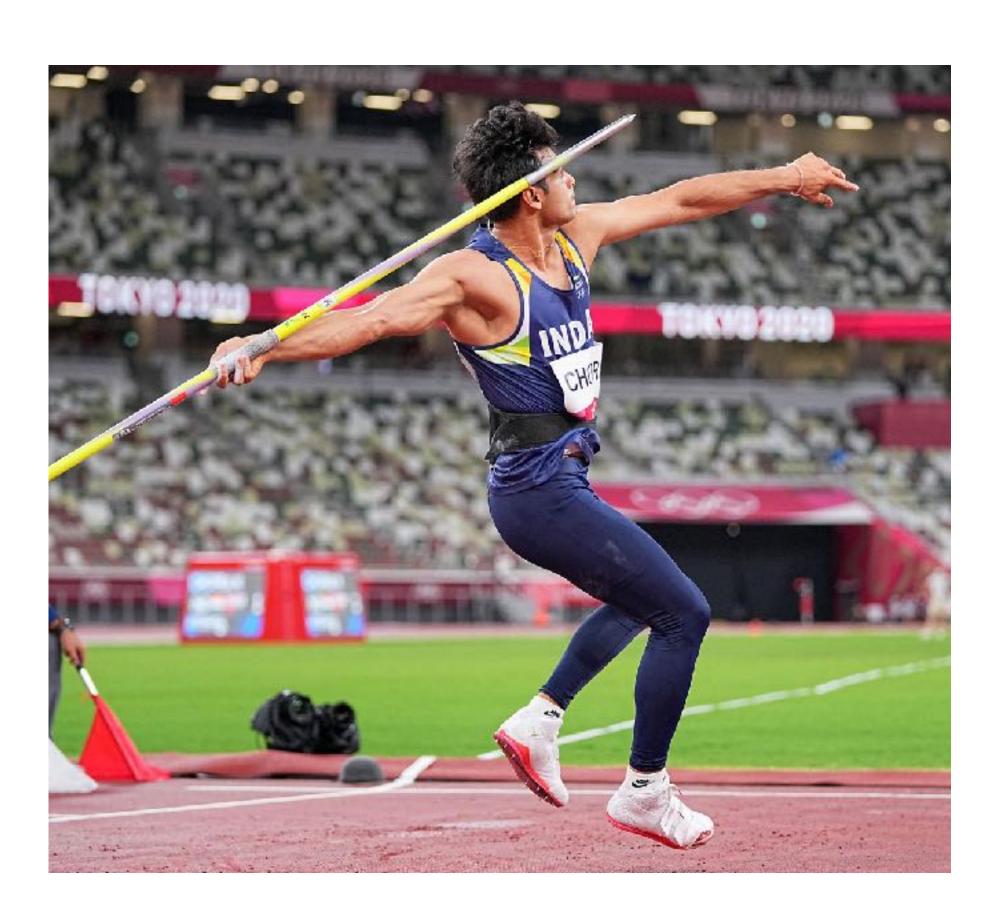
https://gifdb.com/images/high/insect-fly-rubbing-hands-tnpegh6d412vjafu.gif



Human Manipulation







https://media.cnn.com/api/v1/images/stellar/prod/210807101343-restricted-01-neeraj-chopra-olympics-08-07-2021.jpg?q=w 2953,h 1984,x 0,y 0,c fill https://www.espncricinfo.com/photo/shoaib-akhtar-in-action-against-bangladesh-309353?objectId=306979 https://media.gq.com/photos/5e30a0329d87db000817865a/master/w\_1600%2Cc\_limit/03-how-kobe-bryant-changed-sneaker-history-gq-kanuary-2020.jpg



#### Human Manipulation



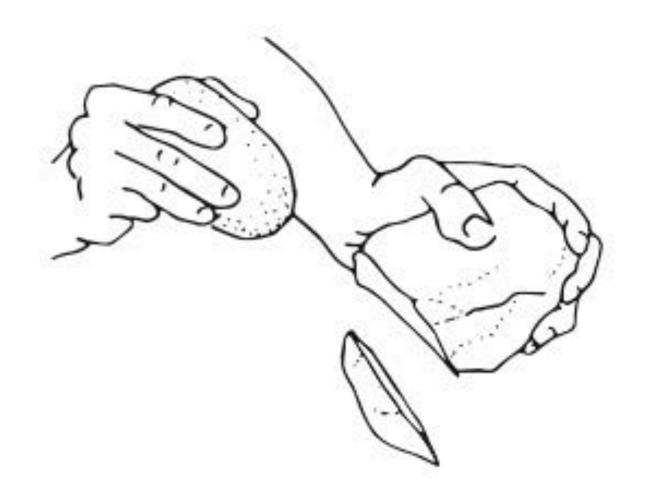


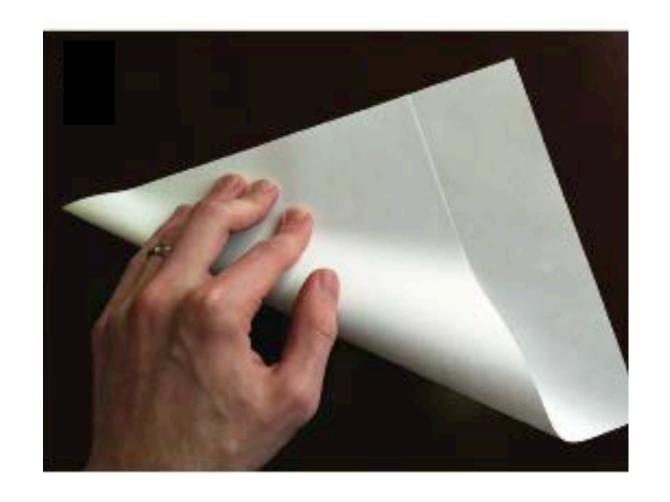






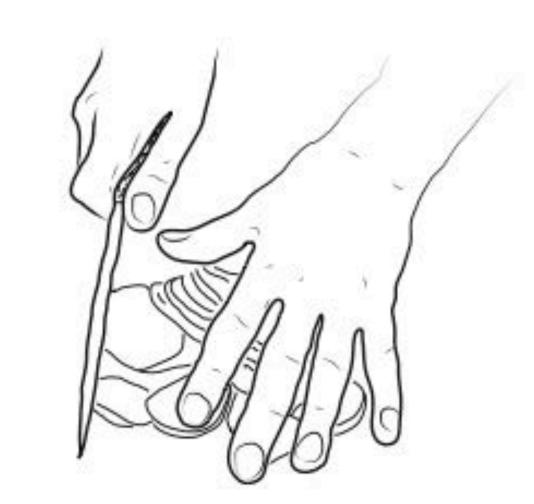
#### Human Manipulation











#### Figure

Examples of human manipulation. (a) Throwing a baseball. (b) Knapping a stone tool. (c) Folding origami. (d) Cutting a potato. (e) Bimanual manipulation of a potato while the knife is parked in an ulnar grasp. (f) Pushing potato slices with a knife and spread fingers. Panel a from video (https://youtu.be/jZKvJY6gDfg) by Power Drive Performance (http://www.pitcherspowerdrive.com), reproduced with permission. Panel b by Helen Beare (https://australianmuseum.net.au/image/stone-tools-initial-reduction-flaking), reproduced with permission from the Australian Museum. Panel c from video by YouTube user kiwiwhispers ASMR (https://youtu.be/SNfLEnnP6Nc), reproduced with permission. Panels d-f adapted from frames of The French Chef (28).

Figure from - Mason, Matthew T. "Toward robotic manipulation." Annual Review of Control, Robotics, and Autonomous Systems 1 (2018): 1-28.



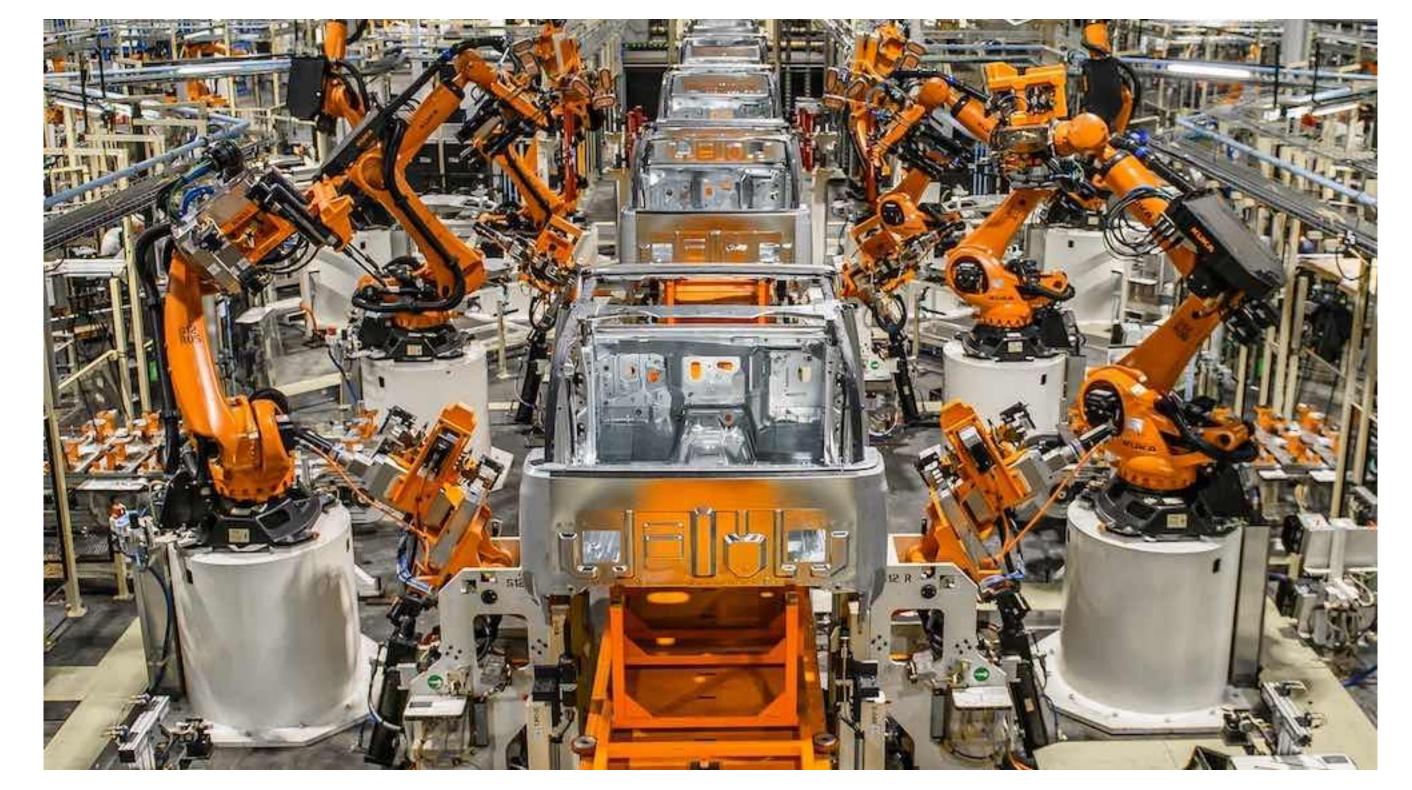
#### Elements of Robotic Manipulation

- Programmed Motion
- Compliant Motion
- Structured pick-and-place manipulation
- Unstructured pick-and-place manipulation
  - Path planning
  - General-purpose grippers
  - Grasp and placement pose planning
- Assembly and task mechanics
- In-hand Manipulation
- Nonprehensile Manipulation
- Whole-X Manipulation



#### Programmed Motion

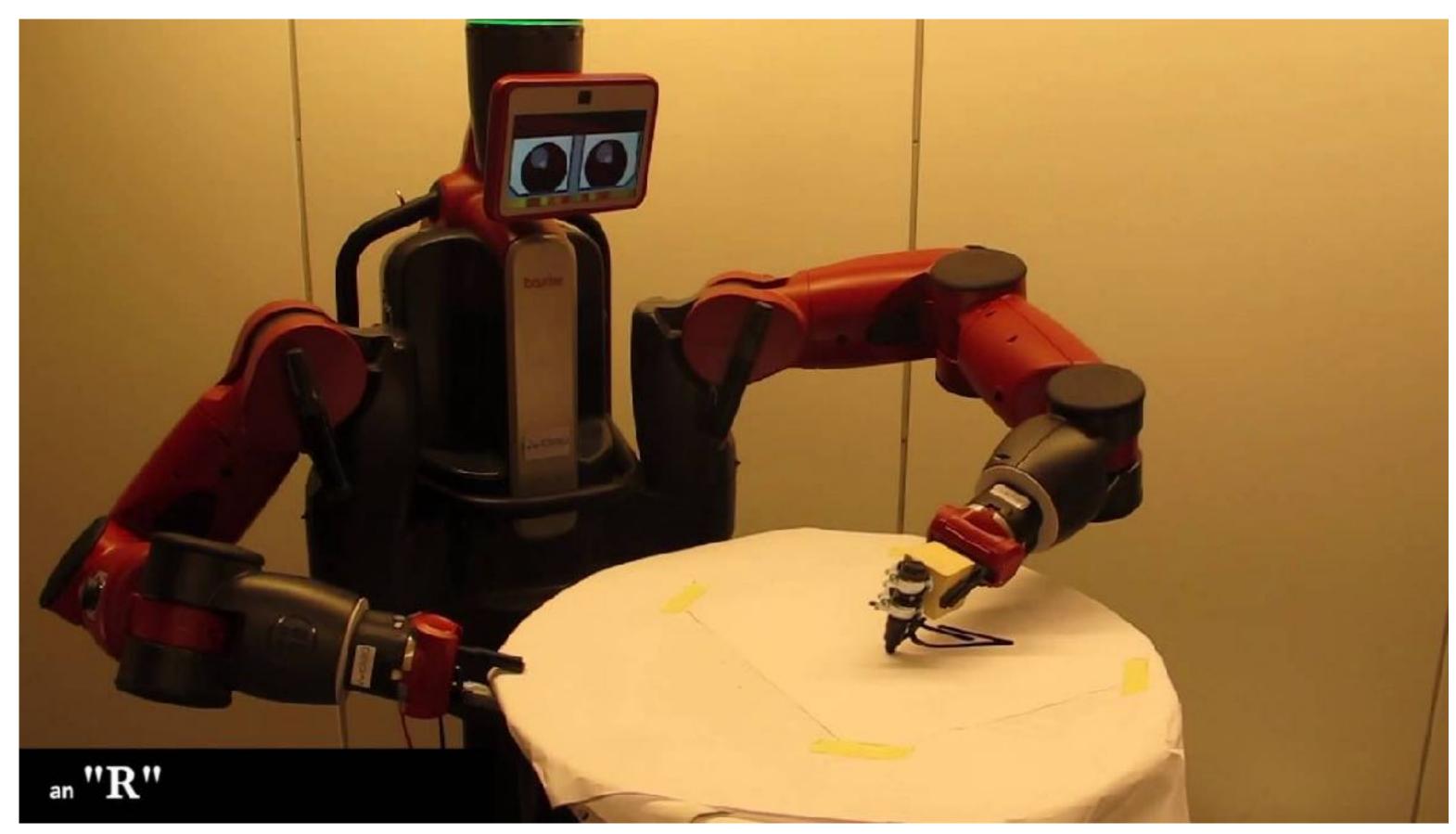
 Rests on the developments in motors, transmissions, encoders, kinematics, mechanism design, dynamic modeling and control





#### Compliant Motion

- Context of teleoperation
- Hybrid-position/force control
- Impedence control



https://www.youtube.com/watch?app=desktop&v=KU--TOMDDFU



#### Structured pick-and-place manipulation

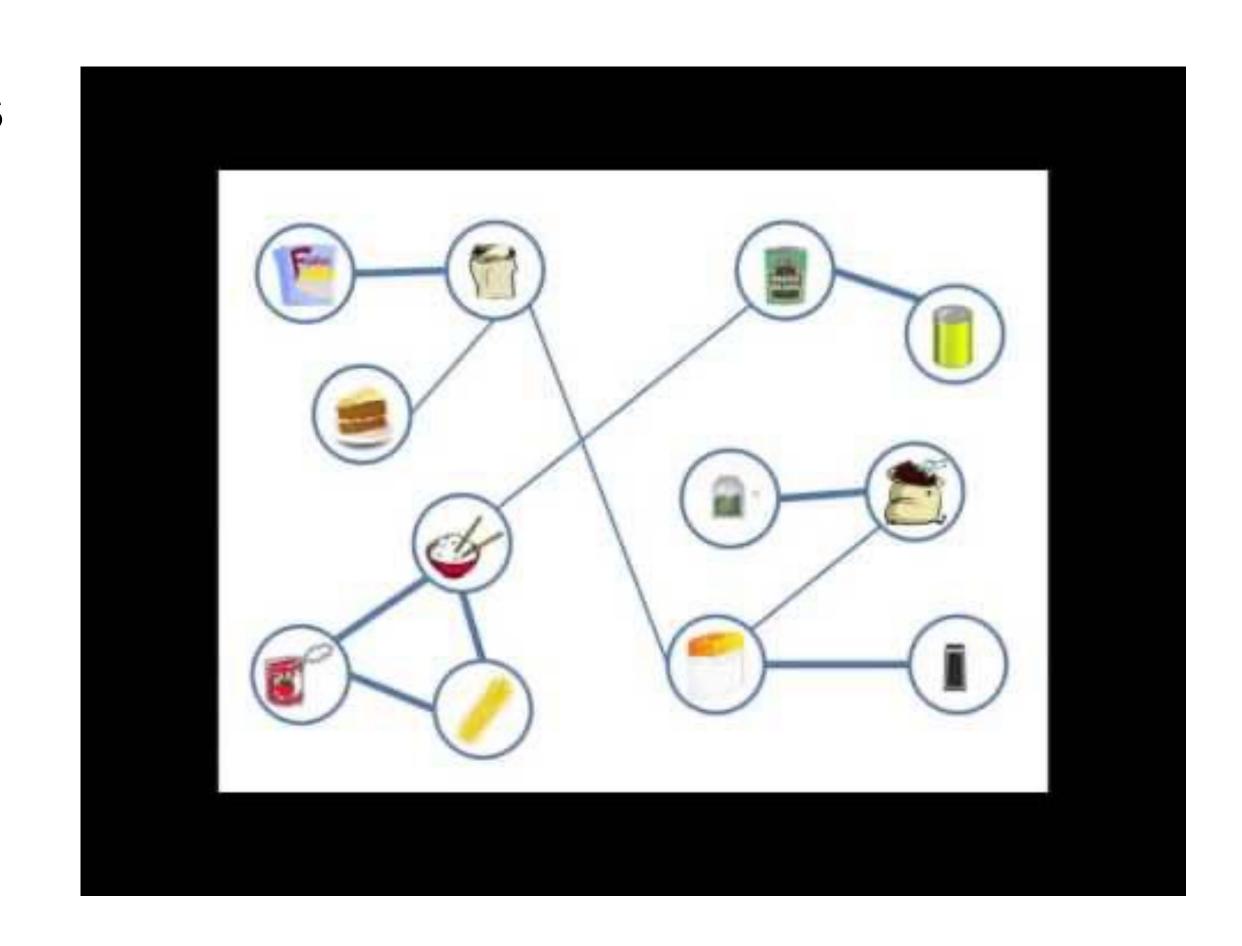
- Moving a sequence of objects one at a time from one place to another.
- Structured environment and scenario
  - Objects are identical
  - Motion is repetitive
  - Gripper design and motion programming is done offline.





#### Unstructured pick-and-place manipulation

- Planning software to produce arm motions
- Grippers that can handle a broad range of objects
- Grasp pose planning
- Stable placement pose planning



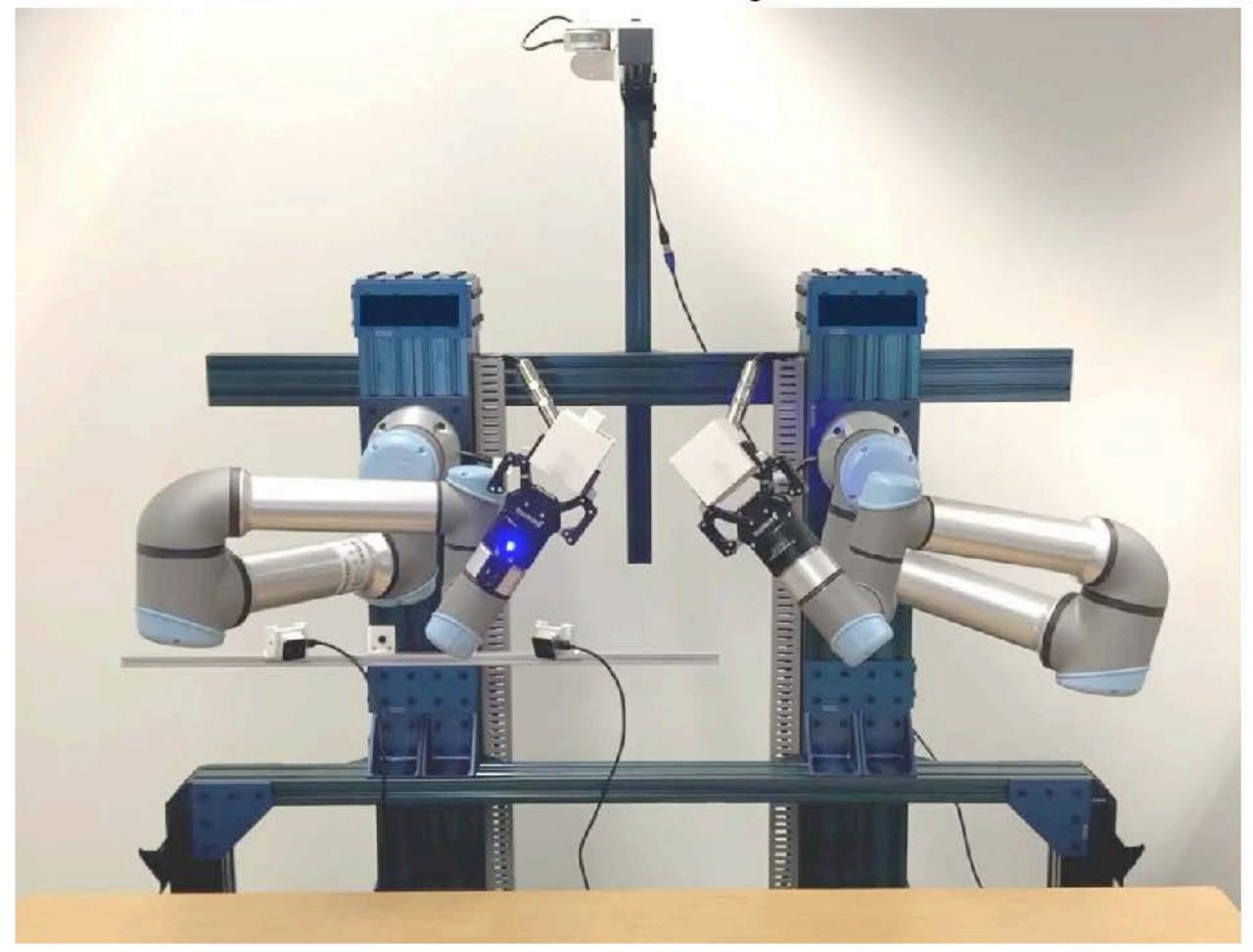
Abdo, Nichola, Cyrill Stachniss, Luciano Spinello, and Wolfram Burgard. "Organizing objects by predicting user preferences through collaborative filtering." *The International Journal of Robotics Research* 35, no. 13 (2016): 1587-1608.

https://www.youtube.com/watch?app=desktop&v=\_icB8QcycMM



#### Robotic Assembly Task

Task: Geometry Informed Object Assembly



Chahyon Ku, Carl Winge, Ryan Diaz, Wentao Yuan, Karthik Desingh "Evaluating Robustness of Visual Representations for Object Assembly Task Requiring Spatio-Geometrical Reasoning, "Accepted ICRA 2024.



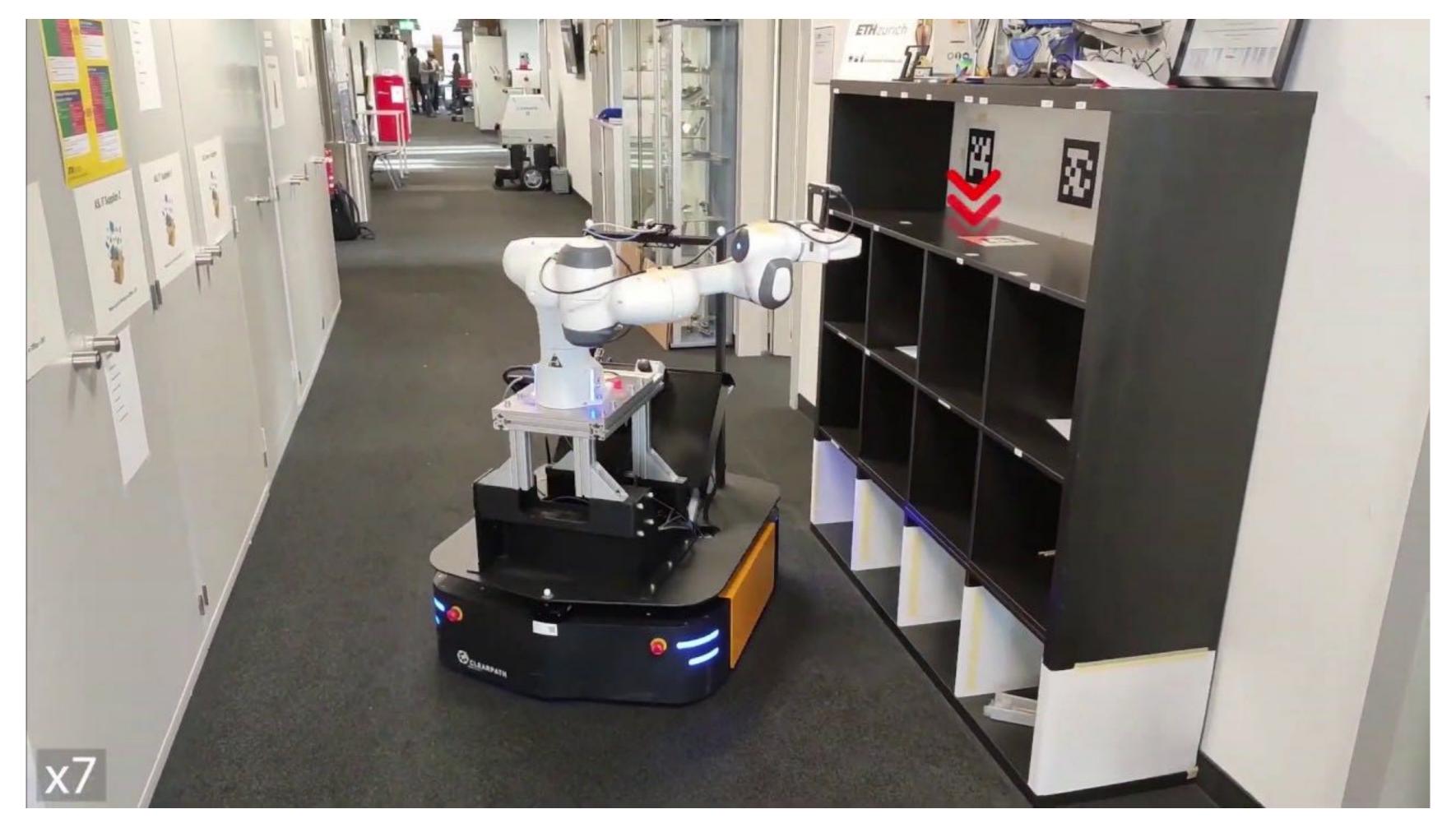
#### In-hand Manipulation



Chen, Tao, Jie Xu, and Pulkit Agrawal. "A system for general in-hand object re-orientation." In Conference on Robot Learning, pp. 297-307. PMLR, 2022.



#### Whole-body manipulation



Kindle, Julien, Fadri Furrer, Tonci Novkovic, Jen Jen Chung, Roland Siegwart, and Juan Nieto. "Whole-body control of a mobile manipulator using end-to-end reinforcement learning." arXiv preprint arXiv:2003.02637 (2020).

https://www.youtube.com/watch?v=3qobNCMUMV4



## Taxonomy of Grasps

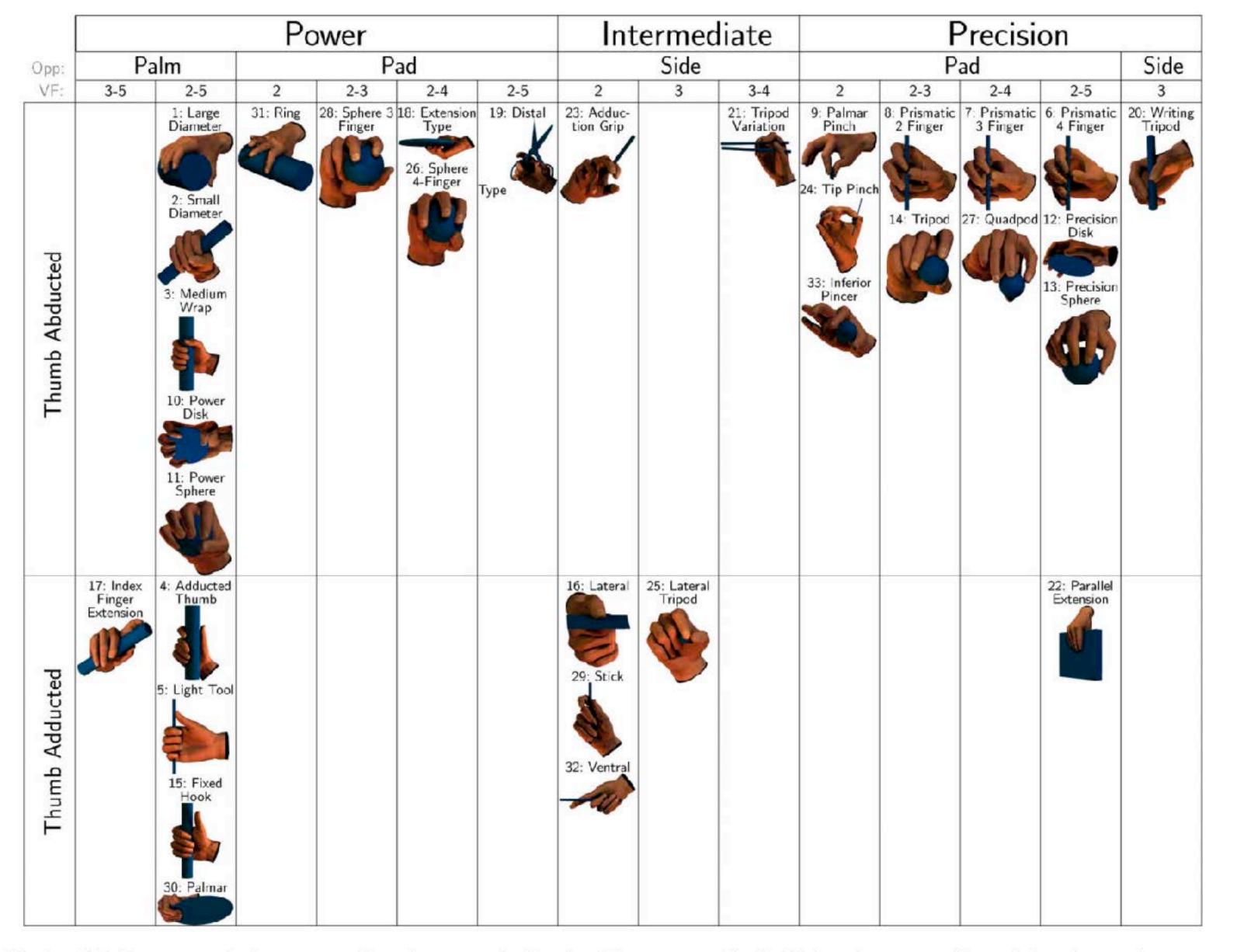


Fig. 4. GRASP taxonomy that incorporates all previous grasp classifications. The grasps are classified in the columns according to their assignment into power, intermediate and precision grasp, the opposition type, and the VF assignment. The assignment of the rows is done be the position of the thumb that can be in an abducted position.

Feix, Thomas, Javier Romero, Heinz-Bodo Schmiedmayer, Aaron M. Dollar, and Danica Kragic. "The grasp taxonomy of human grasp types." IEEE Transactions on human-machine systems 46, no. 1 (2015): 66-77.



#### Why is robot manipulation challenging?

- Mechanism
- Perception
- Modeling and Control
- Planning
- Uncertainty



#### Future research challenges

- 1. Is there a fundamental and precise metric for comparing manipulative behaviors, or for comparing tasks, that would provide a basis for measuring progress in the field?
- 2. How can we best take advantage of advances in machine learning to advance our understanding and improve our technology?
- 3. How do we develop the adaptability, robustness, and breadth of behaviors exhibited by animals and humans?



# Next lecture: Planning

