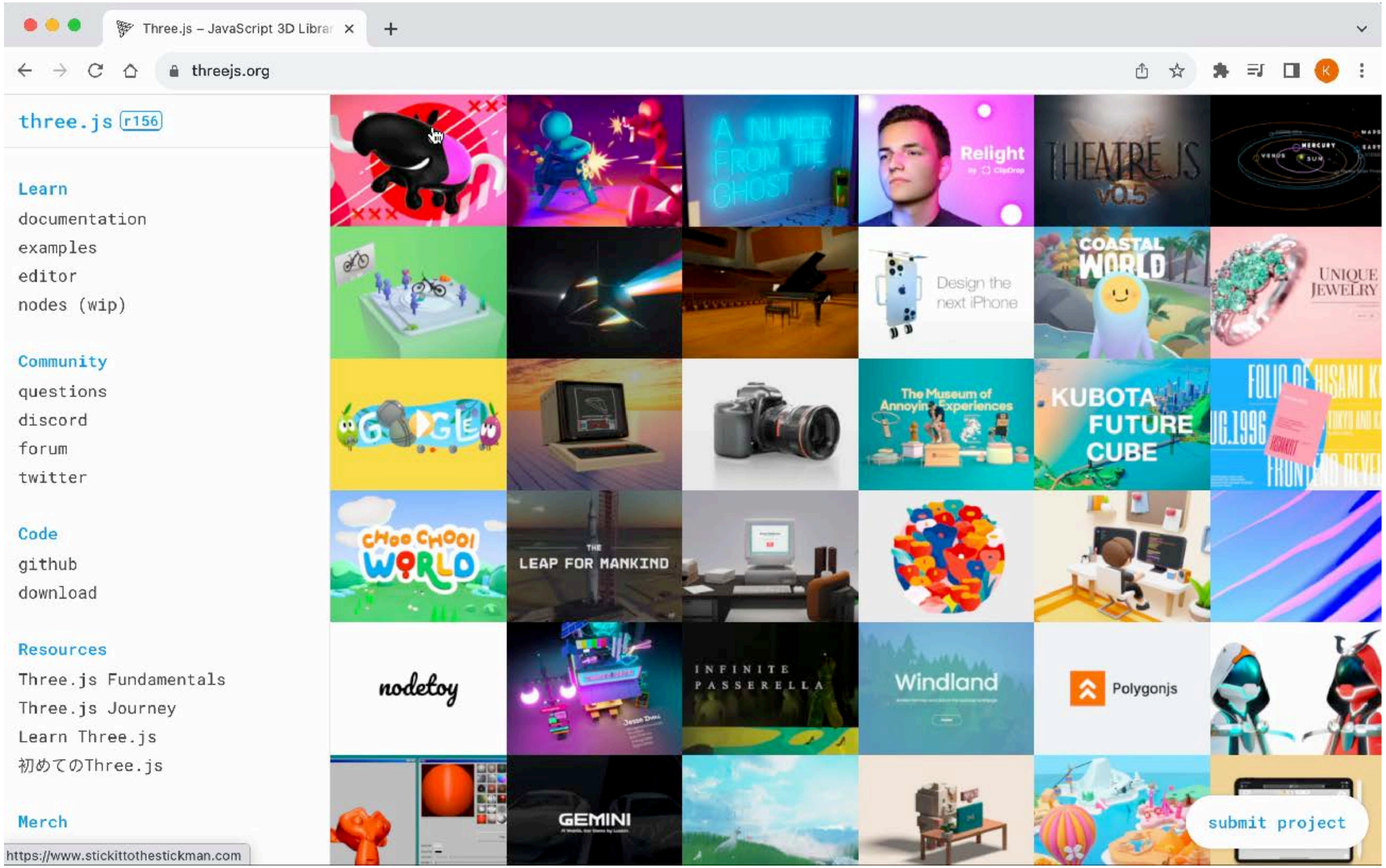
Lecture 04
Representations - I
Transformations

Course Logistics

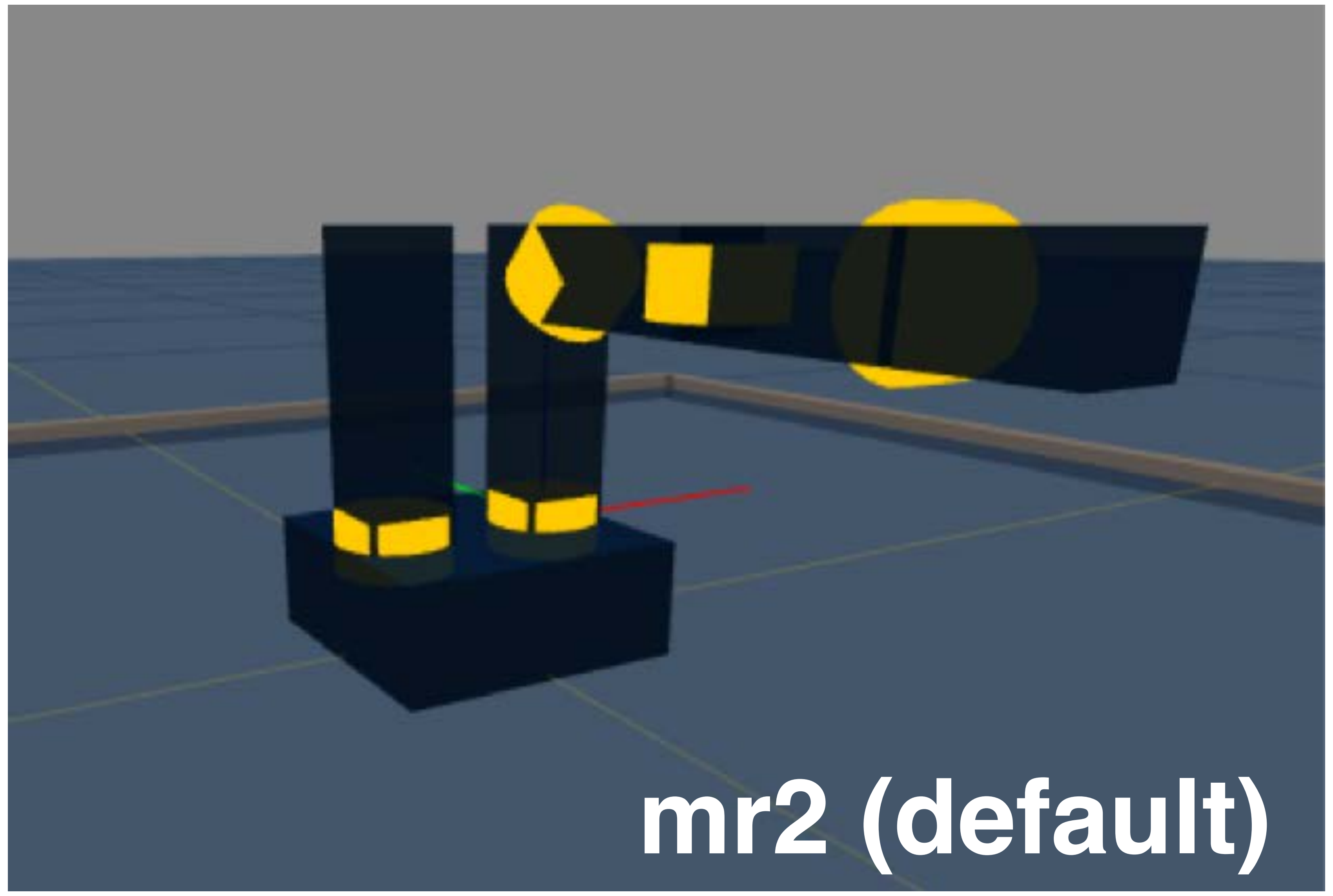
- Everyone should be on Ed discussion board now.
- Everyone should be on Gradescope now.
- Quiz 2 will be posted tomorrow (Tuesday) 6pm and will be due on 01/31 (Wednesday) noon.
- Project 1 is due on **01/31 (Wednesday) 11:59 pm CT**.
- Project 2 will be released on 01/31.
- Autograder is available. Please check to see if you have access to it.
  - 10 submissions per day. This is a good coding practice and we will not increase it.
  - You don't need a valid solution to test autograder. So if you haven't submitted anything just try it first.

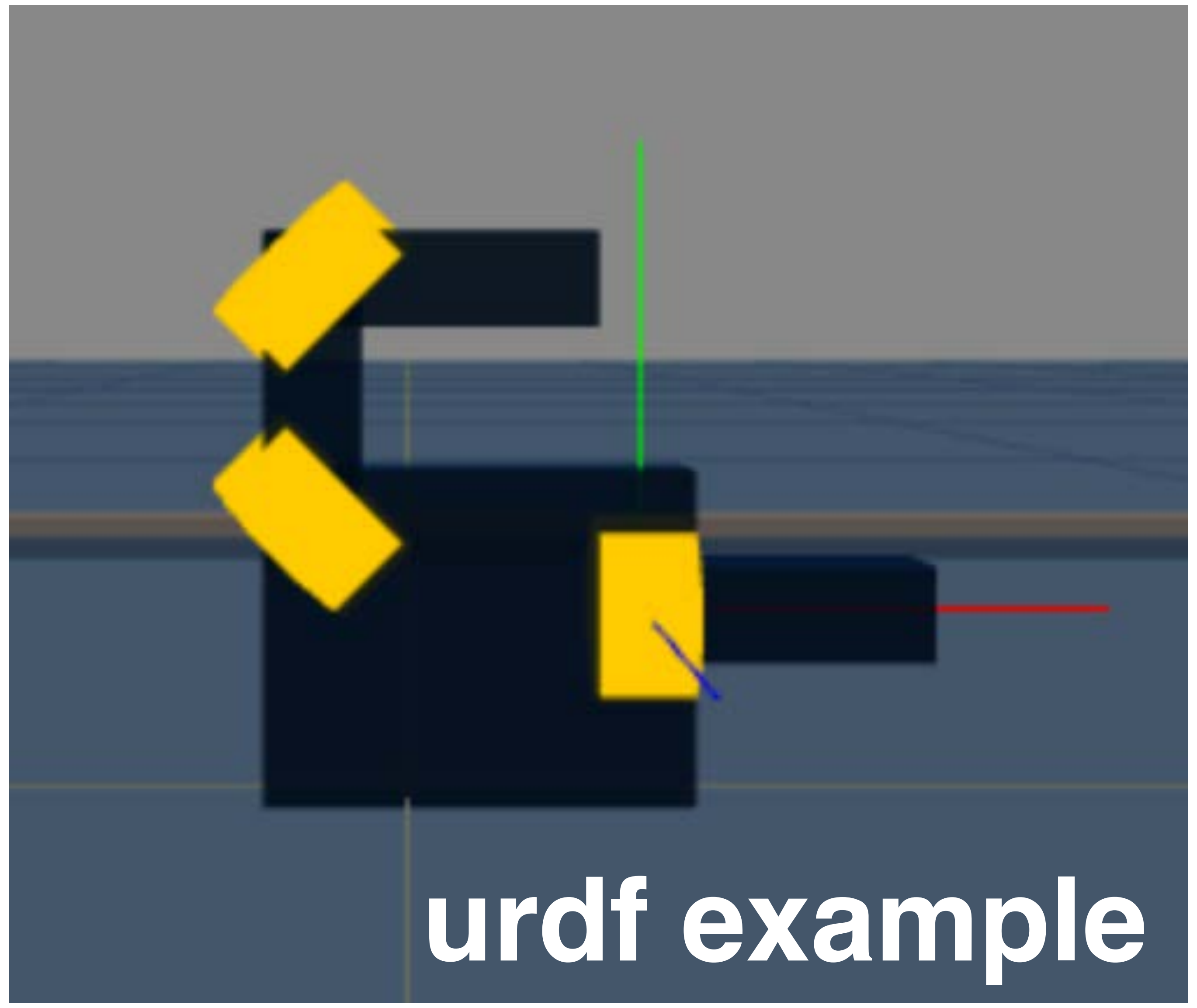
Why JavaScript?

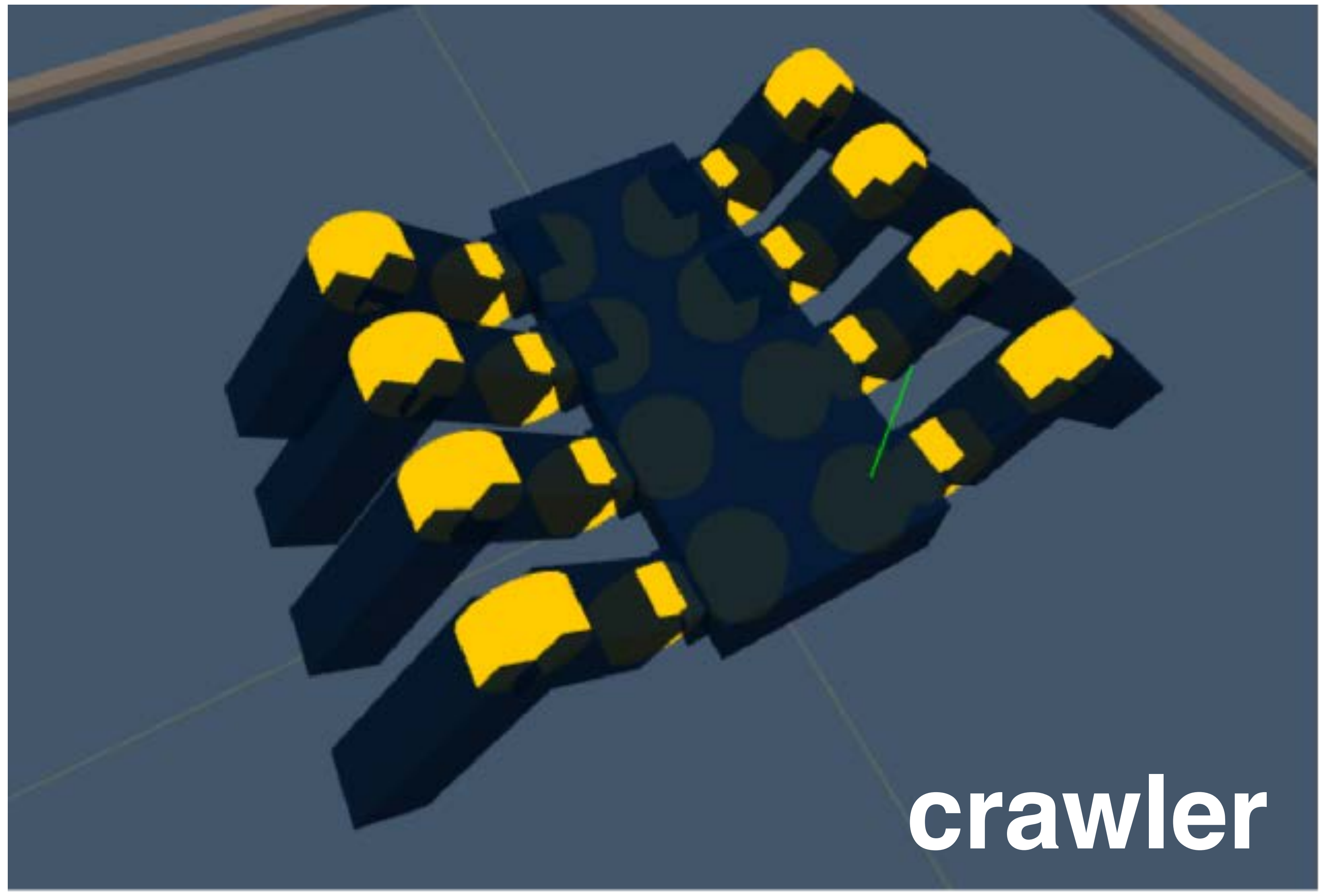
- **ThreeJS**



Why JavaScript?

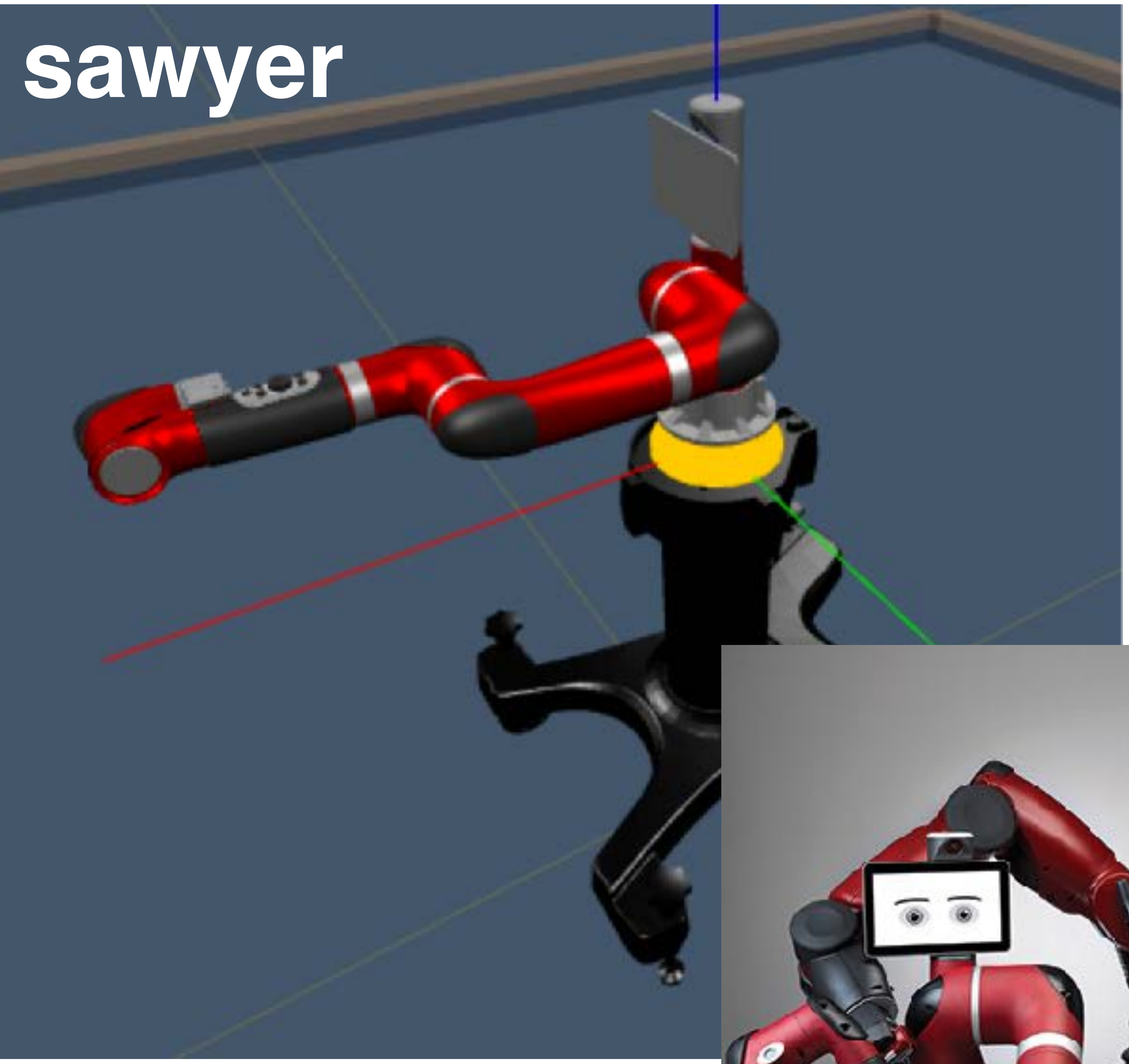


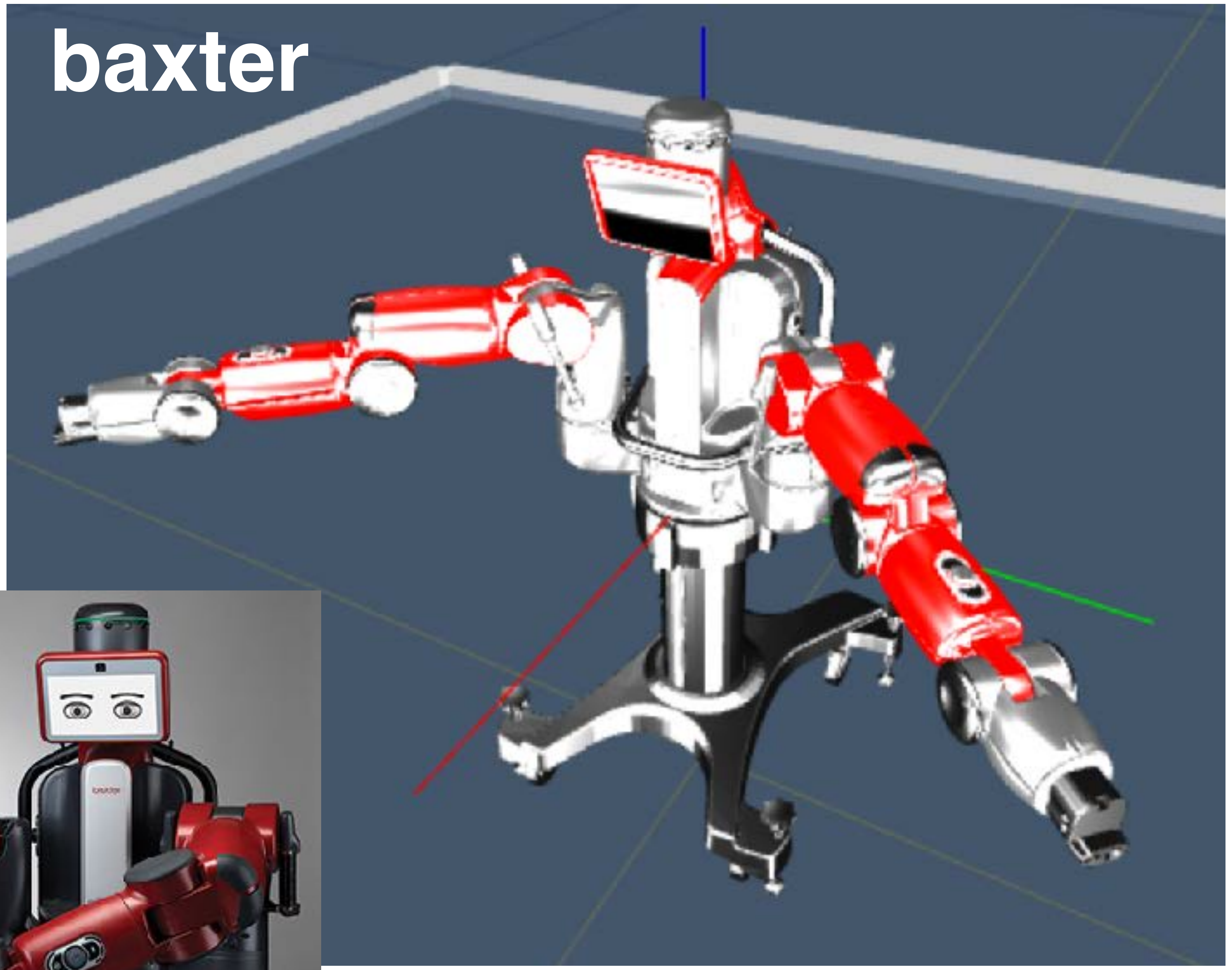


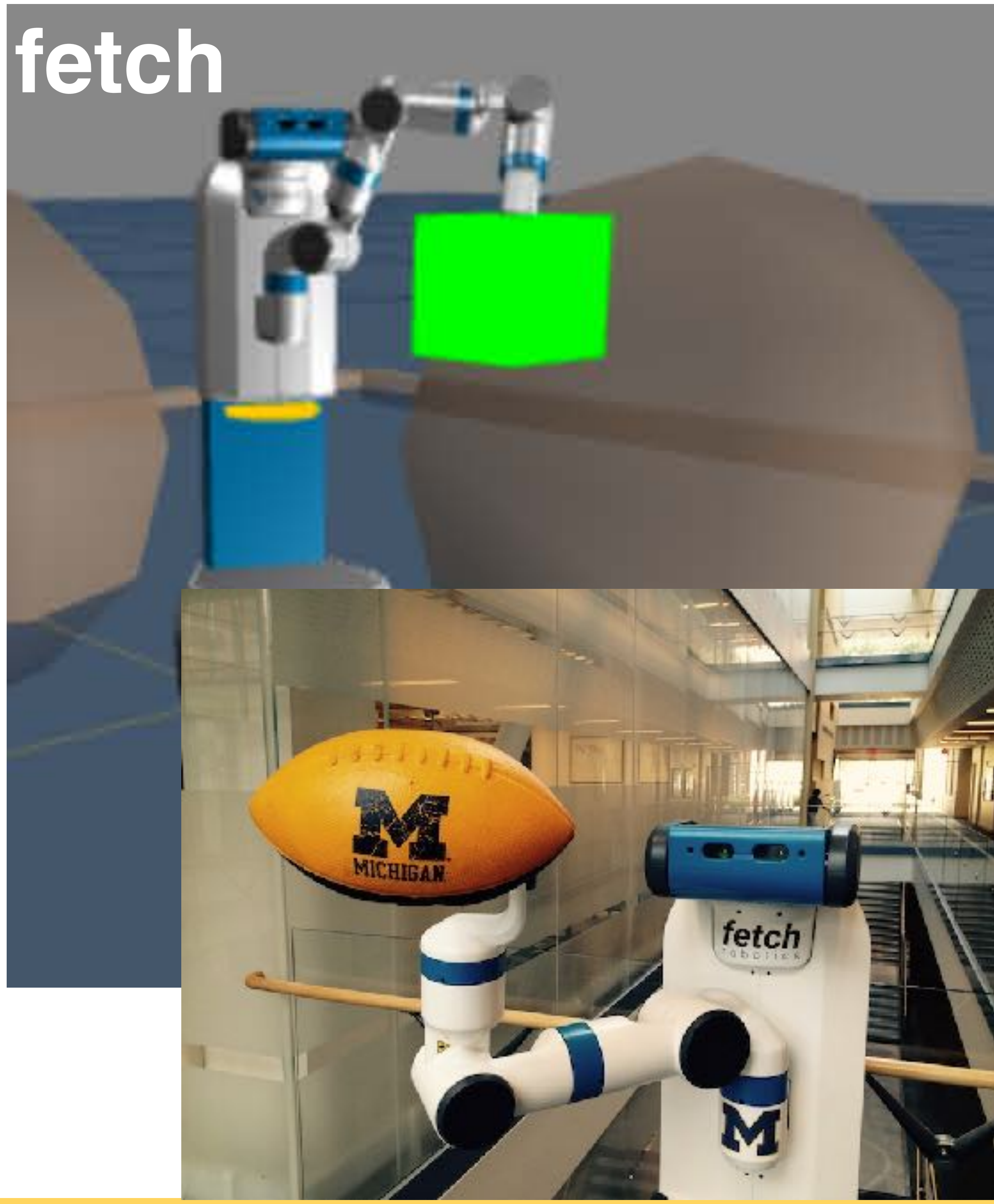


Why JavaScript?

You can load a URDF of a famous robot!

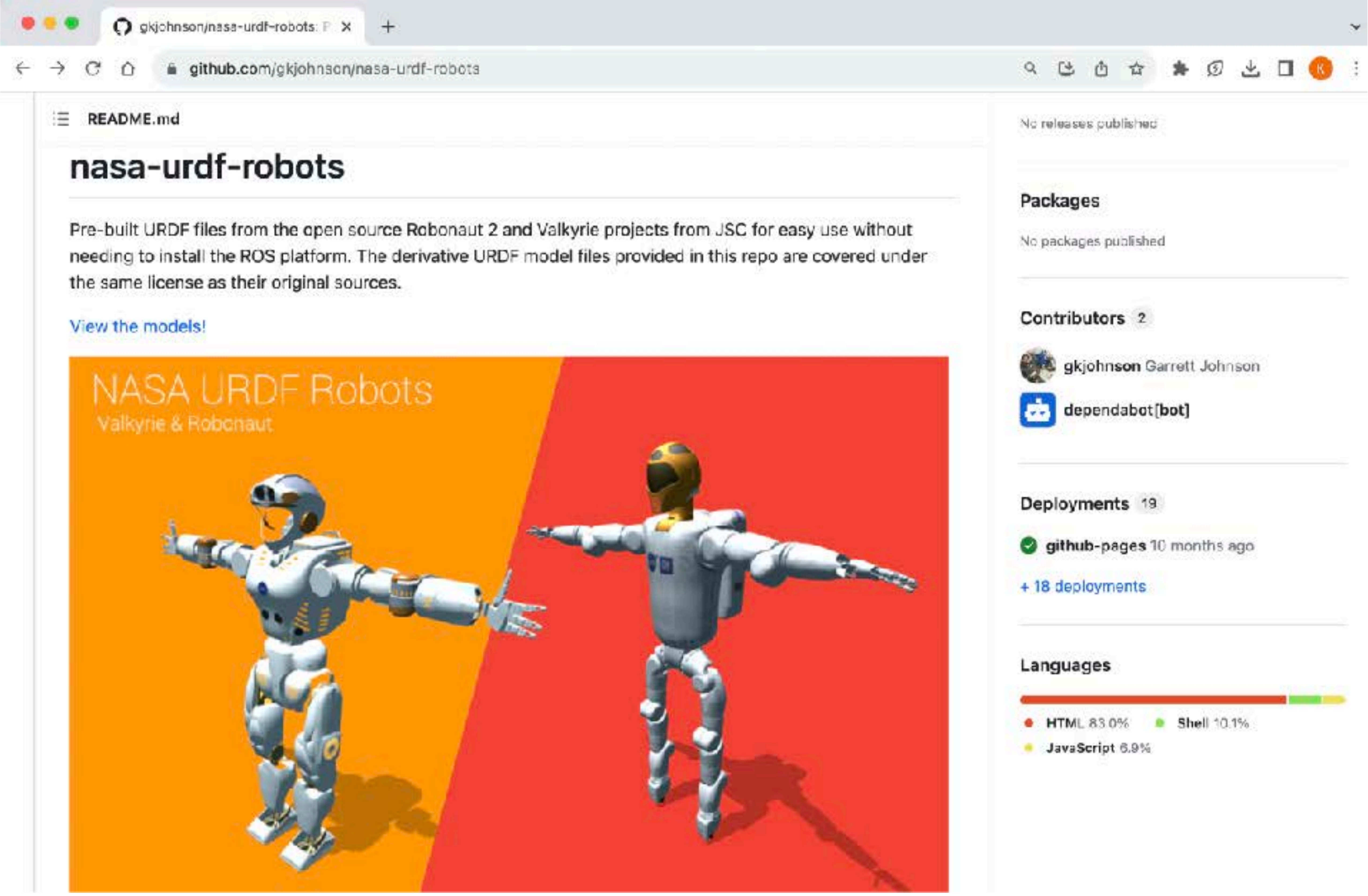


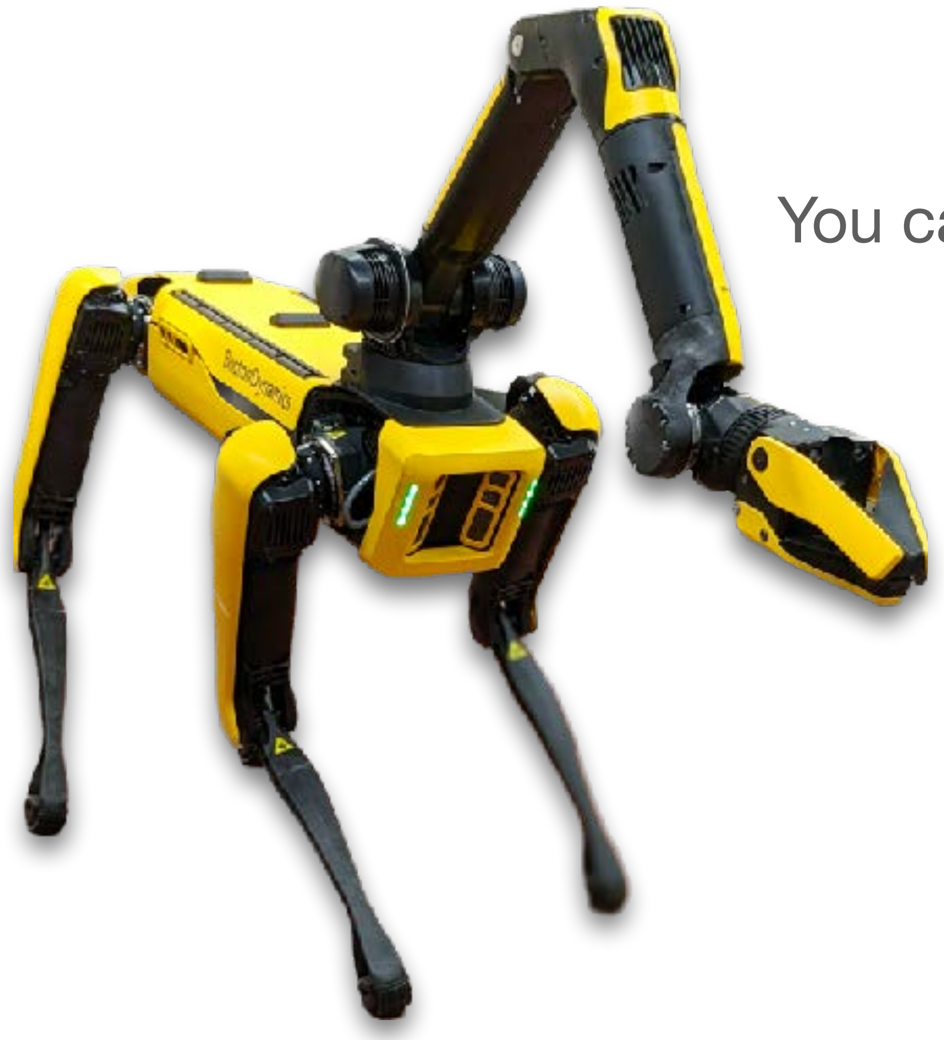




Why JavaScript?

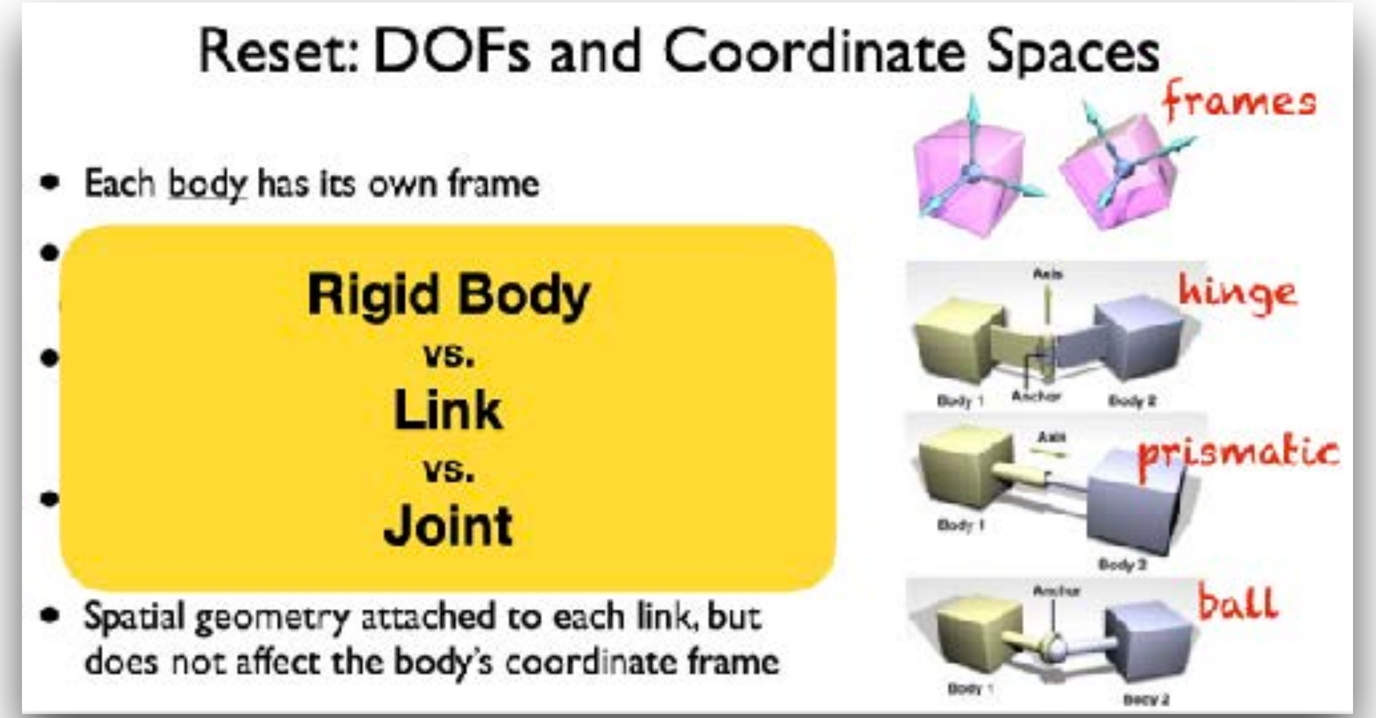
More robot models!!!

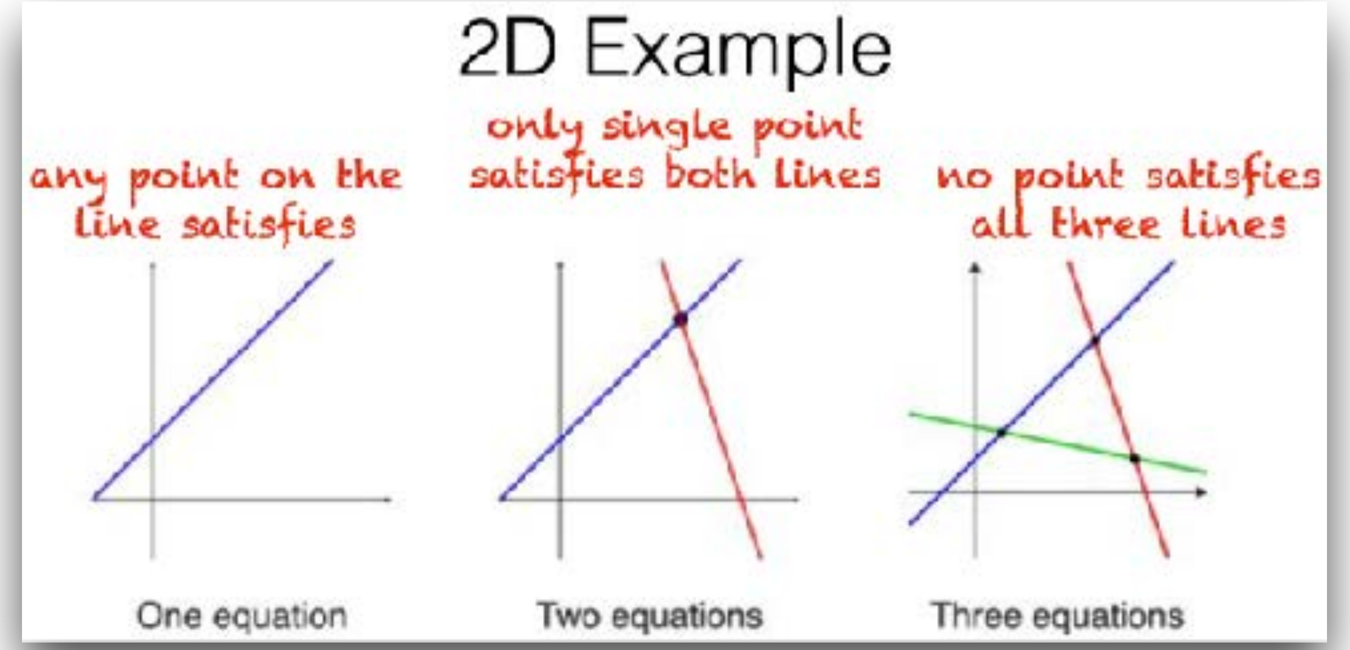


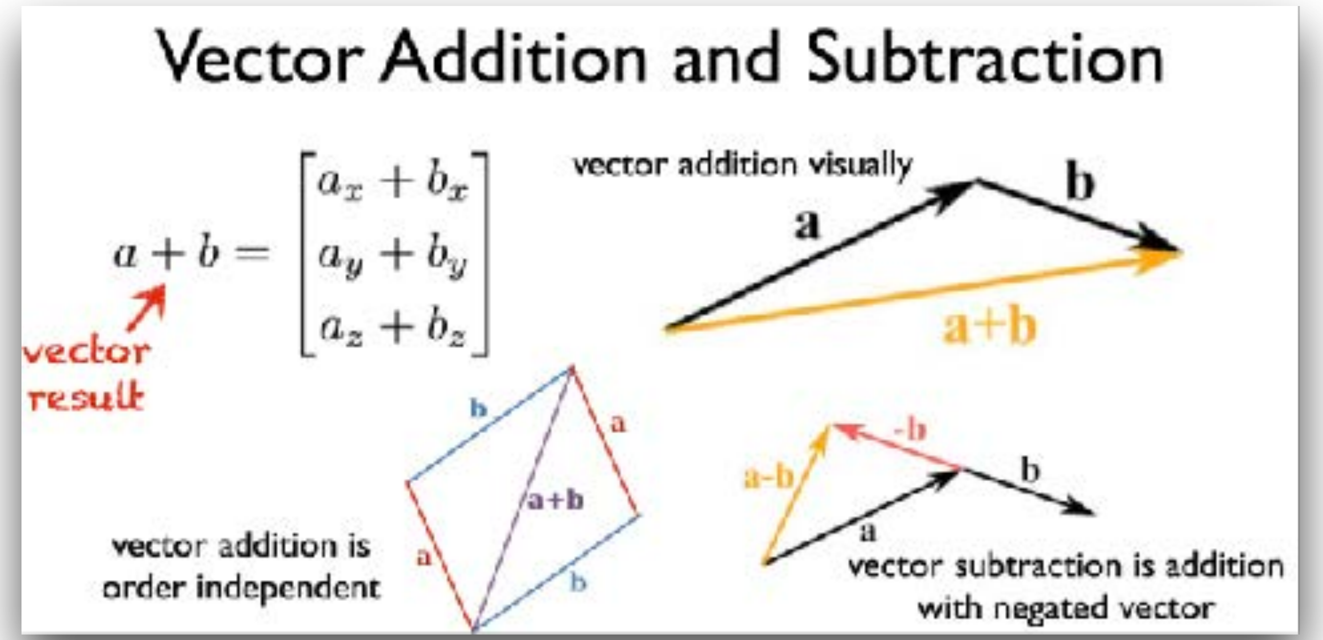


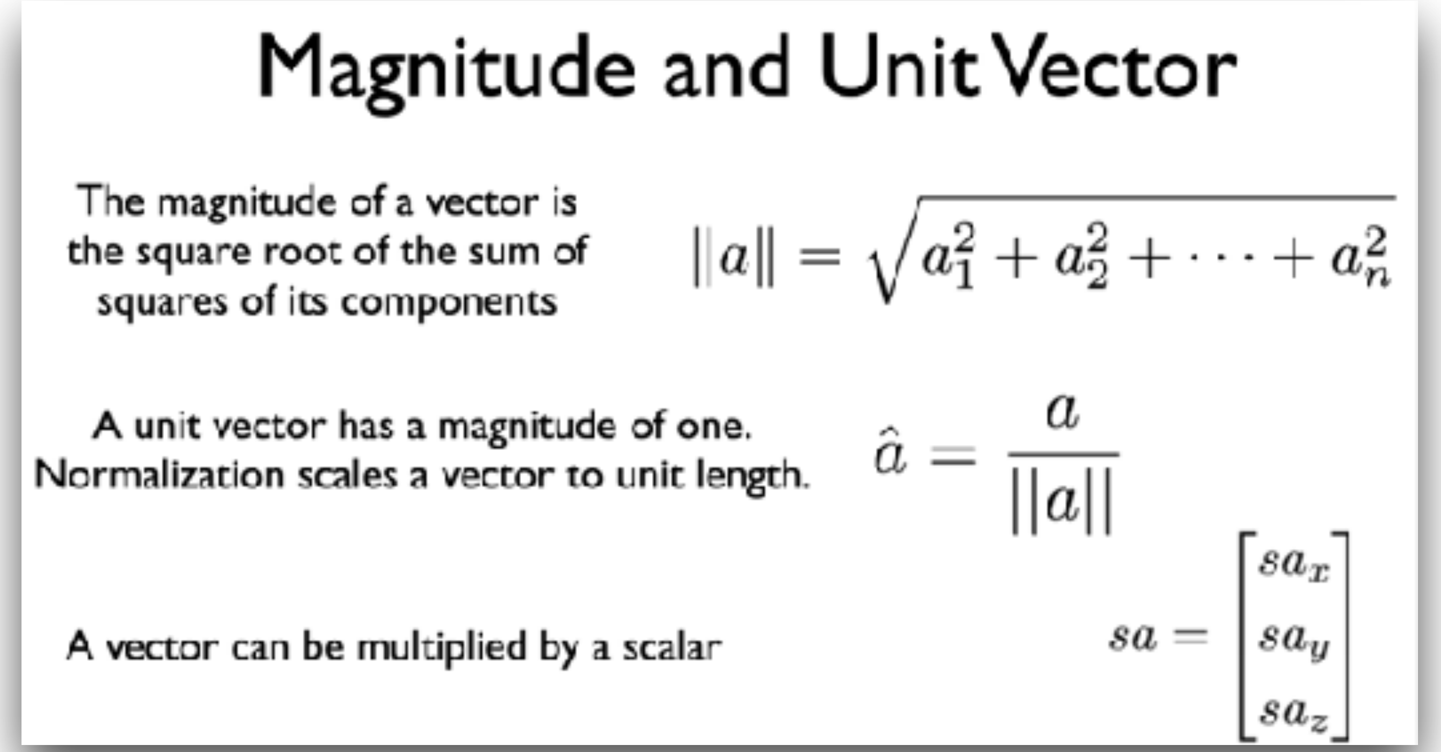
You can try to load URDFs of our lab robots too!
Ask the course staff for the URDFs.

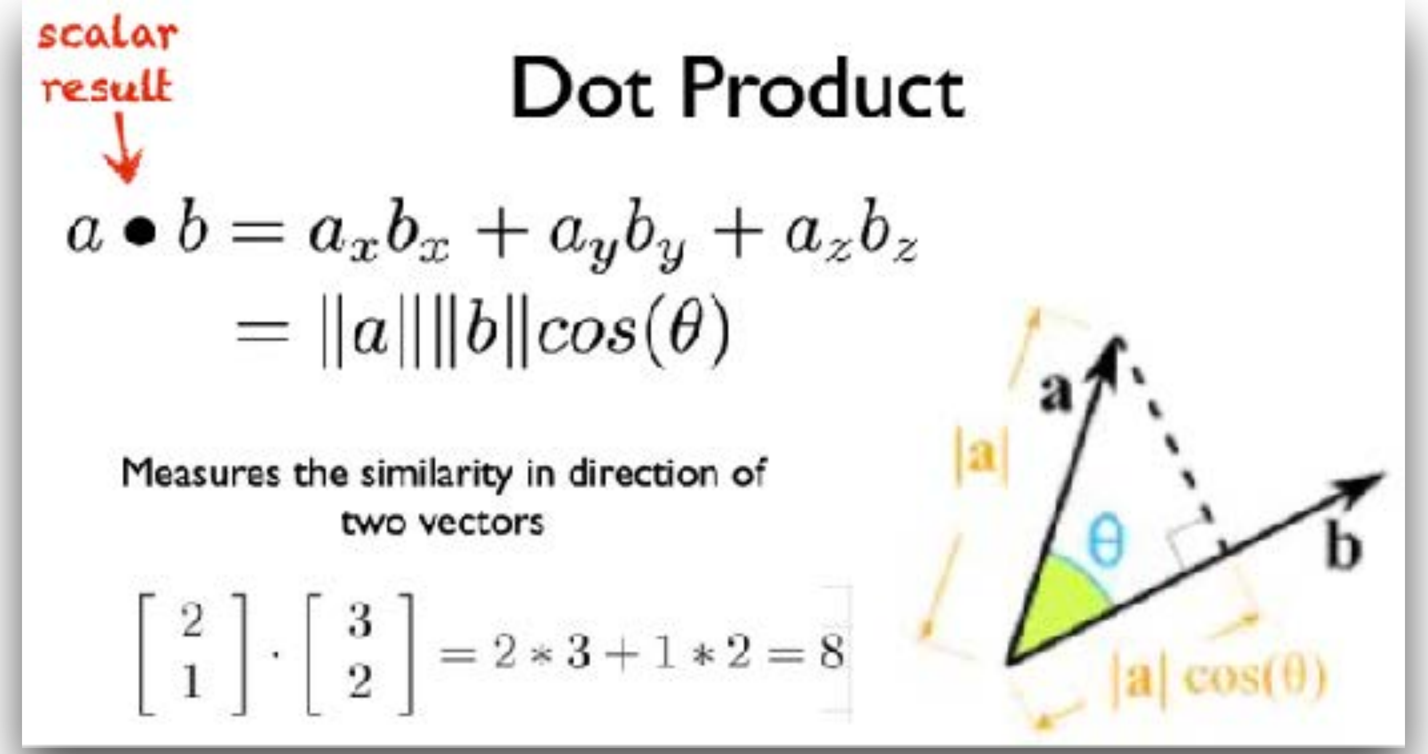
Previously

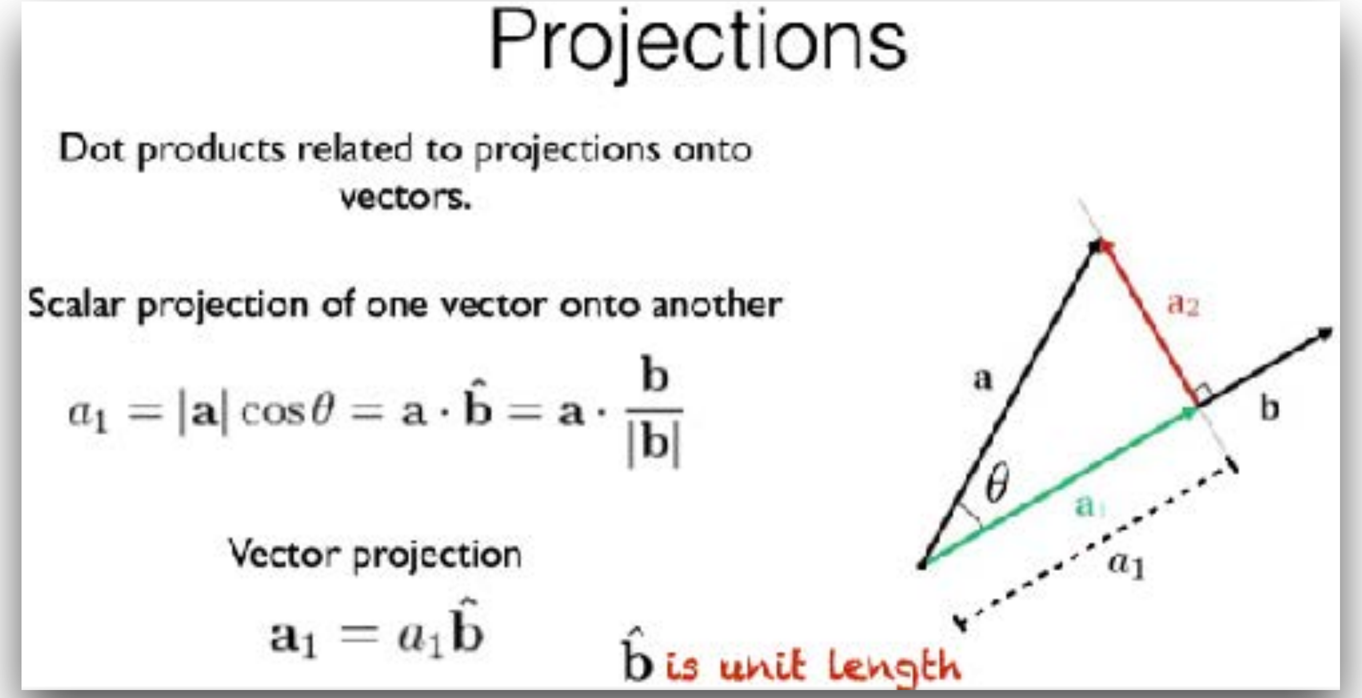

Reset: DOFs and Coordinate Spaces
frames
Each body has its own frame
Rigid Body
vs.
Link
vs.
Joint
hinge
prismatic
ball
Spatial geometry attached to each link, but does not affect the body's coordinate frame

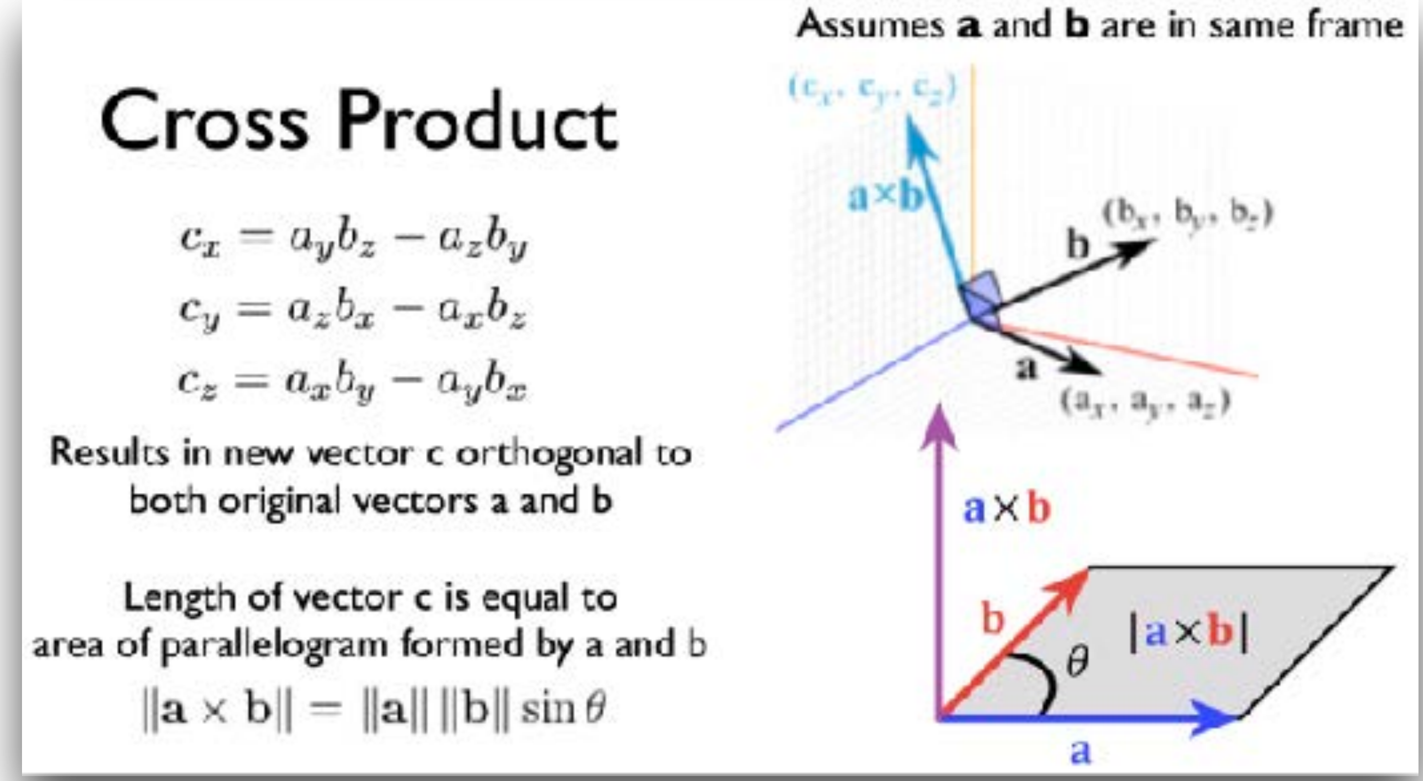

2D Example
any point on the line satisfies
only single point satisfies both lines
no point satisfies all three lines
One equation
Two equations
Three equations

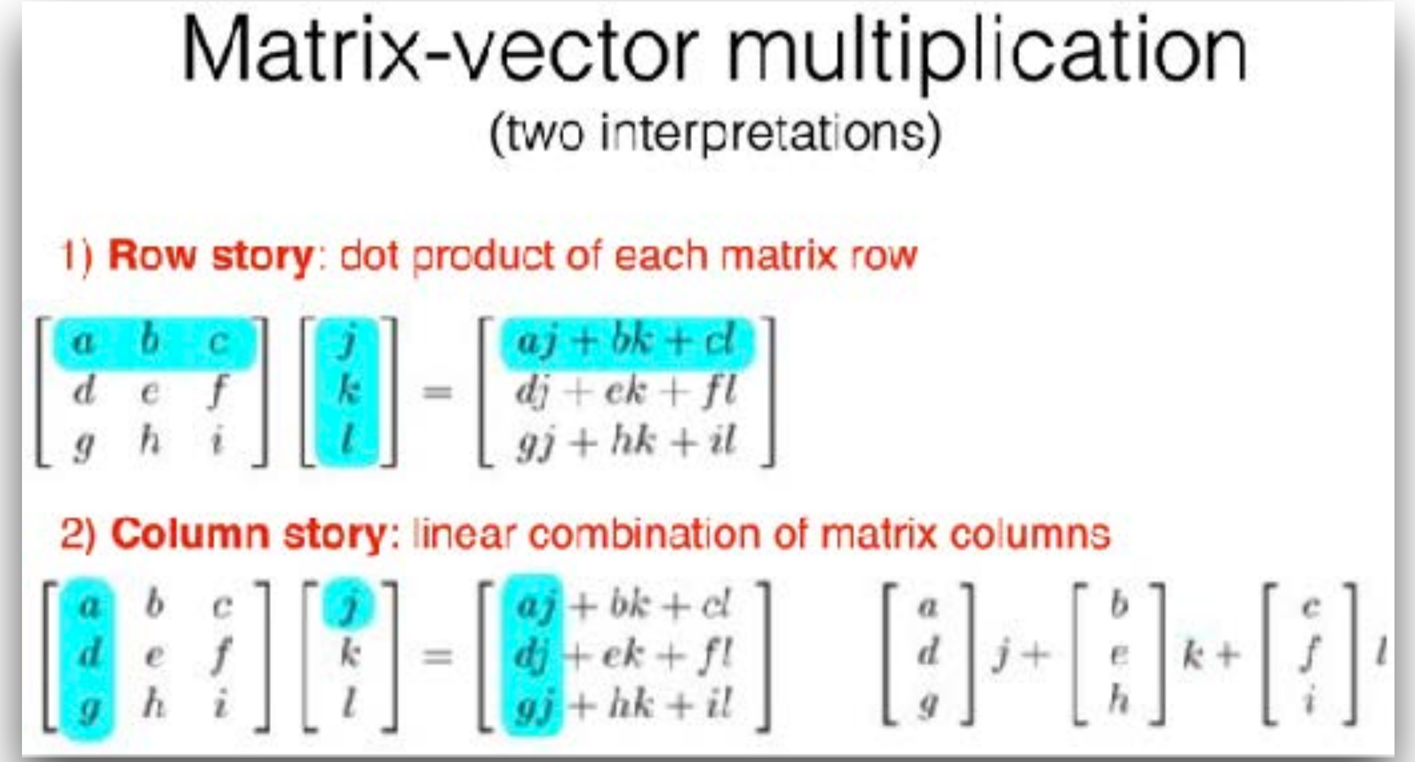

Vector Addition and Subtraction
$a + b = \begin{bmatrix} a_x + b_x \\ a_y + b_y \\ a_z + b_z \end{bmatrix}$
vector result
vector addition visually
a
b
a+b
vector addition is order independent
vector subtraction is addition with negated vector

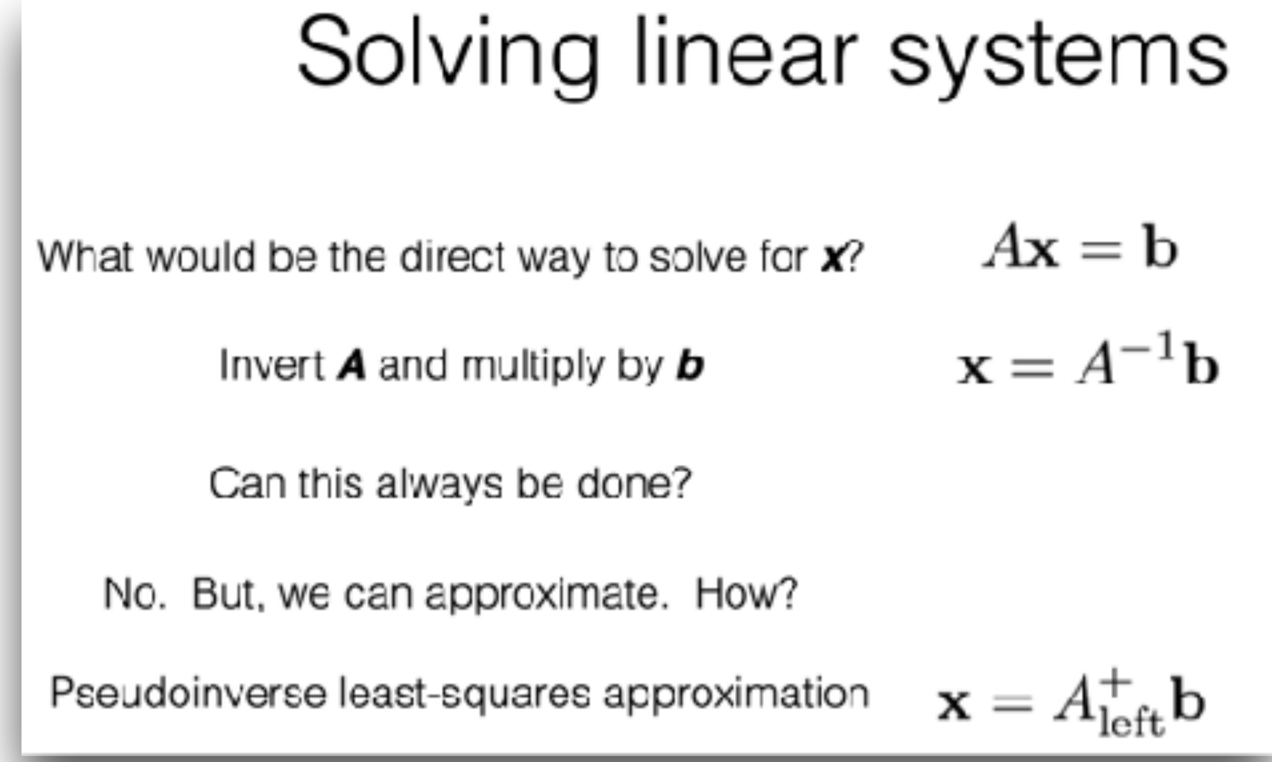

Magnitude and Unit Vector
The magnitude of a vector is the square root of the sum of squares of its components
$||a|| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$
A unit vector has a magnitude of one. Normalization scales a vector to unit length.
$\hat{a} = \frac{a}{||a||}$
A vector can be multiplied by a scalar
$sa = \begin{bmatrix} sa_x \\ sa_y \\ sa_z \end{bmatrix}$


scalar result
Dot Product
$a \bullet b = a_x b_x + a_y b_y + a_z b_z$
$= ||a|| ||b|| cos(\theta)$
Measures the similarity in direction of two vectors
$\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \end{bmatrix} = 2 * 3 + 1 * 2 = 8$
|a|
|a| cos(θ)


Projections
Dot products related to projections onto vectors.
Scalar projection of one vector onto another
$a_1 = |\mathbf{a}| \cos\theta = \mathbf{a} \cdot \hat{\mathbf{b}} = \mathbf{a} \cdot \frac{\mathbf{b}}{|\mathbf{b}|}$
Vector projection
$\mathbf{a}_1 = a_1 \hat{\mathbf{b}}$
$\hat{\mathbf{b}}$ is unit length

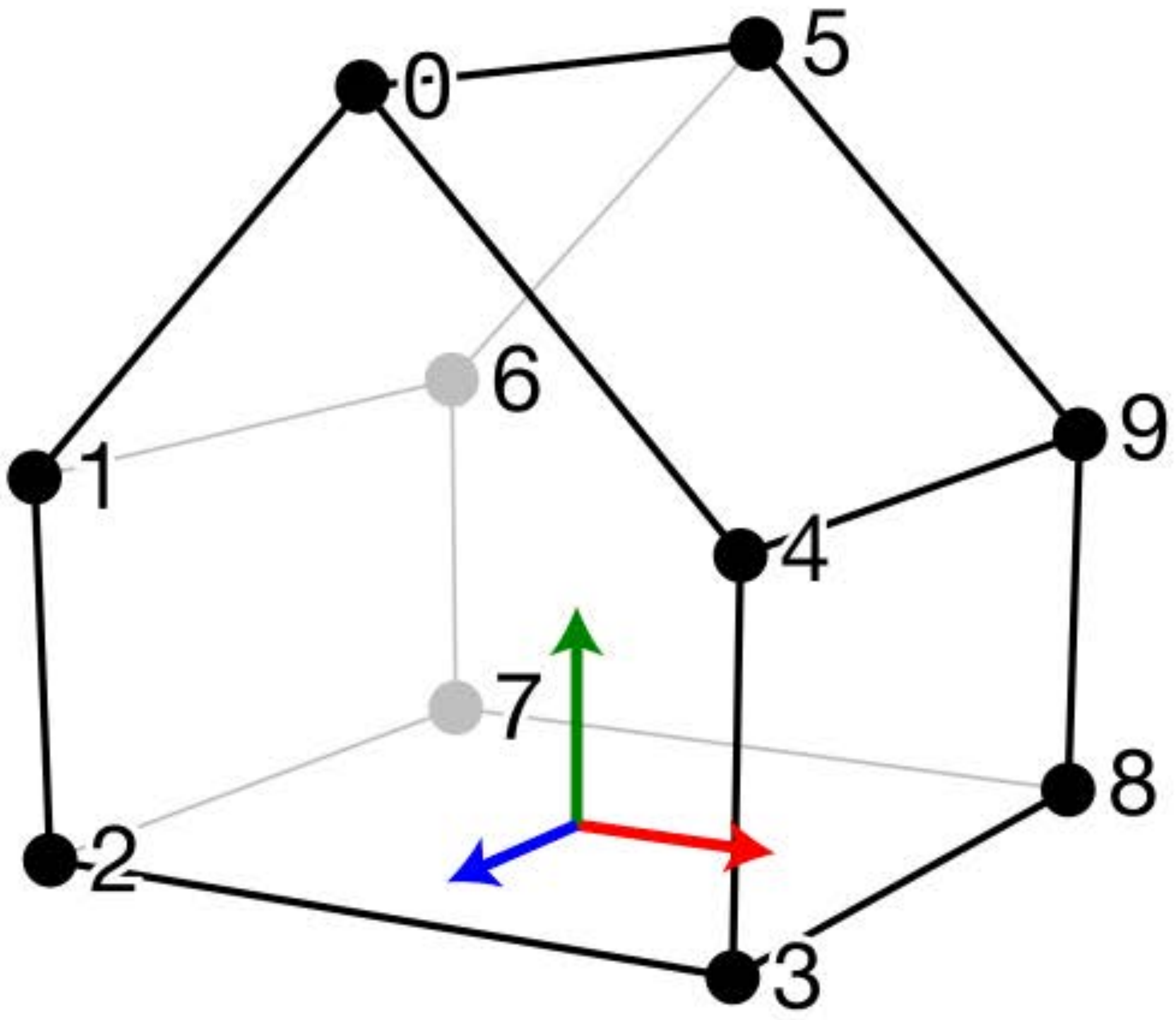

Cross Product
Assumes a and b are in same frame
$c_x = a_y b_z - a_z b_y$
$c_y = a_z b_x - a_x b_z$
$c_z = a_x b_y - a_y b_x$
Results in new vector c orthogonal to both original vectors a and b
Length of vector c is equal to area of parallelogram formed by a and b
$||\mathbf{a} \times \mathbf{b}|| = ||\mathbf{a}|| \, ||\mathbf{b}|| \sin\theta$
a×b
|a×b|


Matrix-vector multiplication
(two interpretations)
1) Row story: dot product of each matrix row
2) Column story: linear combination of matrix columns


Solving linear systems
What would be the direct way to solve for x?
$A\mathbf{x} = \mathbf{b}$
Invert A and multiply by b
$\mathbf{x} = A^{-1}\mathbf{b}$
Can this always be done?
No. But, we can approximate. How?
Pseudoinverse least-squares approximation
$\mathbf{x} = A^{+}_{\text{left}}\mathbf{b}$

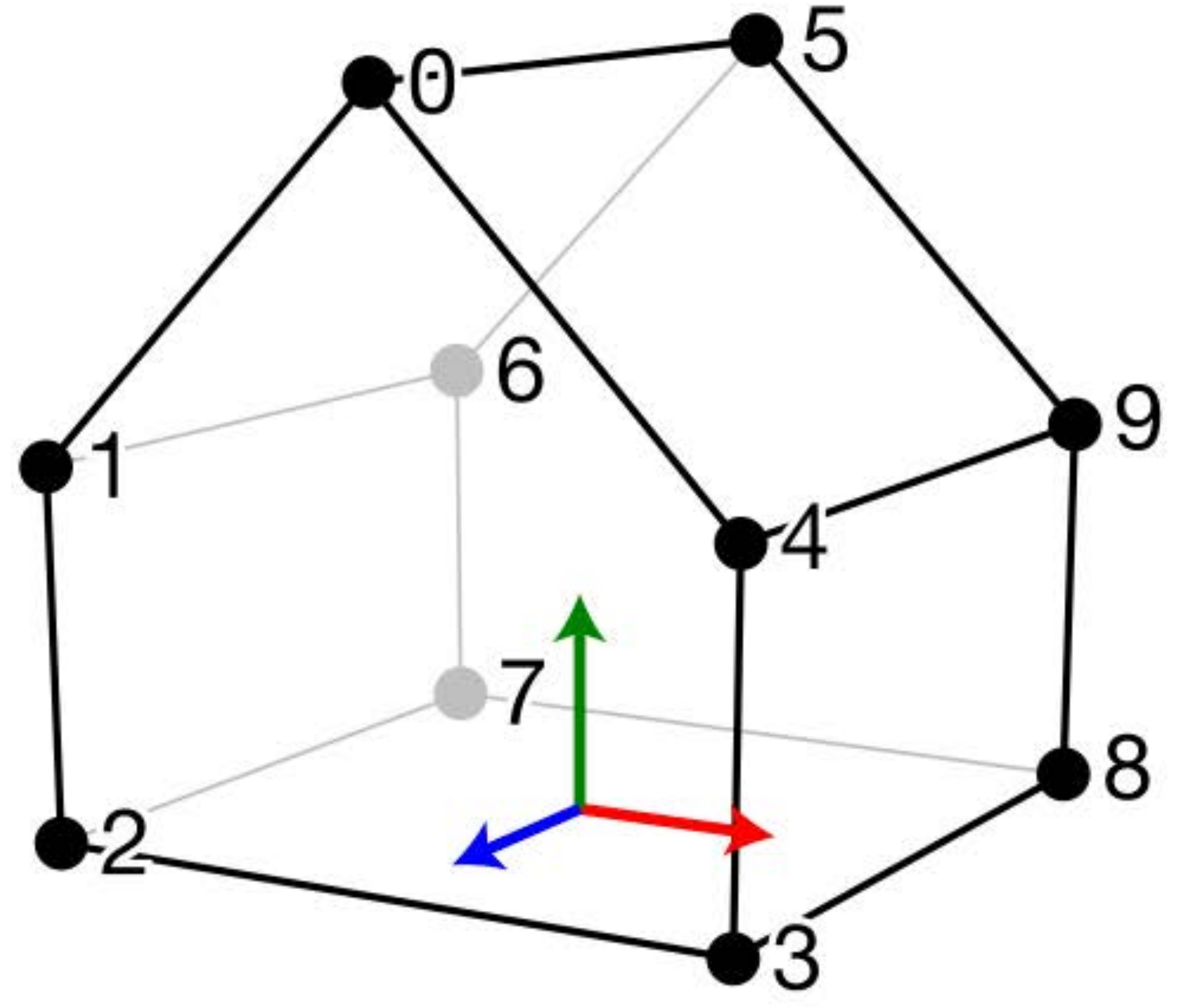
How to define a Link Geometry

Link Geometry



http://csc.lsu.edu/~kooima/courses/csc4356/

Link Geometry



vertex index | vertex location

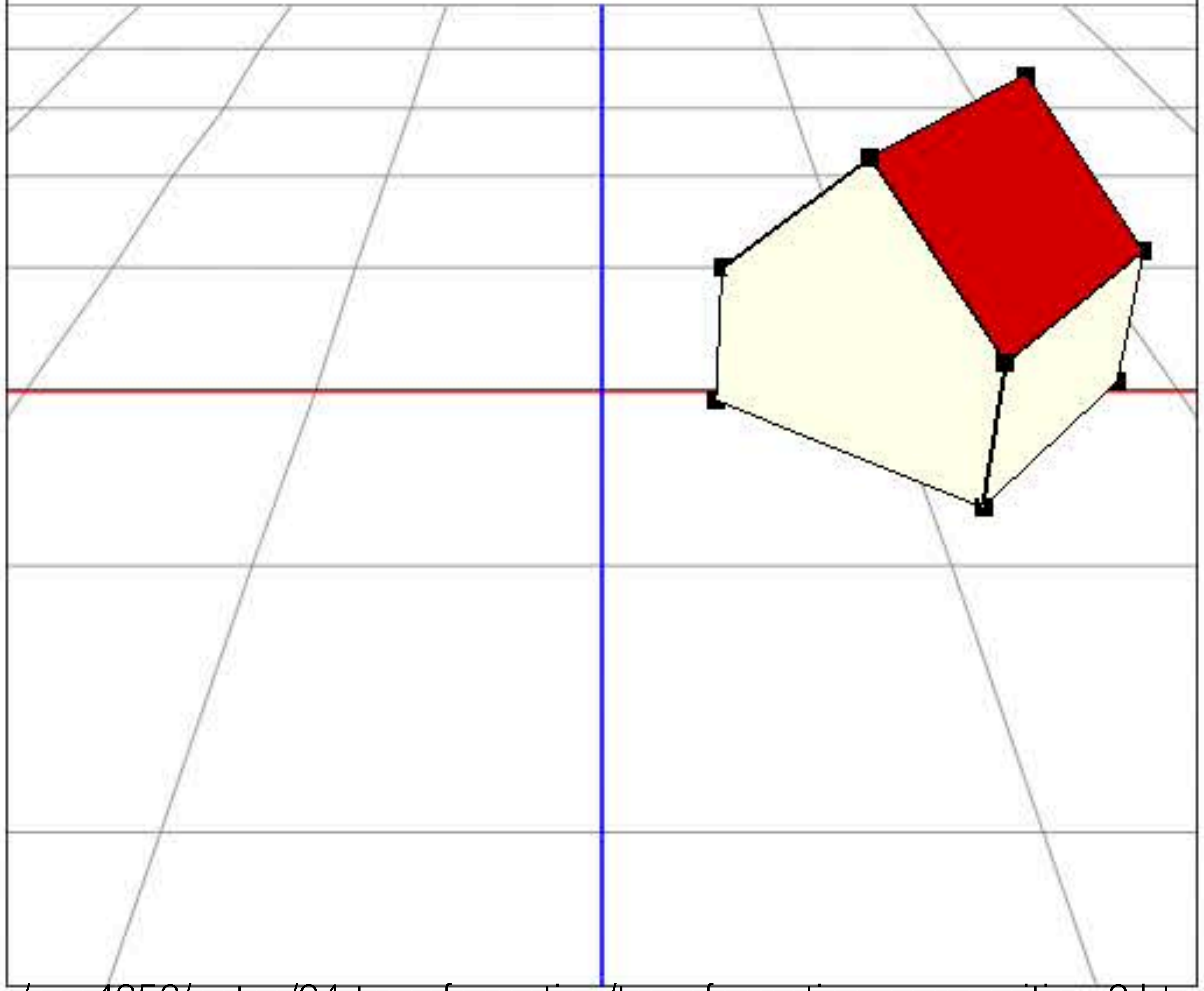
i	x	y	z
0	0.0	1.0	0.5
1	–0.5	0.5	0.5
2	–0.5	0.0	0.5
3	0.5	0.0	0.5
4	0.5	0.5	0.5
5	0.0	1.0	–0.5
6	–0.5	0.5	–0.5
7	–0.5	0.0	–0.5
8	0.5	0.0	–0.5
9	0.5	0.5	–0.5

Each robot link has a geometry specified as 3D vertices.
Vertices are connected into faces of the object's surface.
Vertices are defined wrt. the frame of the robots' link.

http://csc.lsu.edu/~kooima/courses/csc4356/

As the link frame moves, the geometry moves with it.

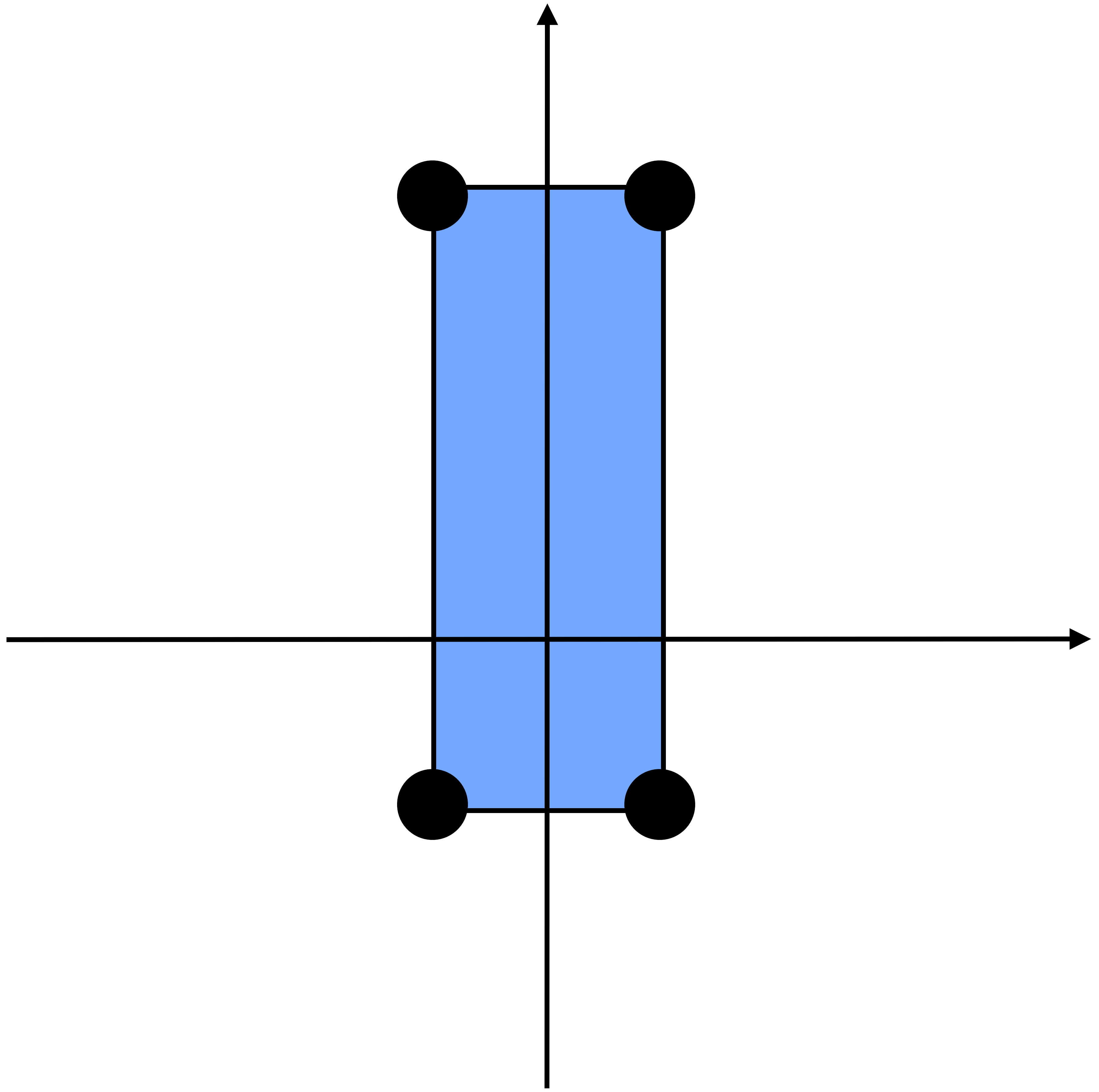
$$A_i = \begin{bmatrix} R_i^{i-1} & o^{i-1} \\ 0 & 1 \end{bmatrix}$$



http://csc.lsu.edu/~kooima/courses/csc4356/notes/04-transformation/transformation-composition-2.html

2D Rotation

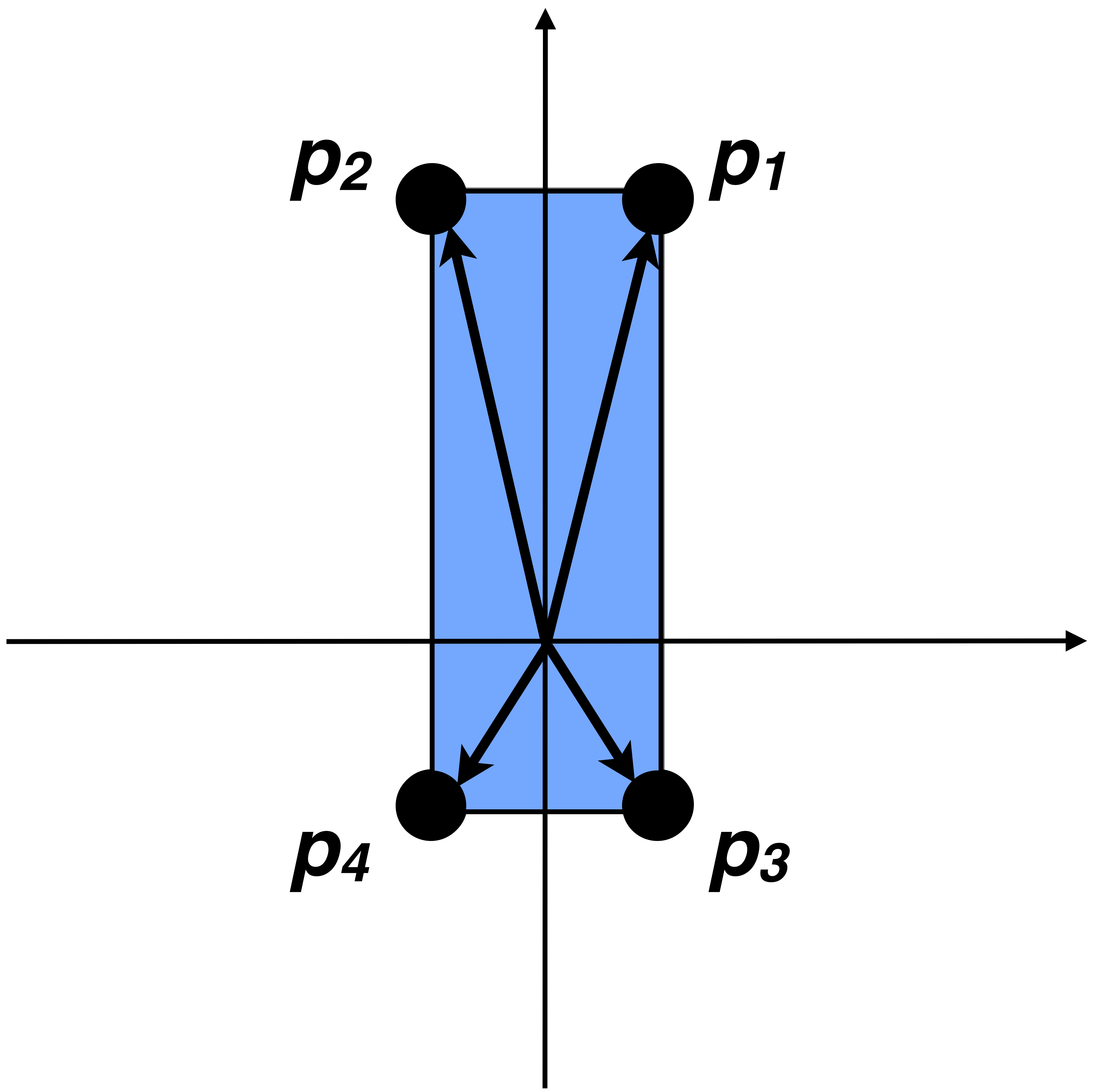
- Consider a link for a 2D robot with a box geometry of 4 vertices



2D Rotation

- Consider a link for a 2D robot with a box geometry of 4 vertices
- Vectors express position of vertices with respect to joint (at origin)

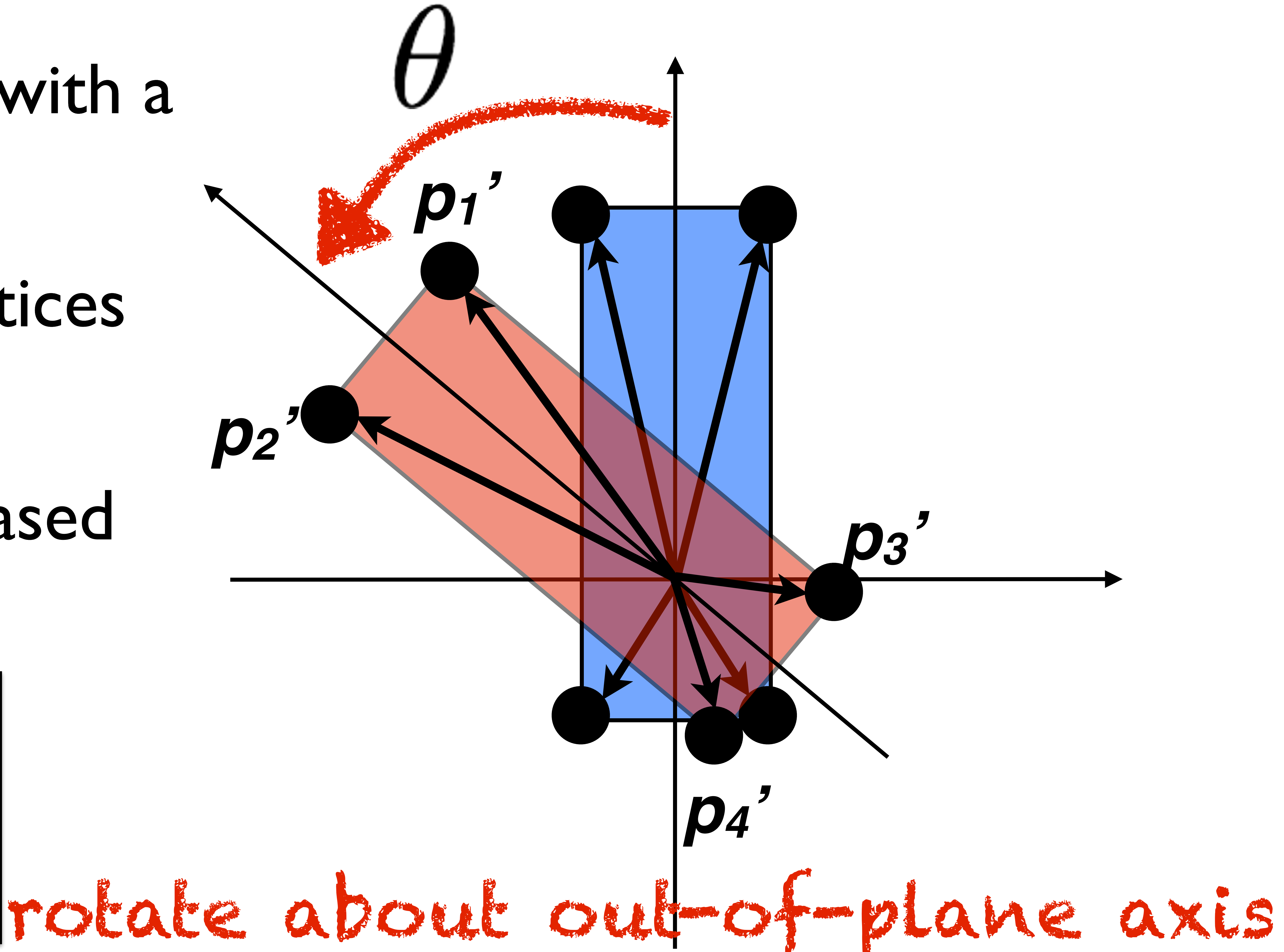
$$\mathbf{p_i} = [x_i, y_i]$$



2D Rotation

- Consider a link for a 2D robot with a box geometry of 4 vertices
- Vectors express position of vertices with respect to joint (at origin)
- How to rotate link geometry based on movement of the joint?

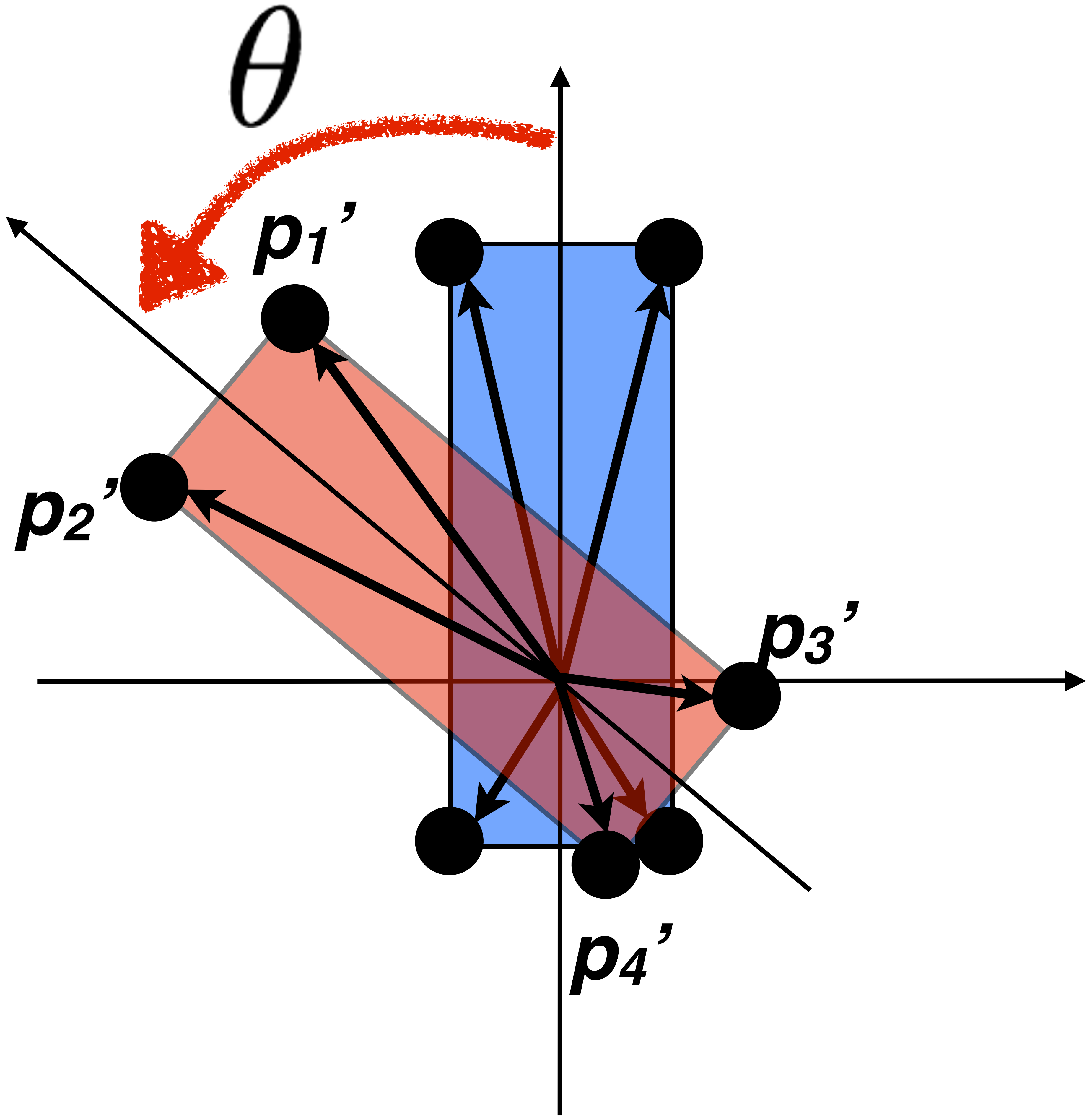
[redacted]



2D Rotation

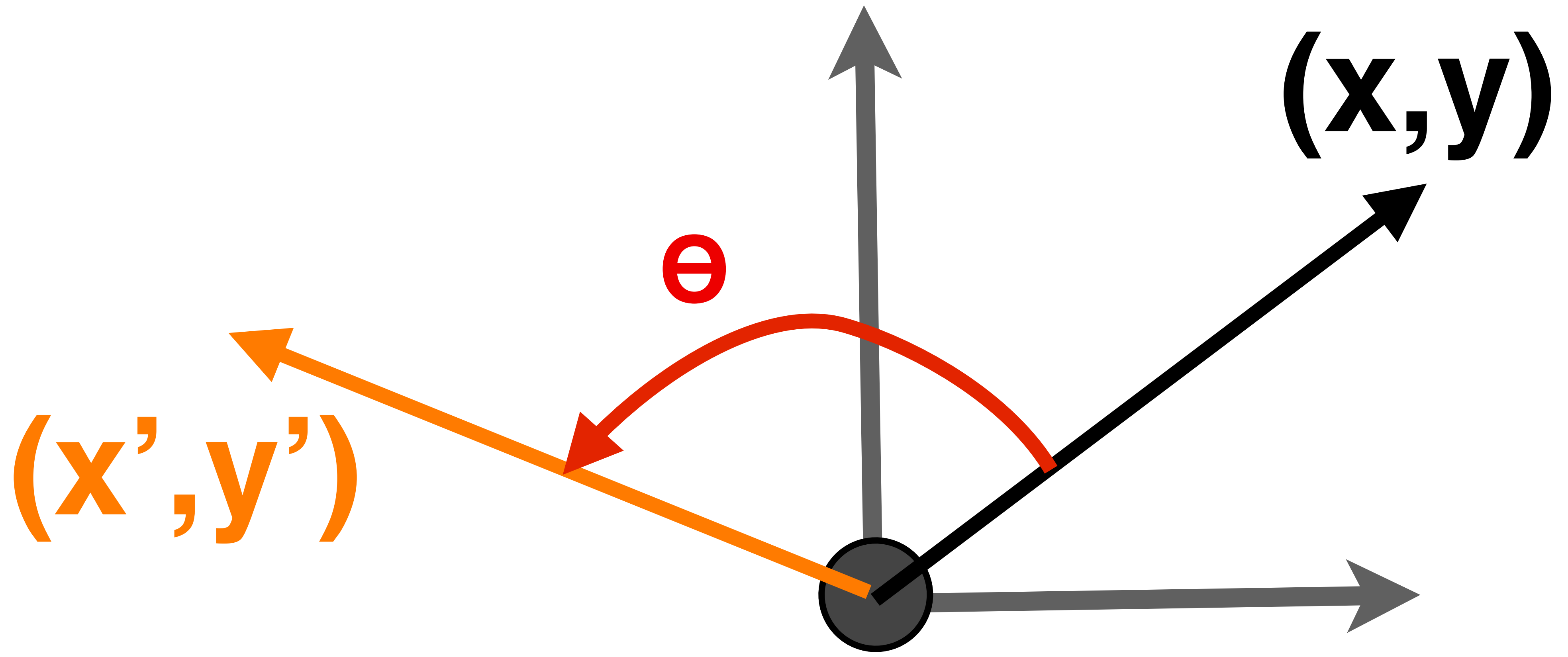
- Consider a link for a 2D robot with a box geometry of 4 vertices
- Vectors express position of vertices with respect to joint (at origin)
- How to rotate link geometry based on movement of the joint?

$$x' = x \cdot \cos(\theta) - y \cdot \sin(\theta)$$
$$y' = x \cdot \sin(\theta) + y \cdot \cos(\theta)$$



2D Rotation Matrix

(counterclockwise)

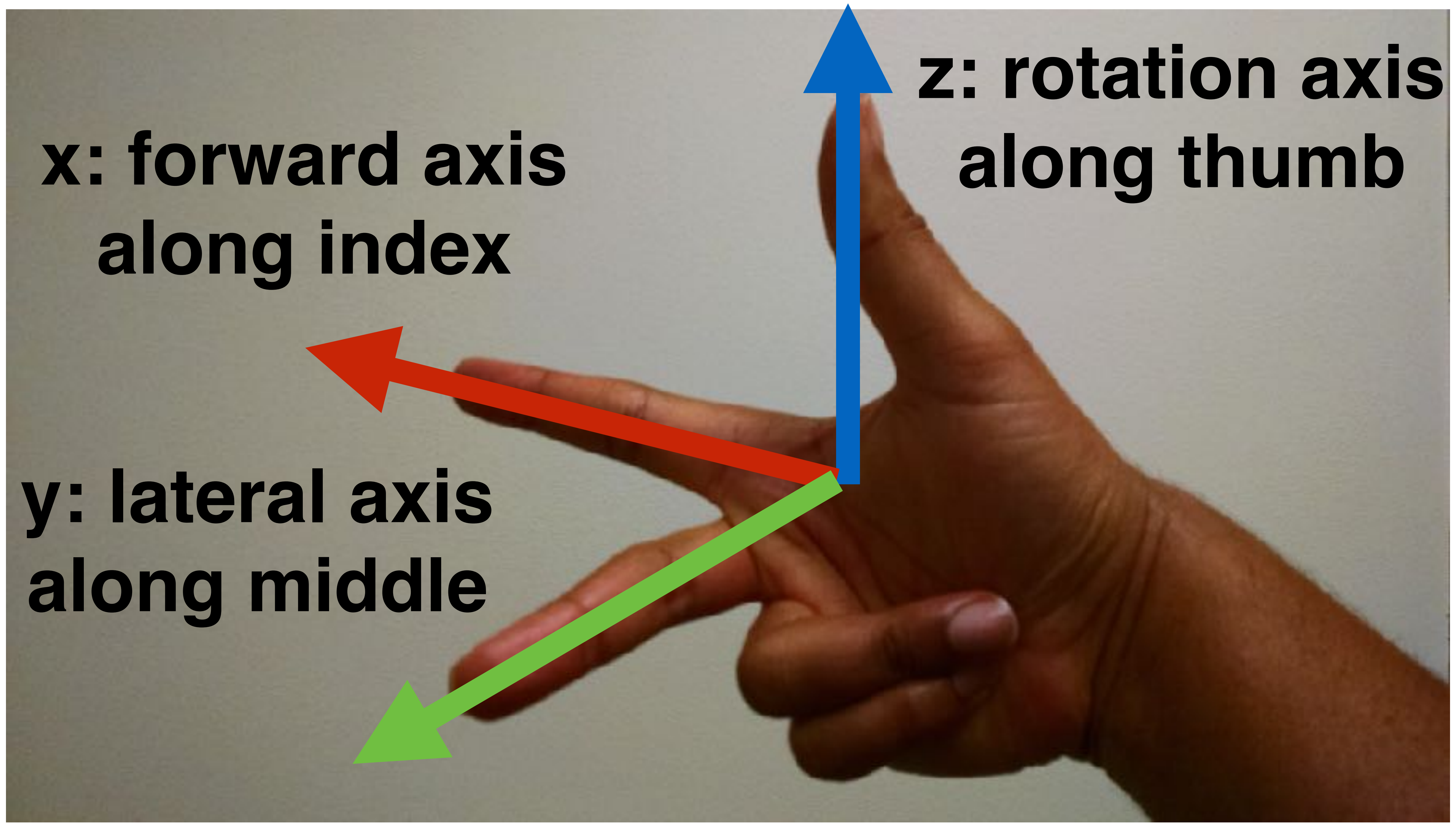


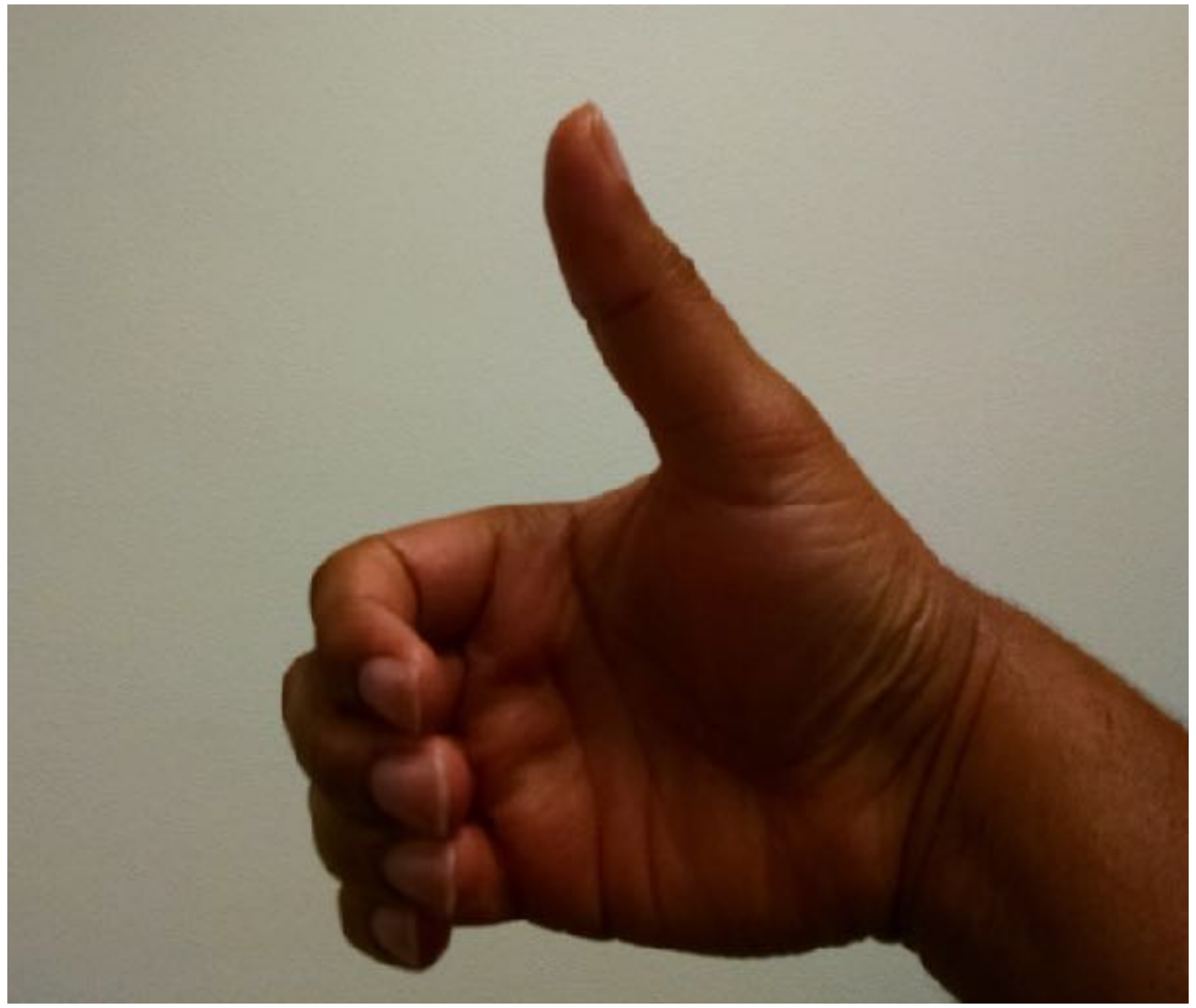
R(θ)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Matrix multiply vector by 2D rotation matrix R
- Matrix parameterized by rotation angle θ
- Remember: this rotation is counterclockwise

Right-hand Rule





rotation occurs about axis from forward towards lateral,
or the "curl" of the fingers

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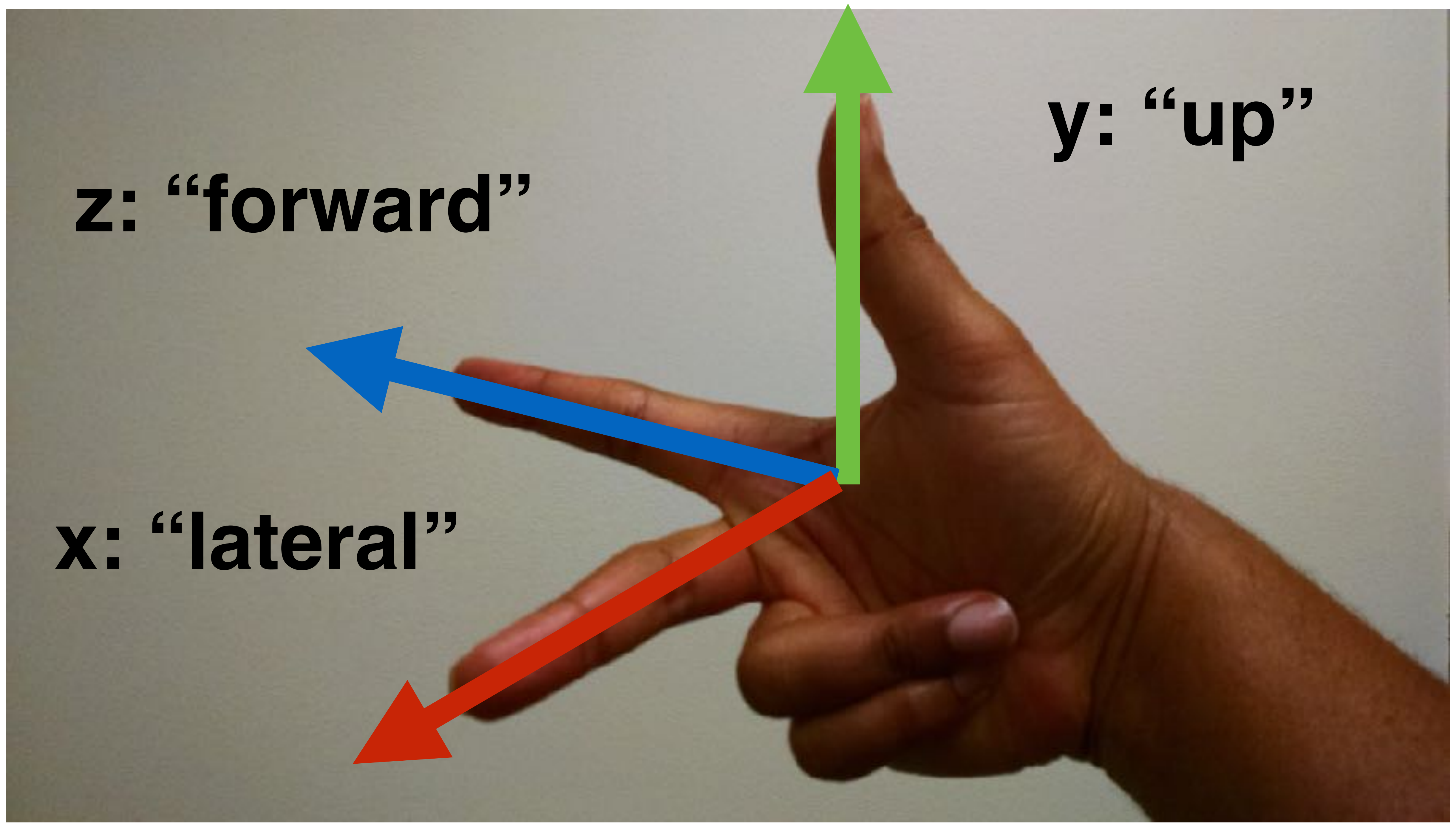
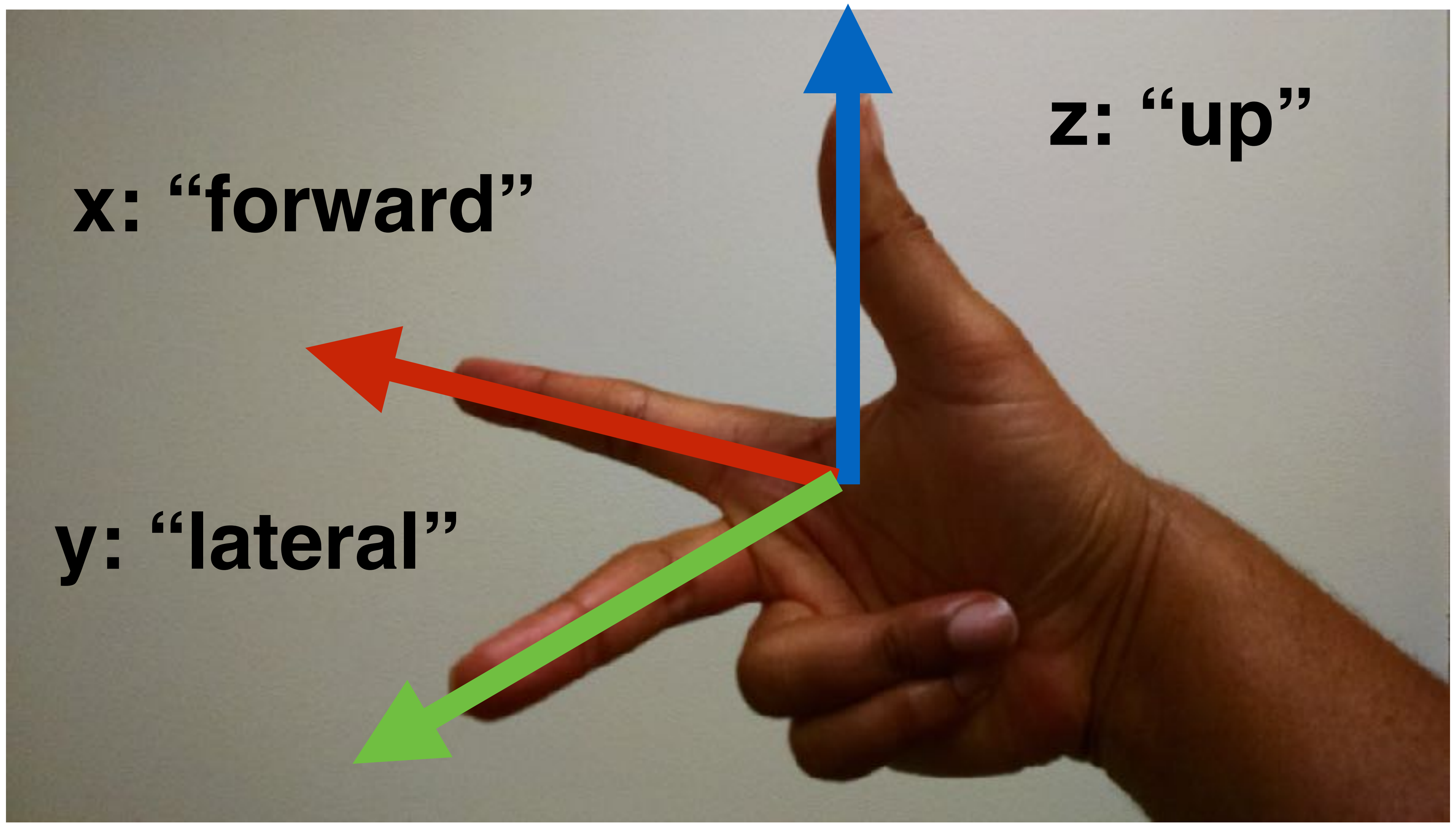
Coordinate conventions



threejs and KinEval
(used in the browser)

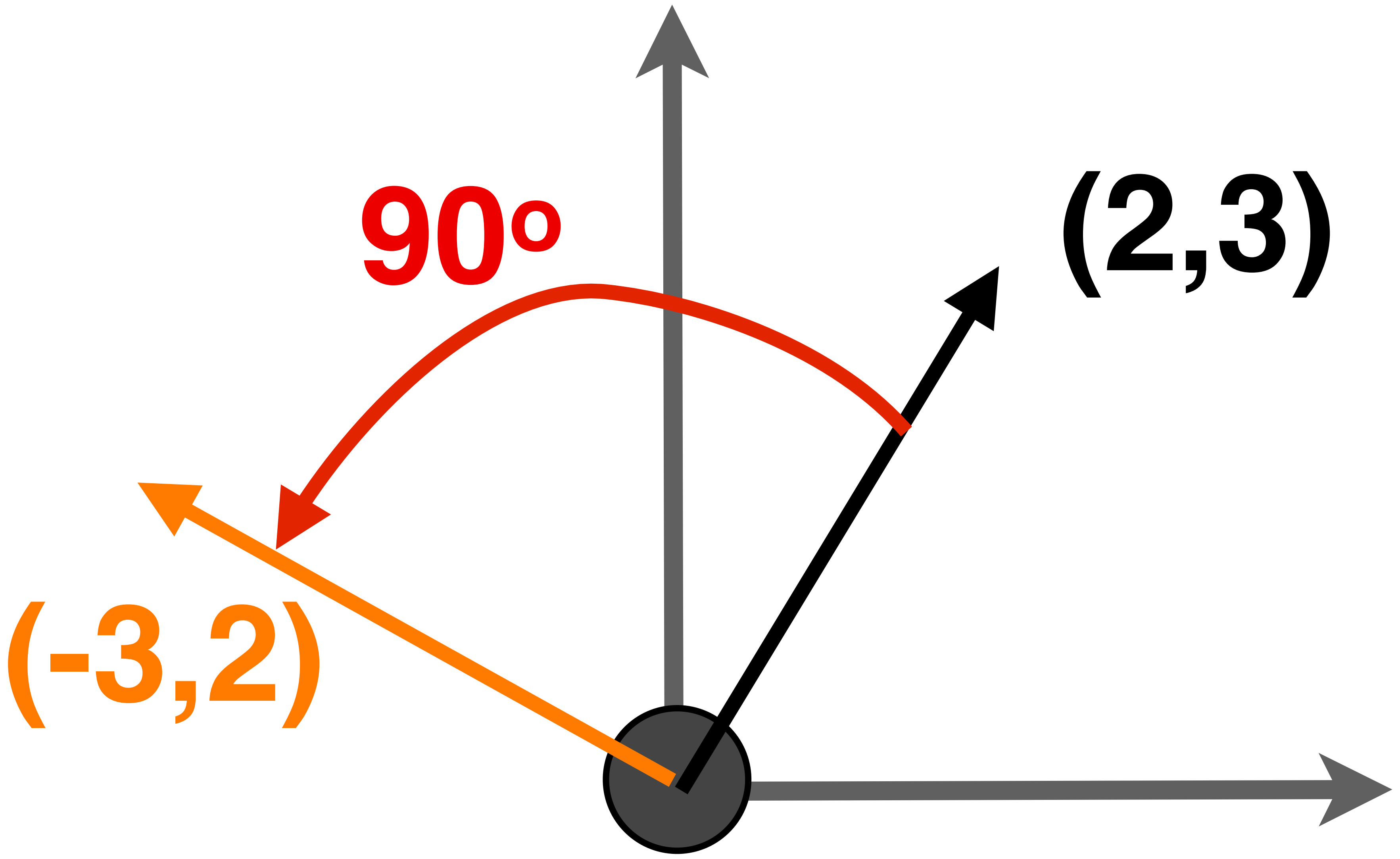


ROS and most of robotics
(used in URDF and rosbridge)

Checkpoint

- What is the 2D matrix for a rotation by 0 degrees?
- What is the 2D matrix for a rotation by 90 degrees?

Example

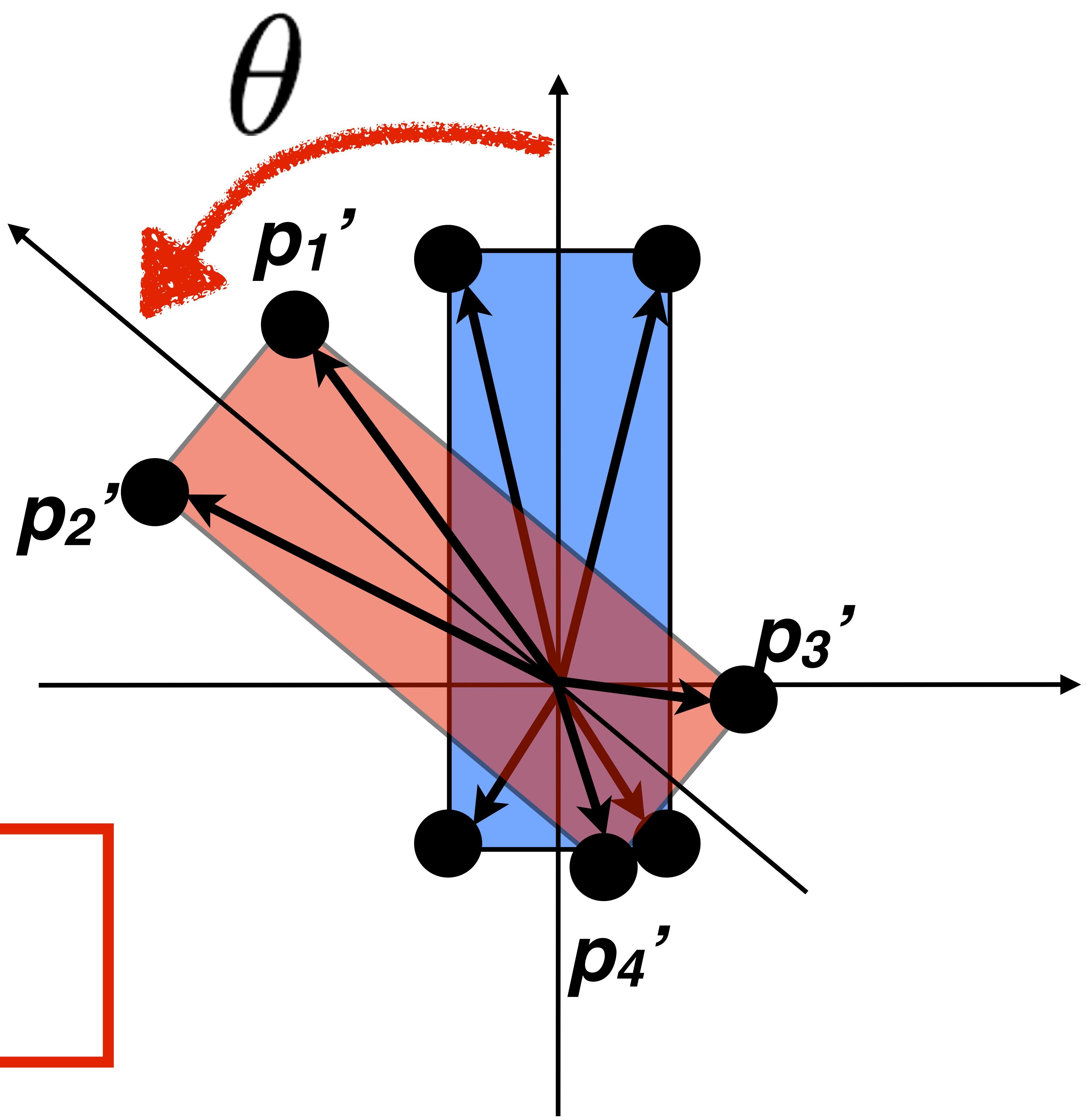


cos(90o) = 0

sin(90o) = 1

$$\begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

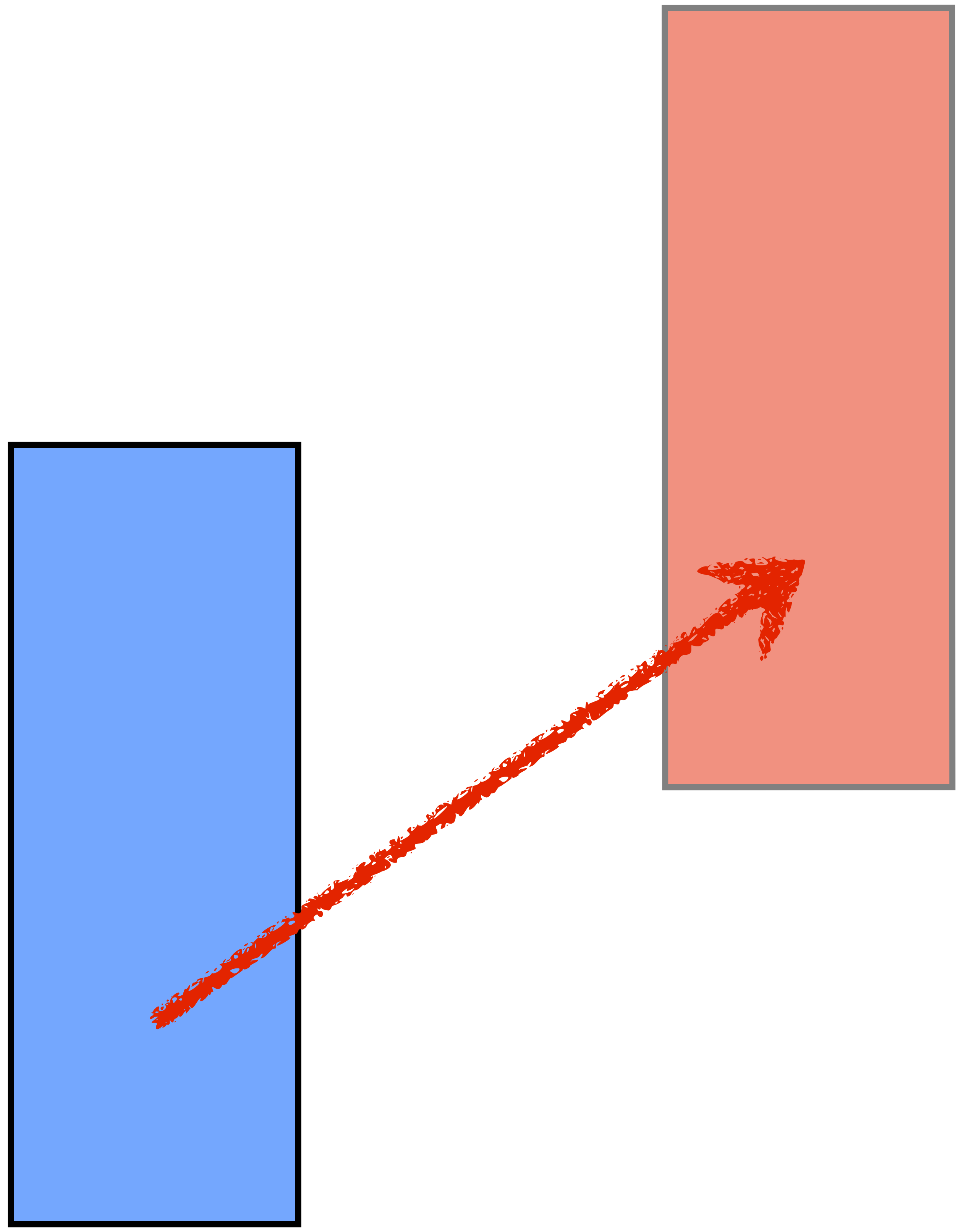
R(90o)



Note: one matrix multiply can transform all vertices

$$\begin{bmatrix} p'_{1x} & p'_{2x} & p'_{3x} & p'_{4x} \\ p'_{1y} & p'_{2y} & p'_{3y} & p'_{4y} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} p_{1x} & p_{2x} & p_{3x} & p_{4x} \\ p_{1y} & p_{2y} & p_{3y} & p_{4y} \end{bmatrix}$$

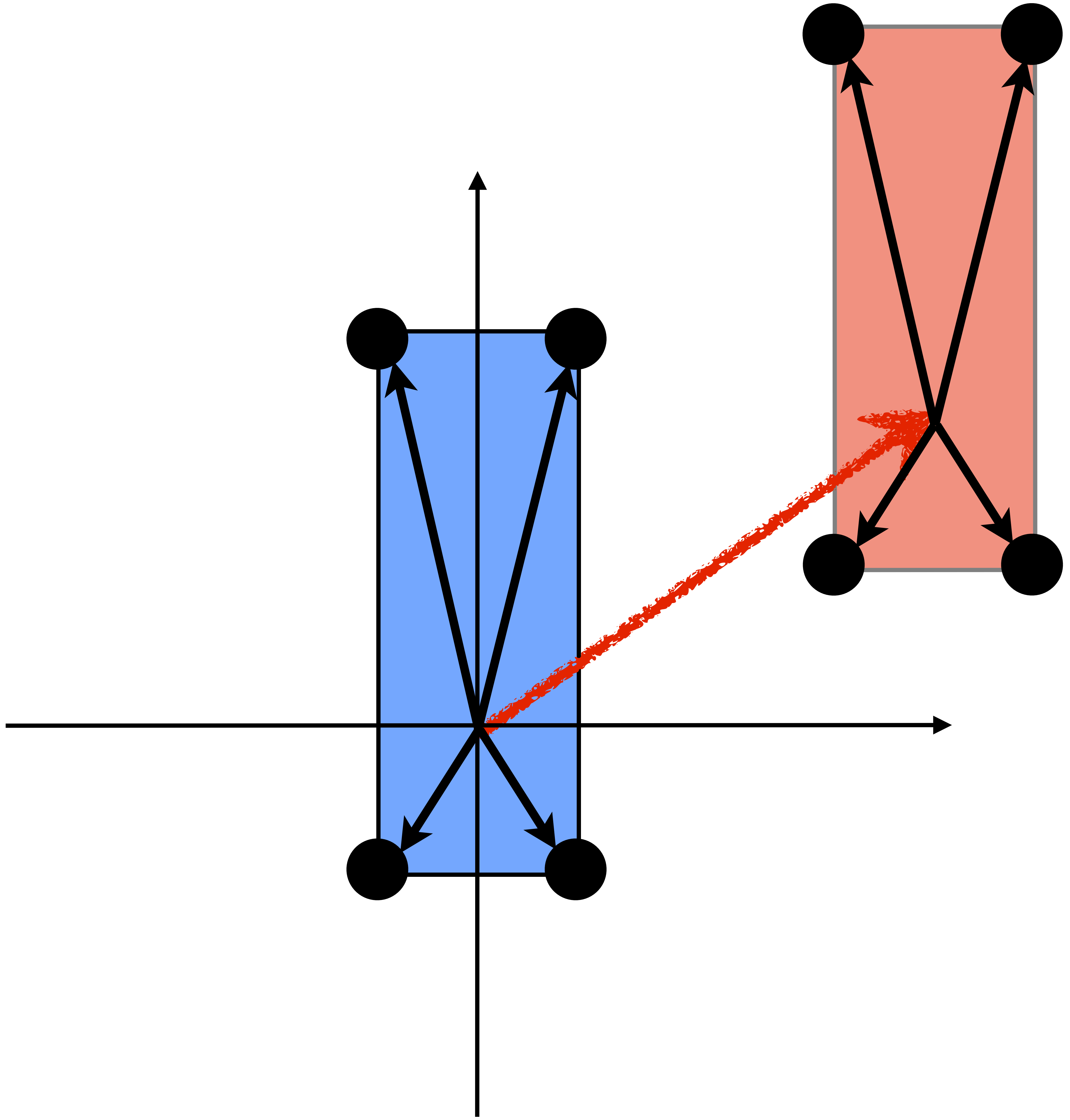
We can rotate.
Can we also translate?



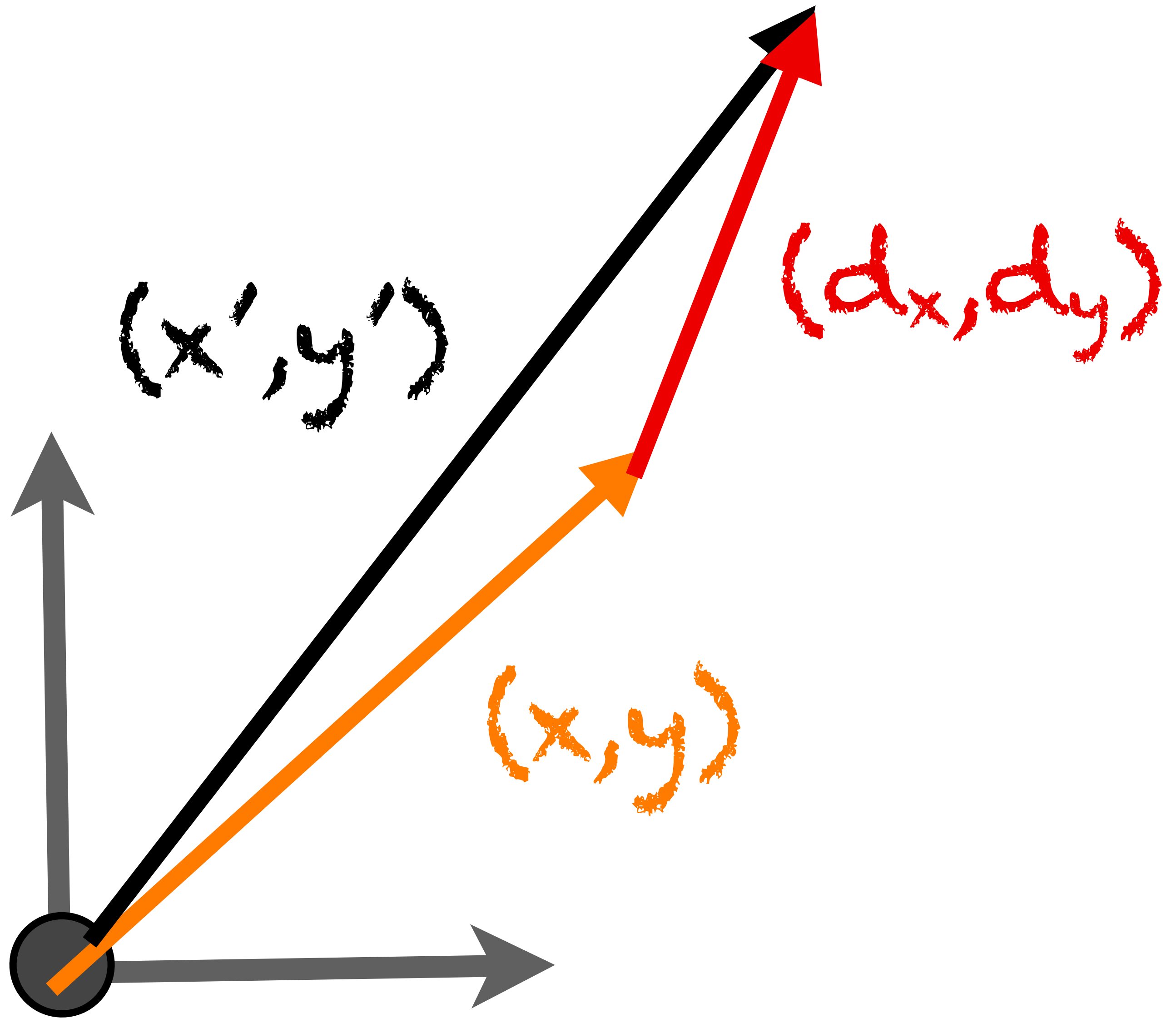
2D Translation

- Consider a link for a 2D robot with a box geometry of 4 vertices
- Vectors express position of vertices with respect to joint (at origin)
- How to translate link geometry to new location?

$$x' = x + d_x$$
$$y' = y + d_y$$



2D Translation Matrix



D(dx,dy)

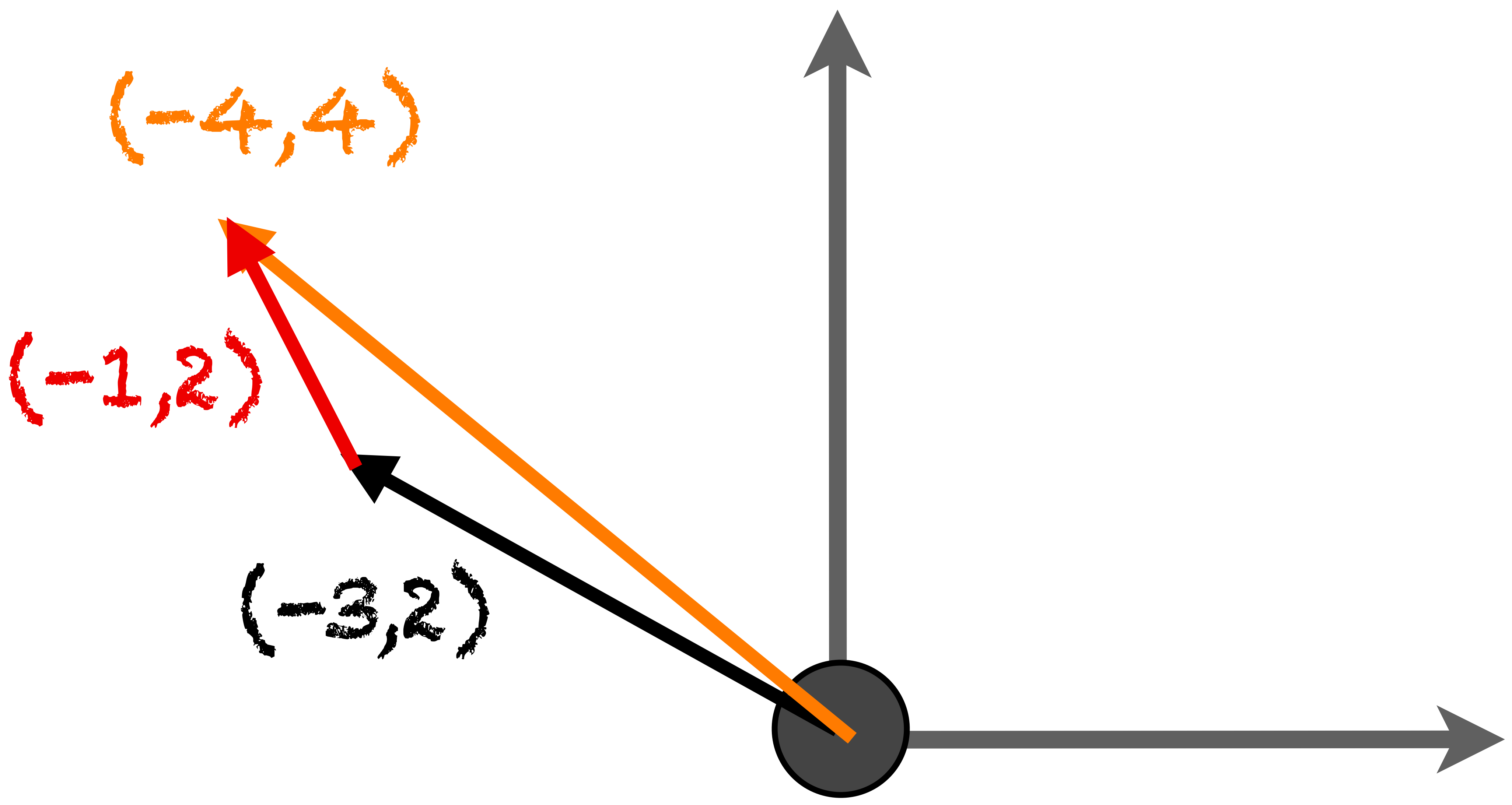
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x + d_x \\ y + d_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Requires homogeneous coordinates
  - 3D vector of 2D position concatenated with a 1
  - A plane at *z=1* in a three dimensional space
- Matrix parameterized by horizontal and vertical displacement (d_x, d_y)

Checkpoint

- What is the 2D matrix for a translation by [-1,2]?

Example

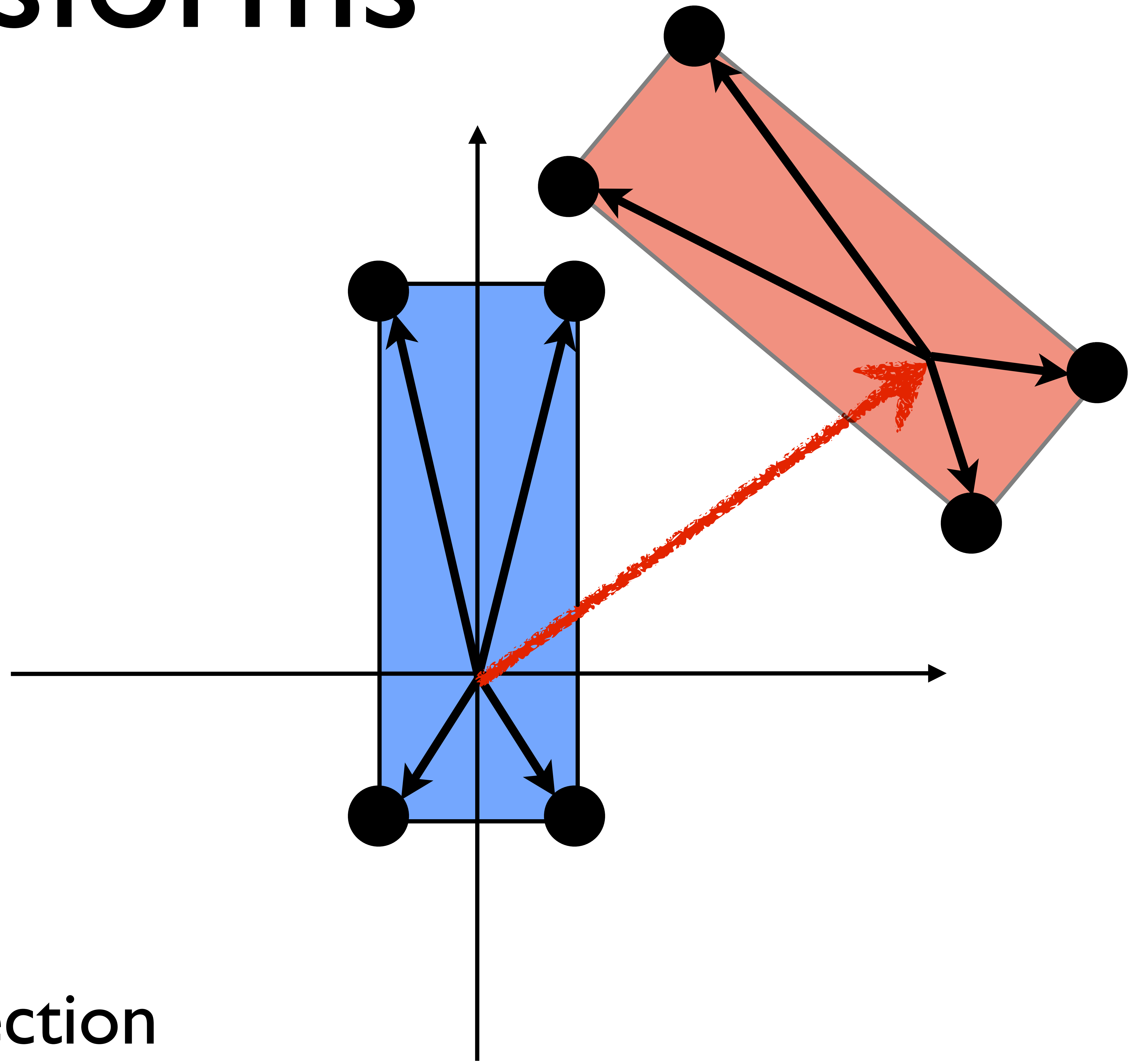


$$\begin{bmatrix} -4 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

D(-1,2)

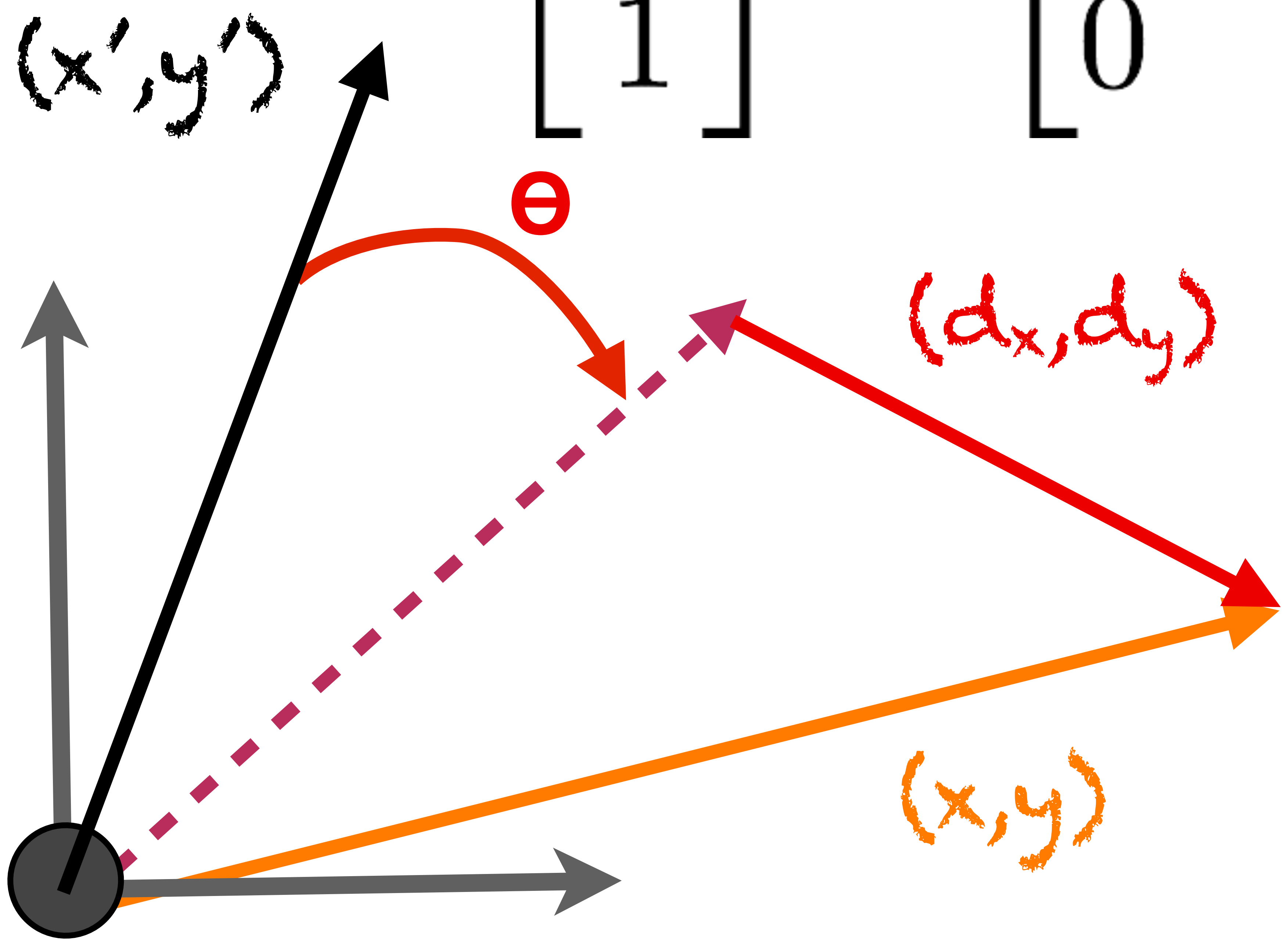
Rigid motions and Affine transforms

- Consider a link for a 2D robot with a box geometry of 4 vertices
- Vectors express position of vertices with respect to joint (at origin)
- How to both rotate and translate link geometry?
    - Rigid motion: rotate then translate
    - Affine transform: allows for rotation, translation, scaling, shearing, and reflection



Composition of Rotation and Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



homogeneous rotation matrix

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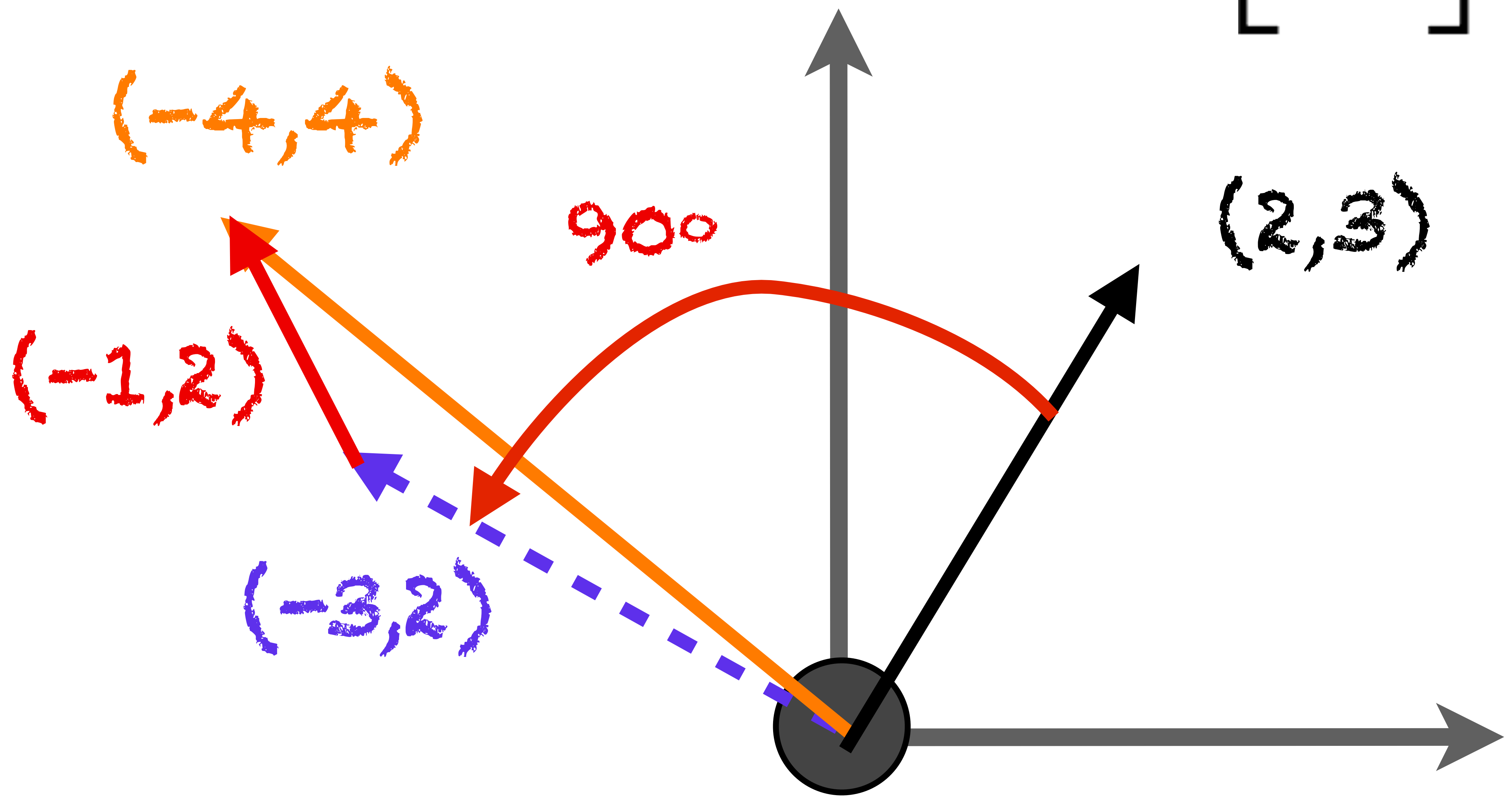
Example

$$\begin{bmatrix} -4 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

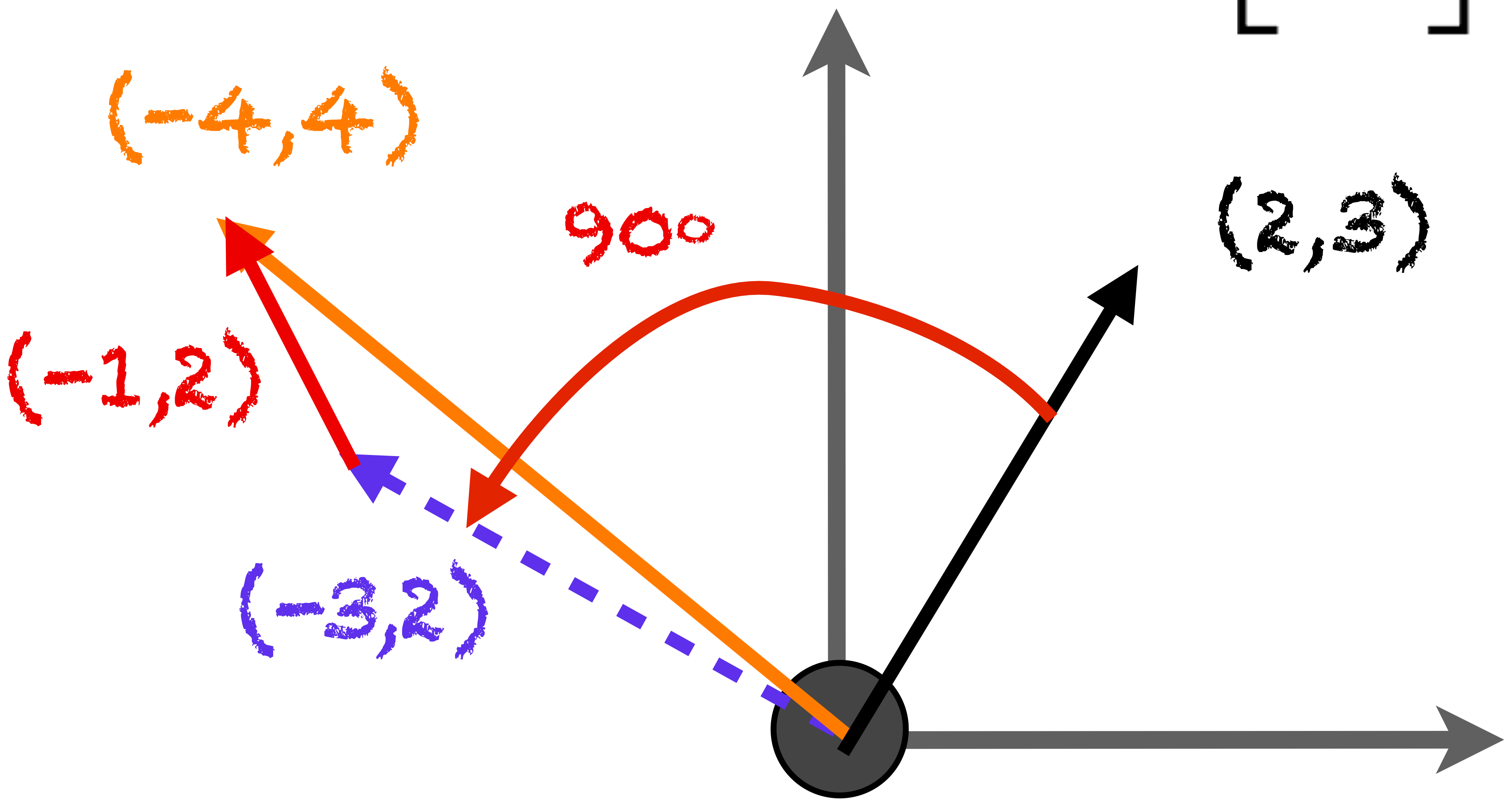
D(-1,2) R(90°)



Example

$$\begin{bmatrix} -4 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

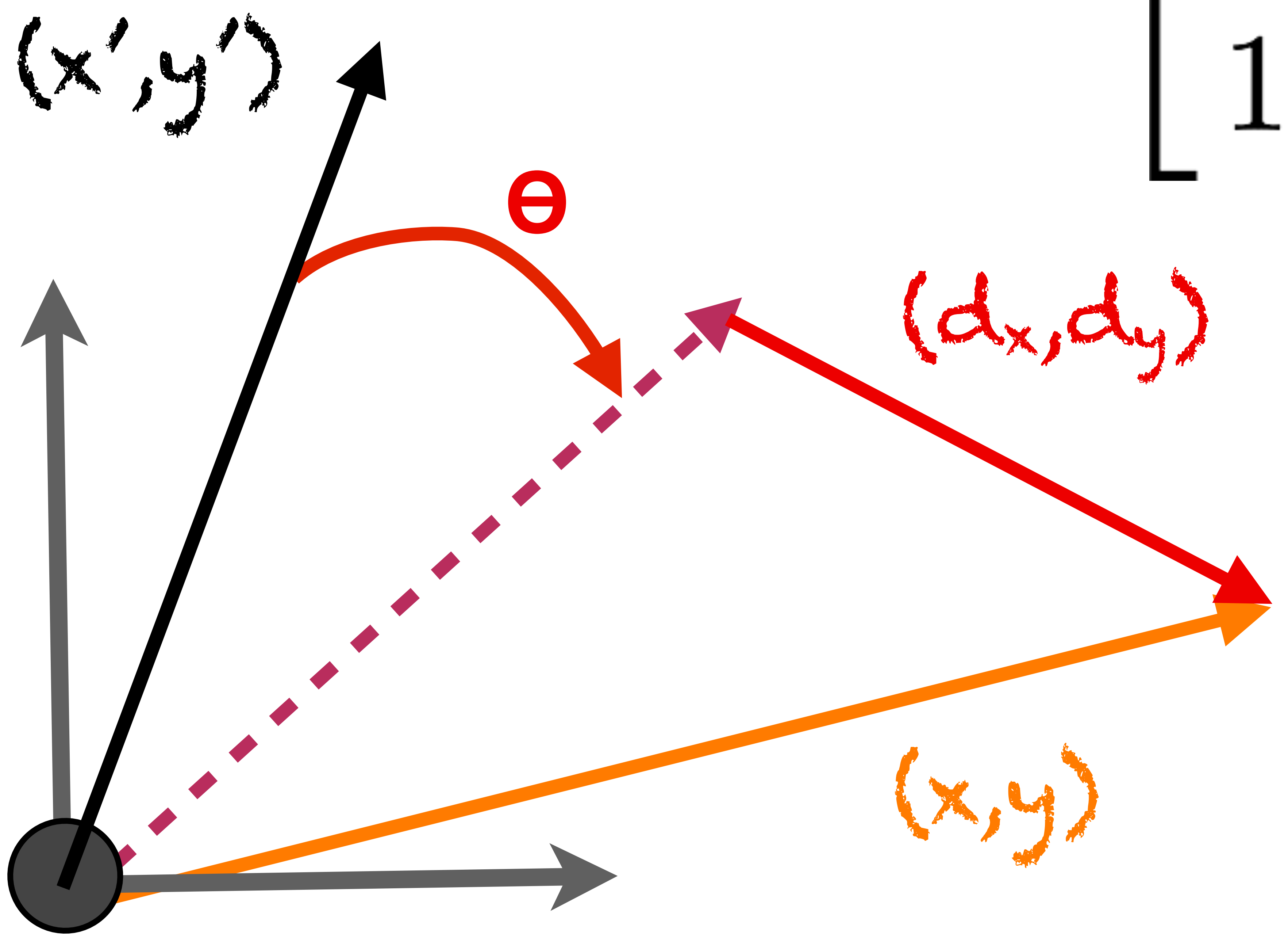
D(-1,2) R(90°)



$$\begin{bmatrix} -4 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

D(-1,2)R(90°)

Homogeneous Transform:
Composition of Rotation and Translation



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & d_x \\ \sin(\theta) & \cos(\theta) & d_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} R_i^{i-1} & o^{i-1} \\ 0 & 1 \end{bmatrix}$$

Homogeneous Transform

defines SE(2): Special Euclidean Group 2

$$H = \begin{bmatrix} R_{00} & R_{01} & d_x \\ R_{10} & R_{11} & d_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{2\times 2} & \mathbf{d}_{2\times 1} \\ \mathbf{0}_{1\times 2} & 1 \end{bmatrix}$$

Homogeneous Transform

defines SE(2): Special Euclidean Group 2

$$H = \begin{bmatrix} R_{00} & R_{01} & d_x \\ R_{10} & R_{11} & d_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{2\times 2} & \mathbf{d}_{2\times 1} \\ \mathbf{0}_{1\times 2} & 1 \end{bmatrix}$$

$$H \in SE(2)$$

Homogeneous Transform

defines SE(2): Special Euclidean Group 2

$$H = \begin{bmatrix} R_{00} & R_{01} & d_x \\ R_{10} & R_{11} & d_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{2\times2} & \mathbf{d}_{2\times1} \\ \mathbf{0}_{1\times2} & 1 \end{bmatrix}$$

$$H \in SE(2) \quad \mathbf{R}_{2\times2} \in SO(2)$$

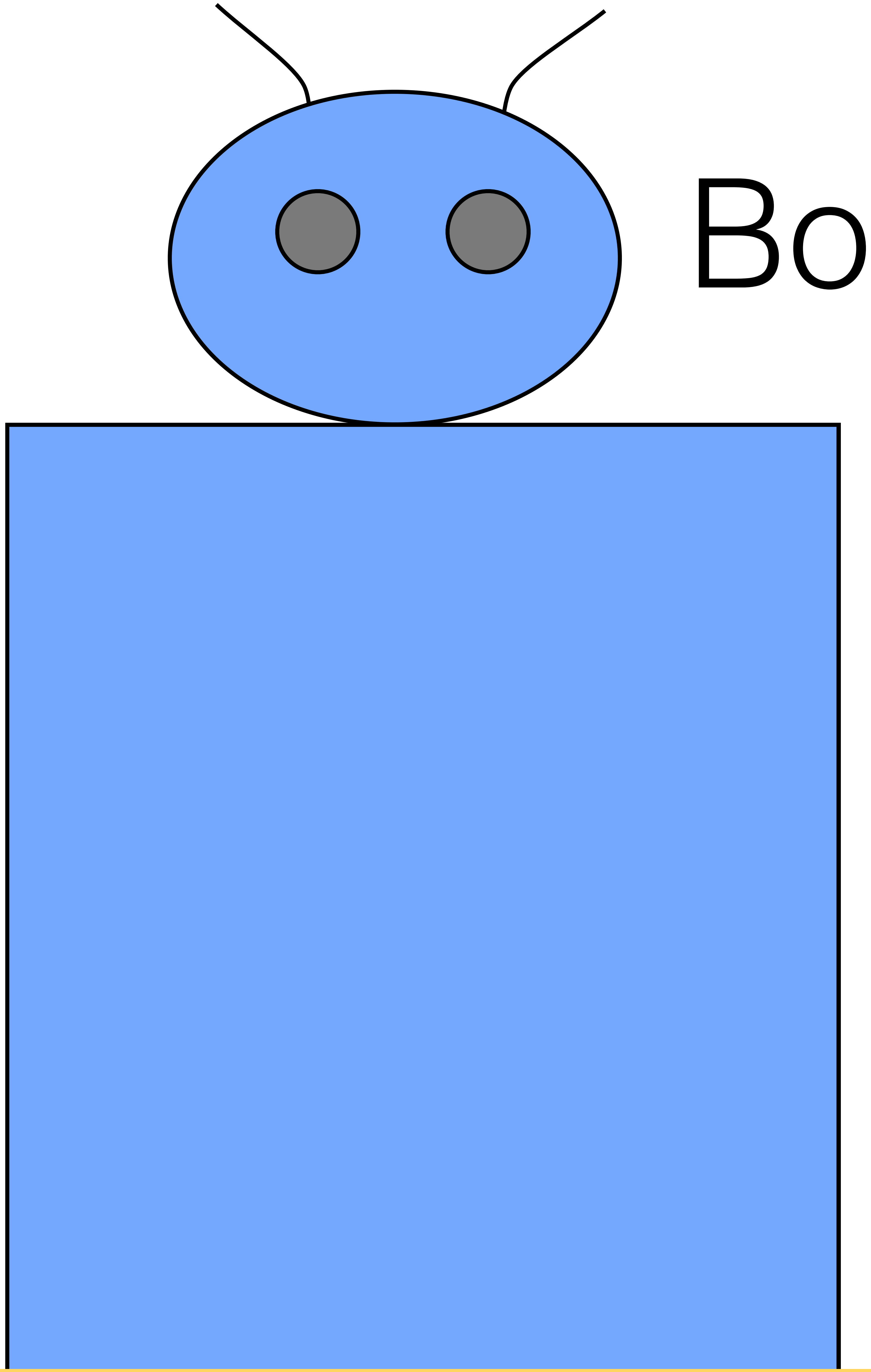
Homogeneous Transform

defines SE(2): Special Euclidean Group 2

$$H = \begin{bmatrix} R_{00} & R_{01} & d_x \\ R_{10} & R_{11} & d_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{2\times2} & \mathbf{d}_{2\times1} \\ \mathbf{0}_{1\times2} & 1 \end{bmatrix}$$

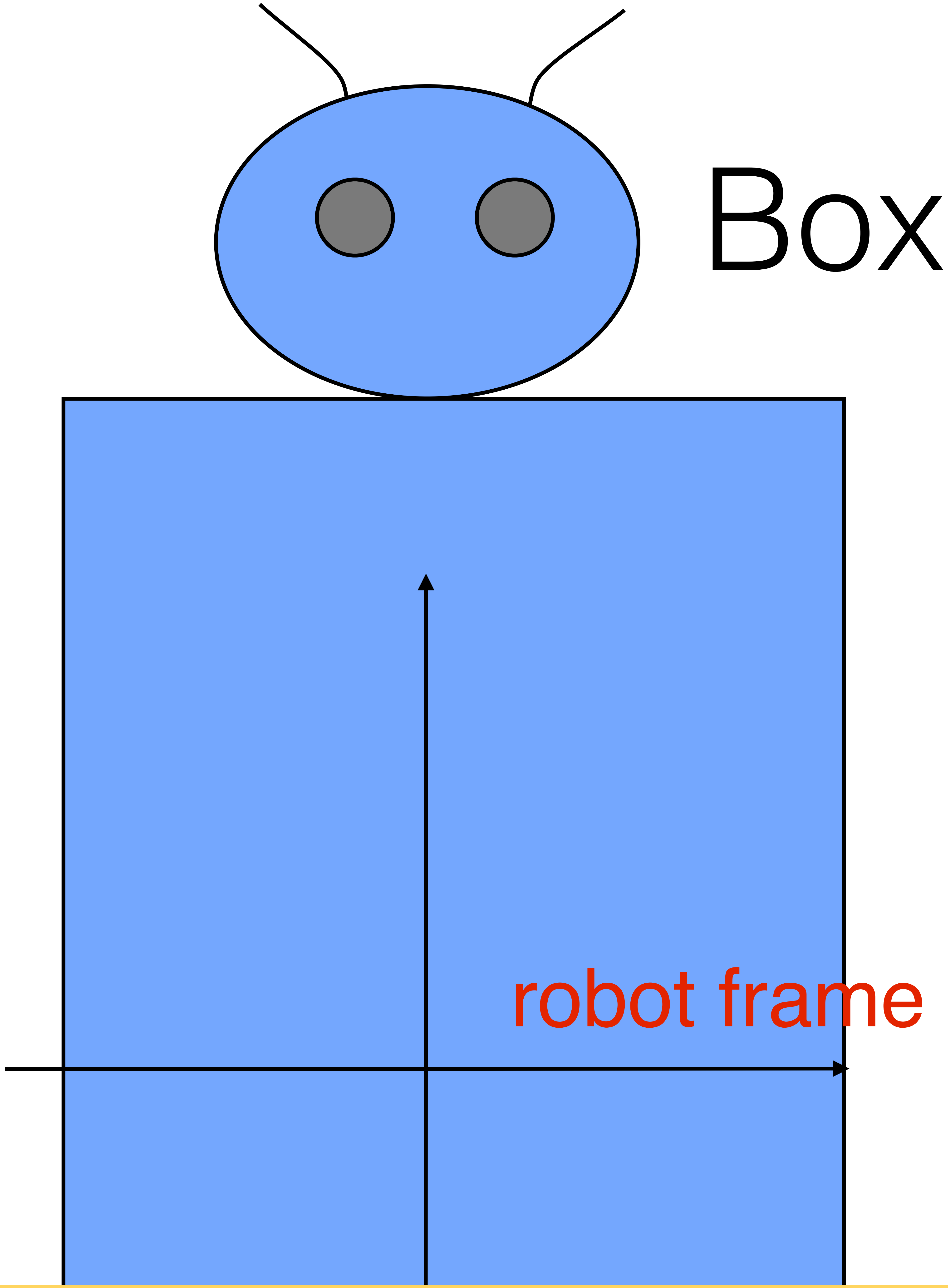
$H \in SE(2)$ $\quad \mathbf{R}_{2\times2} \in SO(2)$ $\quad \mathbf{d}_{2\times1} \in \Re^2$

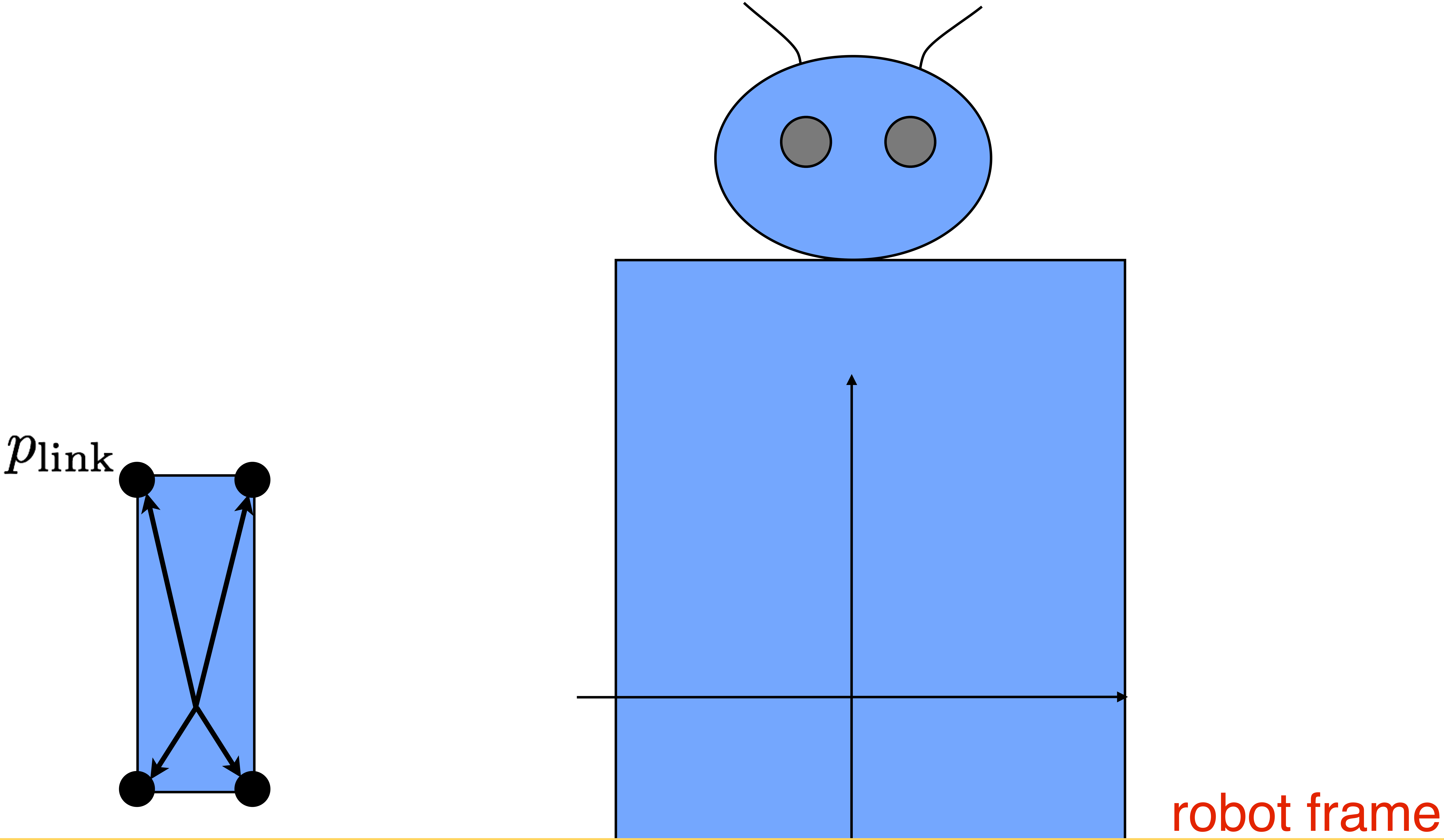
Example:
Let's put an arm link on Boxy



Boxy the robot

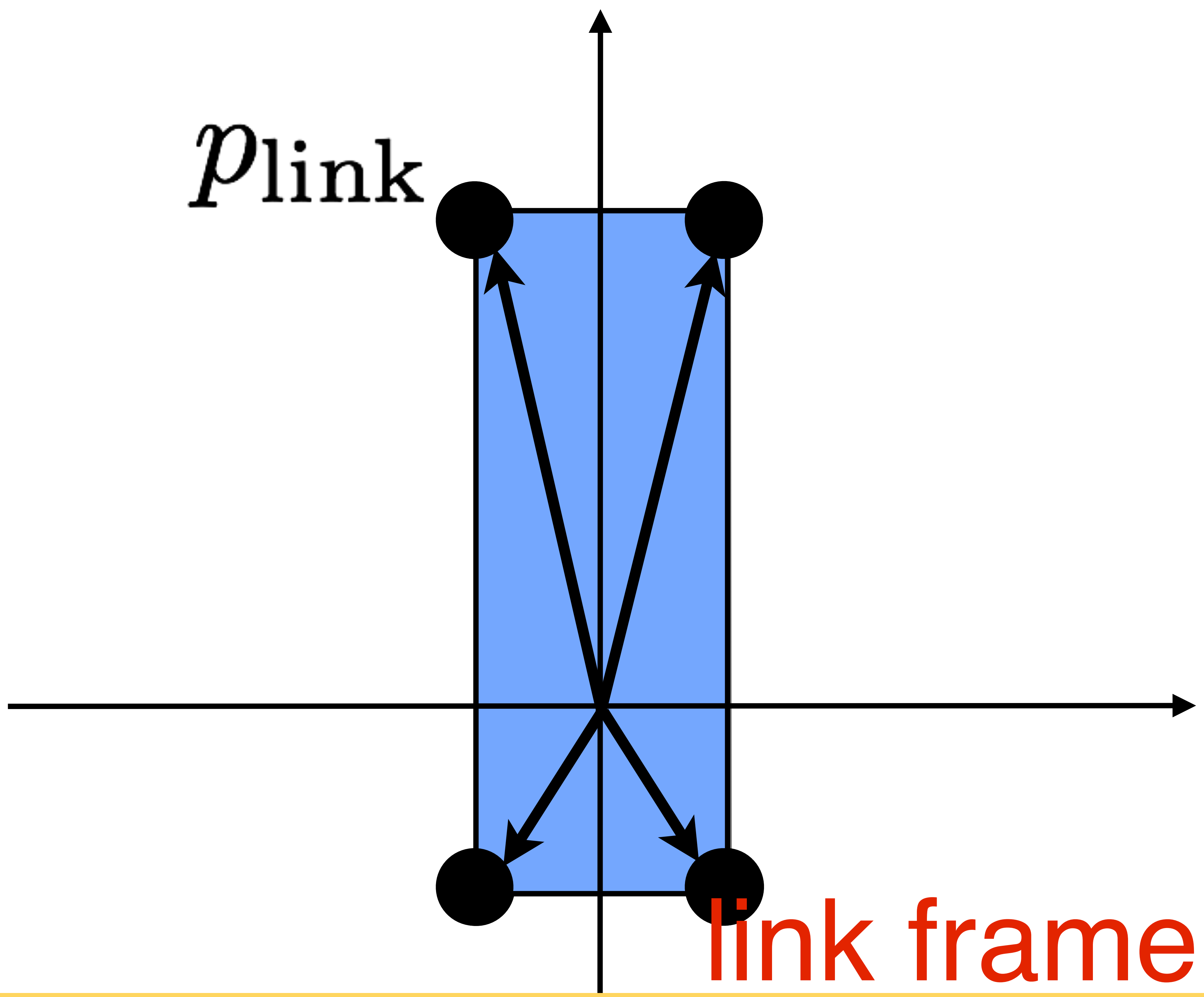
Boxy the robot

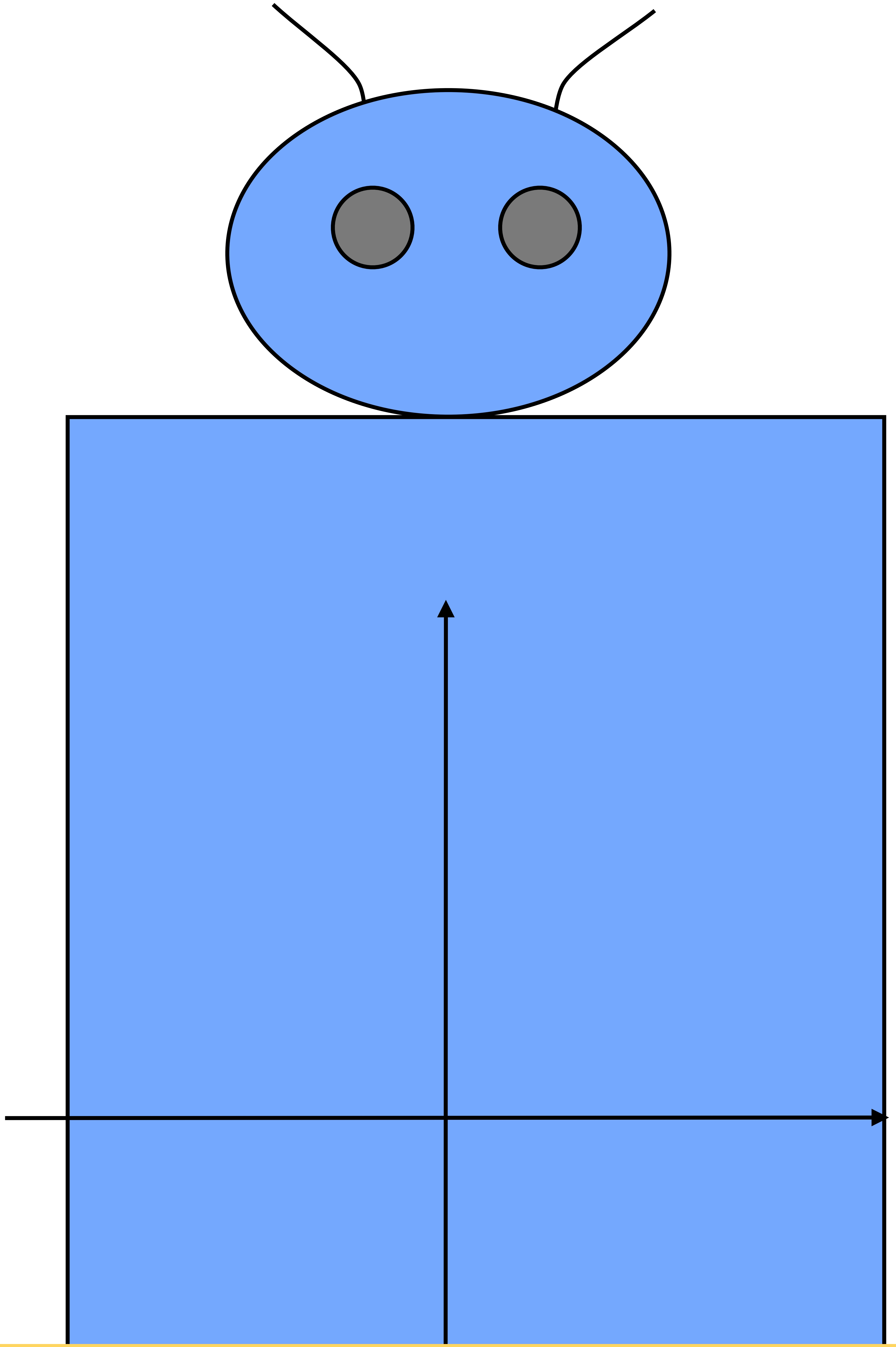



p_{link}
robot frame

Transform the link frame and its vertices into the robot frame

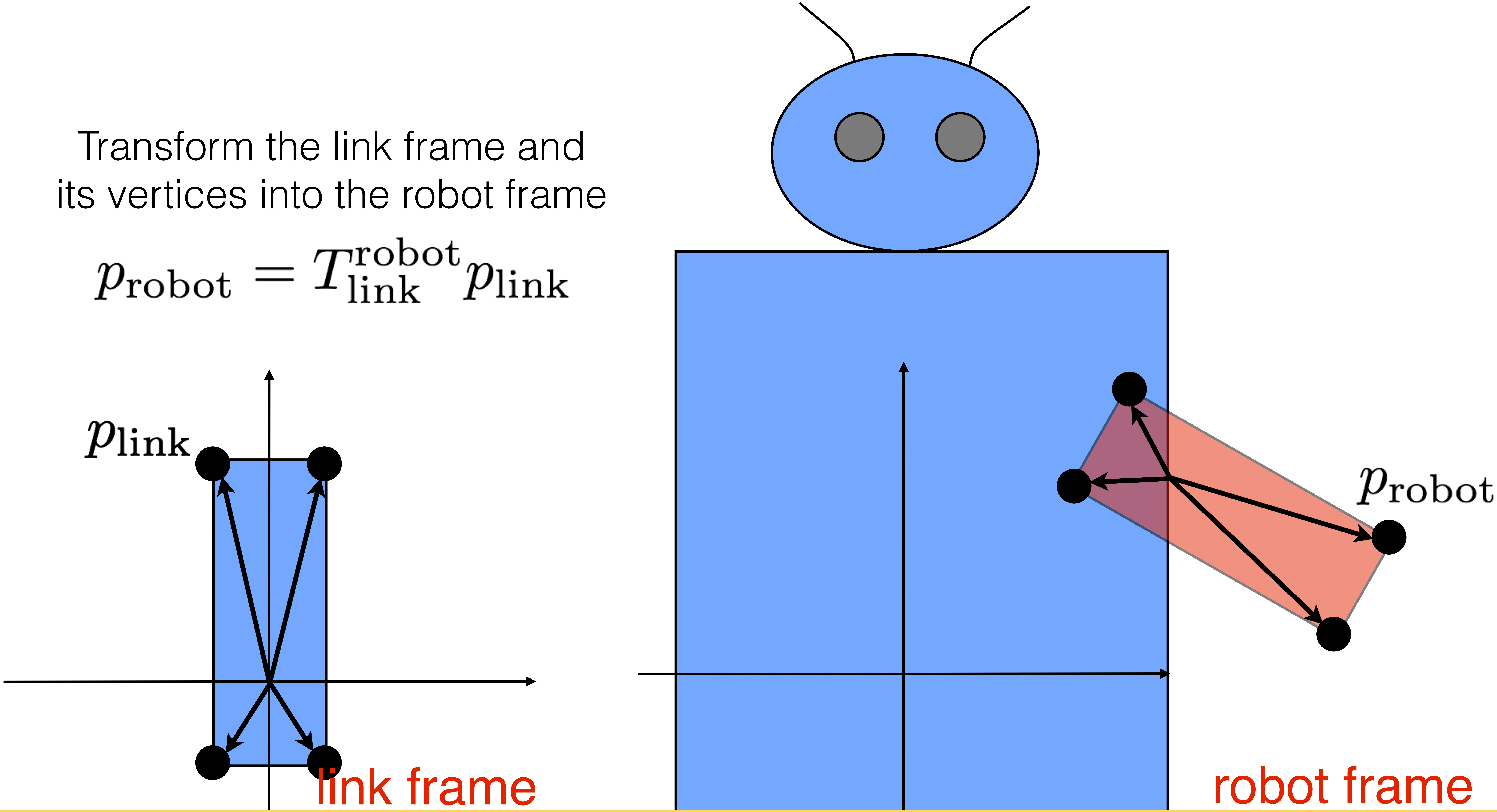
$$p_{\text{robot}} = T_{\text{link}}^{\text{robot}} p_{\text{link}}$$





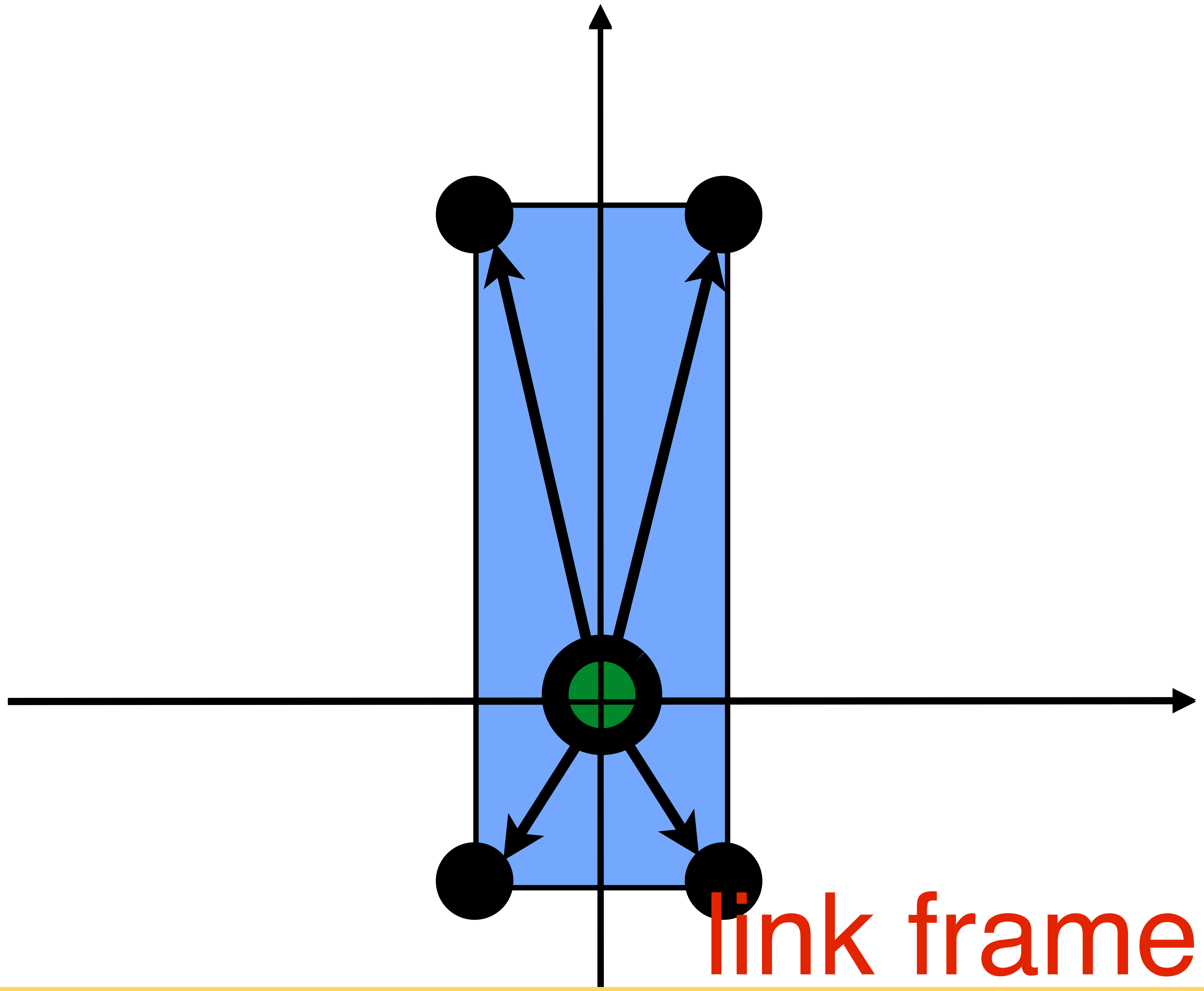
Transform the link frame and its vertices into the robot frame

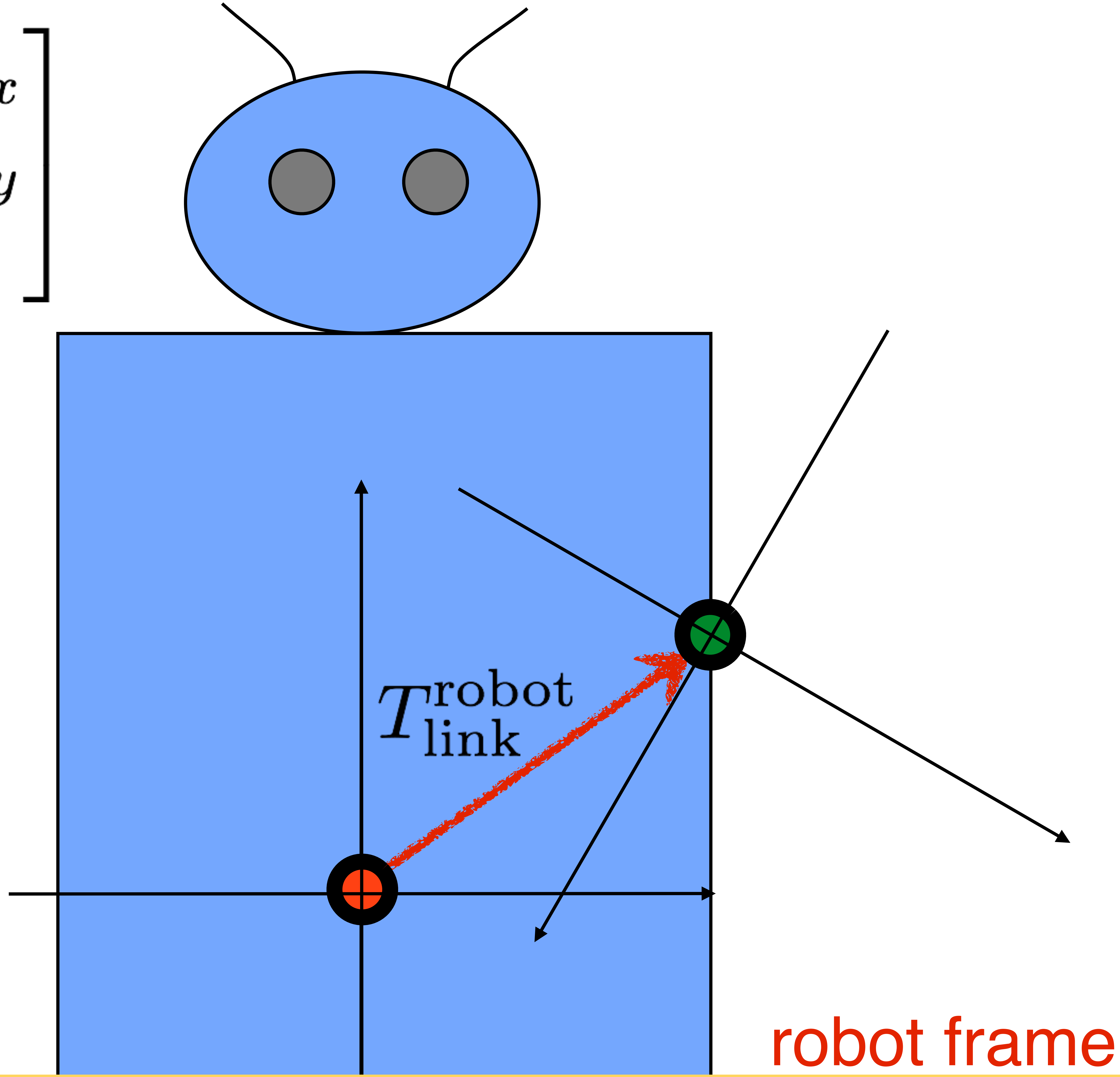
$$p_{\text{robot}} = T_{\text{link}}^{\text{robot}} p_{\text{link}}$$



$$T^{\text{robot}}_{\text{link}} = \begin{bmatrix} R_{00} & R_{01} & d_x \\ R_{10} & R_{11} & d_y \\ 0 & 0 & 1 \end{bmatrix}$$

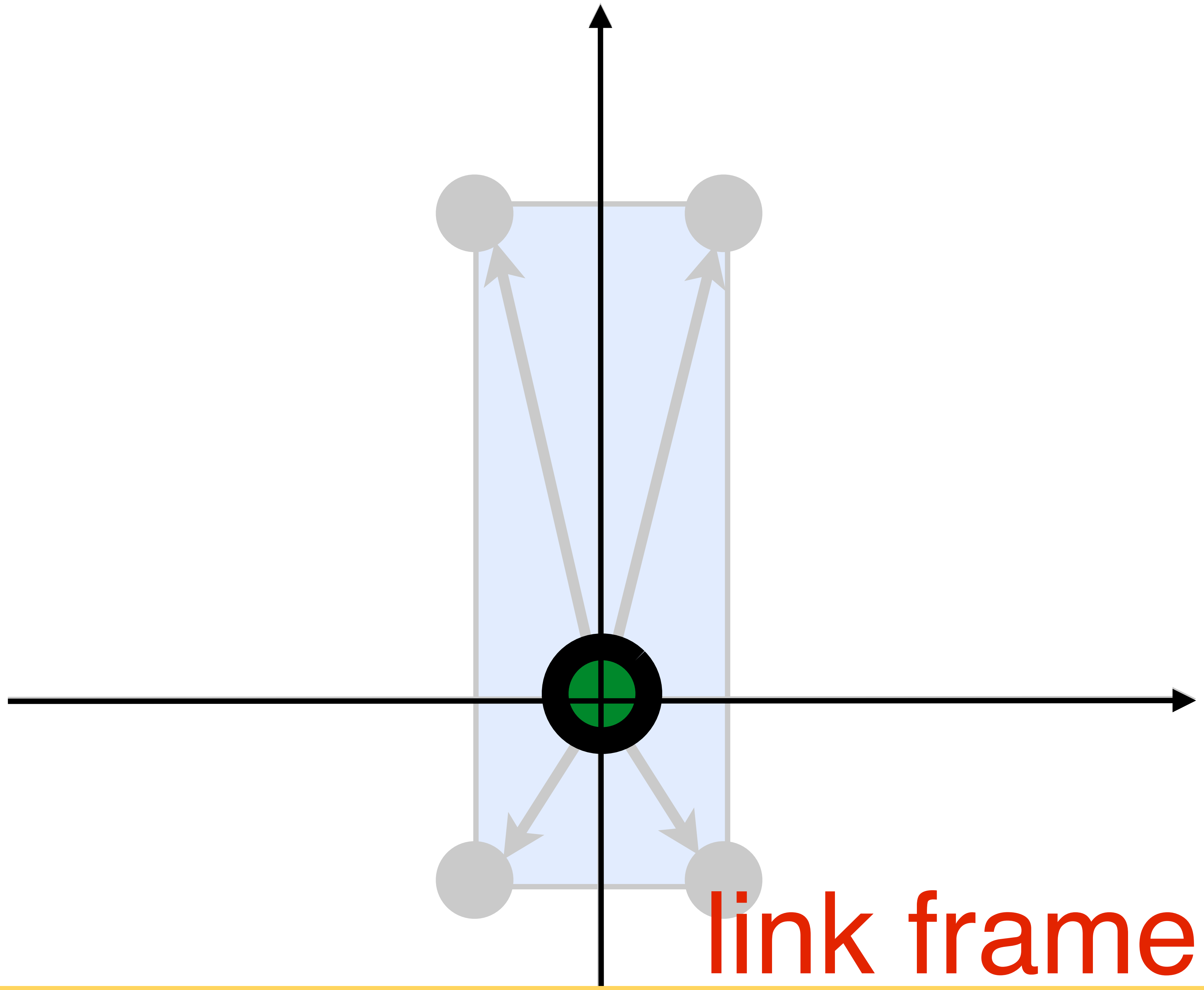
Can we think about this frame relation in steps?

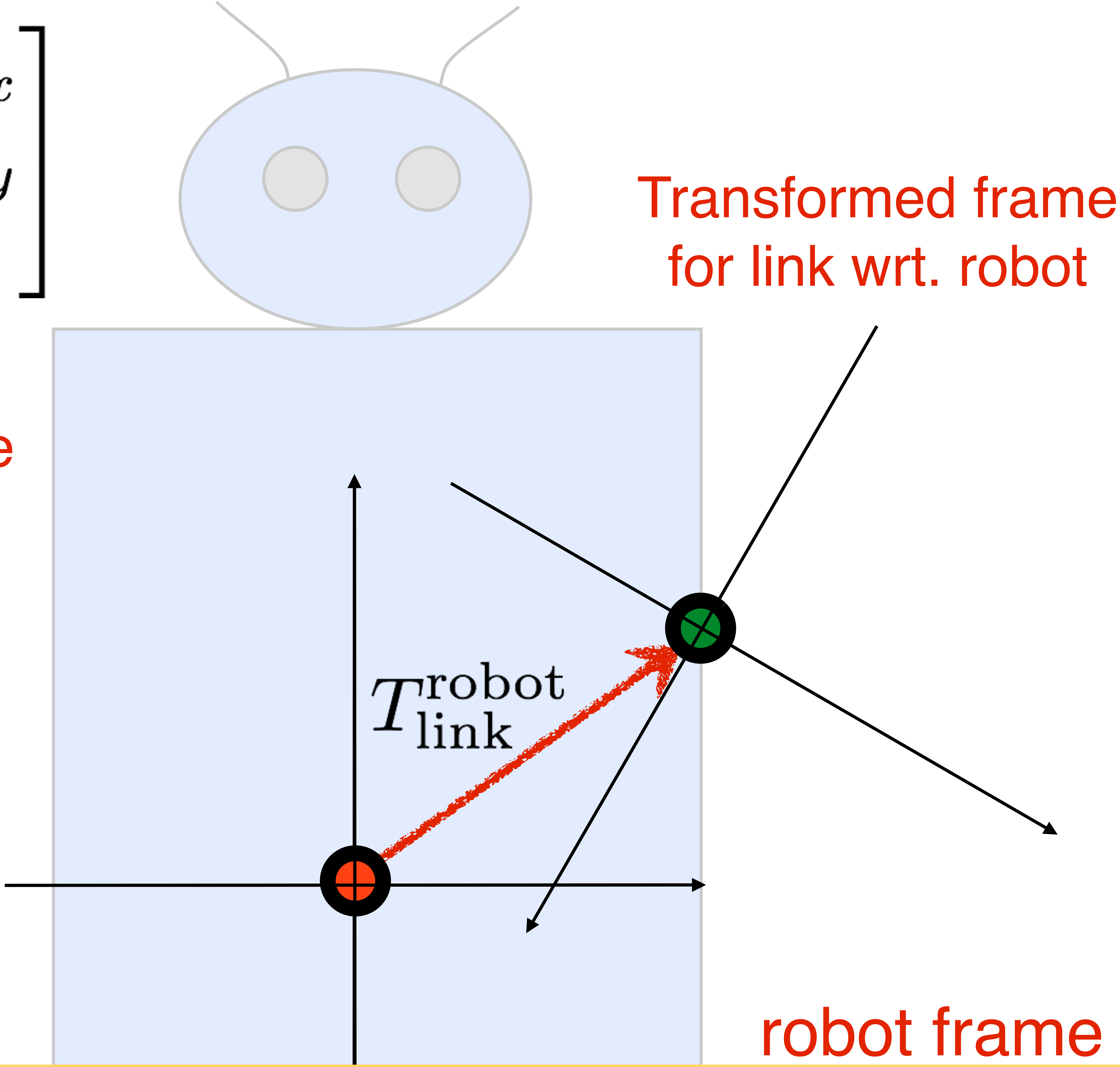




$$T_{\text{link}}^{\text{robot}} = \begin{bmatrix} R_{00} & R_{01} & d_x \\ R_{10} & R_{11} & d_y \\ 0 & 0 & 1 \end{bmatrix}$$

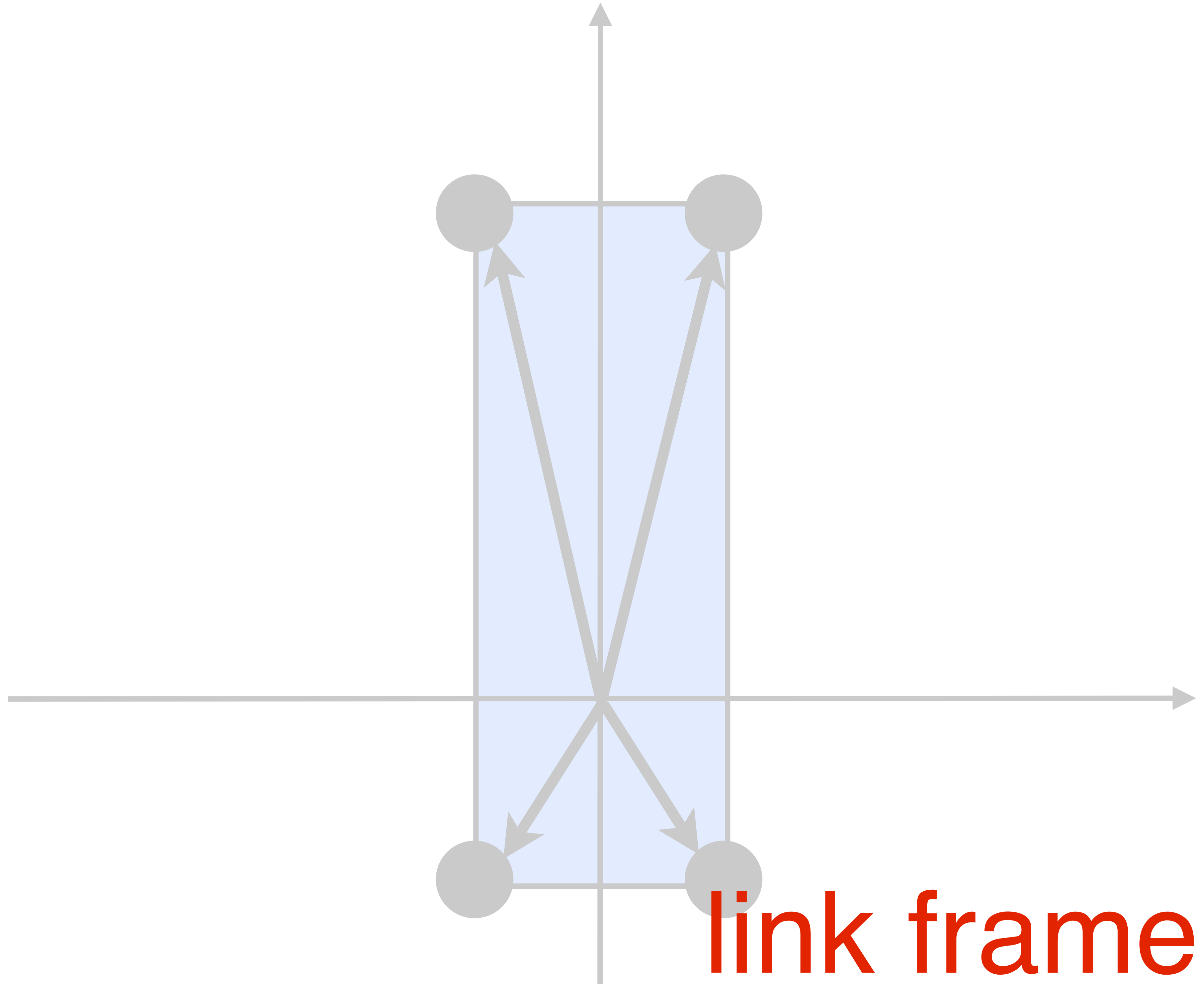
First consider link in its own frame



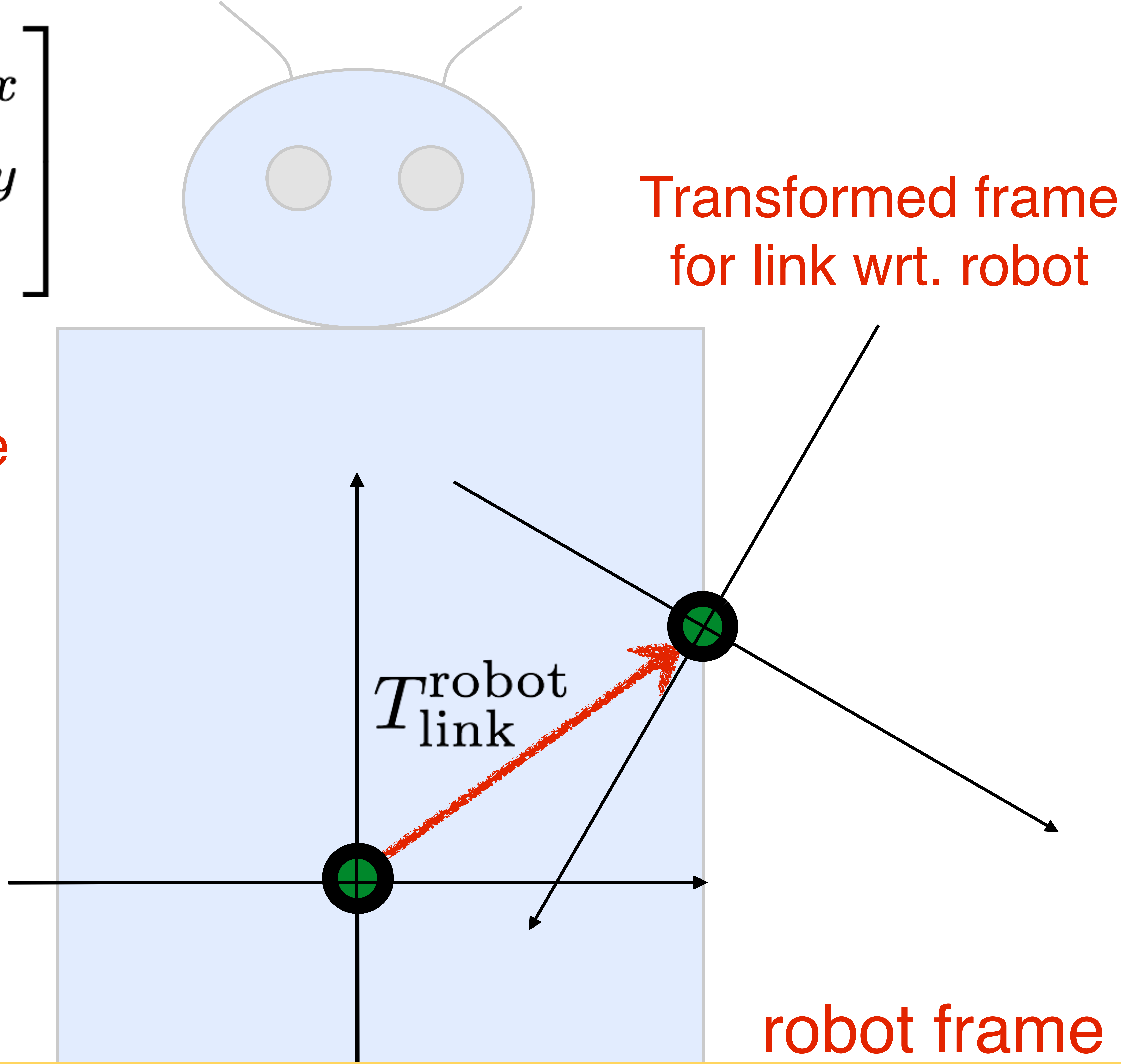


$$T_{\text{link}}^{\text{robot}} = \begin{bmatrix} R_{00} & R_{01} & d_x \\ R_{10} & R_{11} & d_y \\ 0 & 0 & 1 \end{bmatrix}$$

as aligned with robot base frame



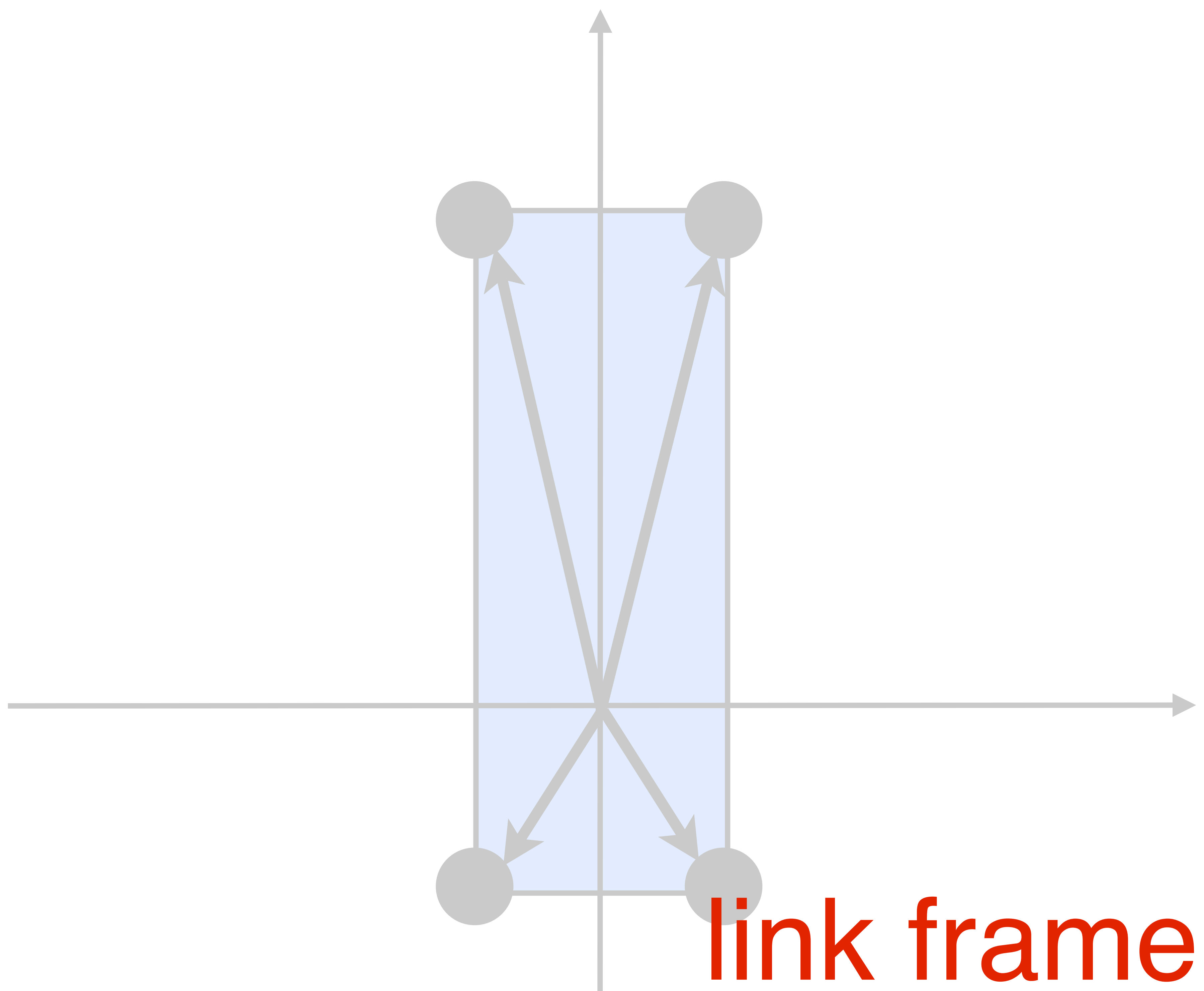
Transformed frame for link wrt. robot

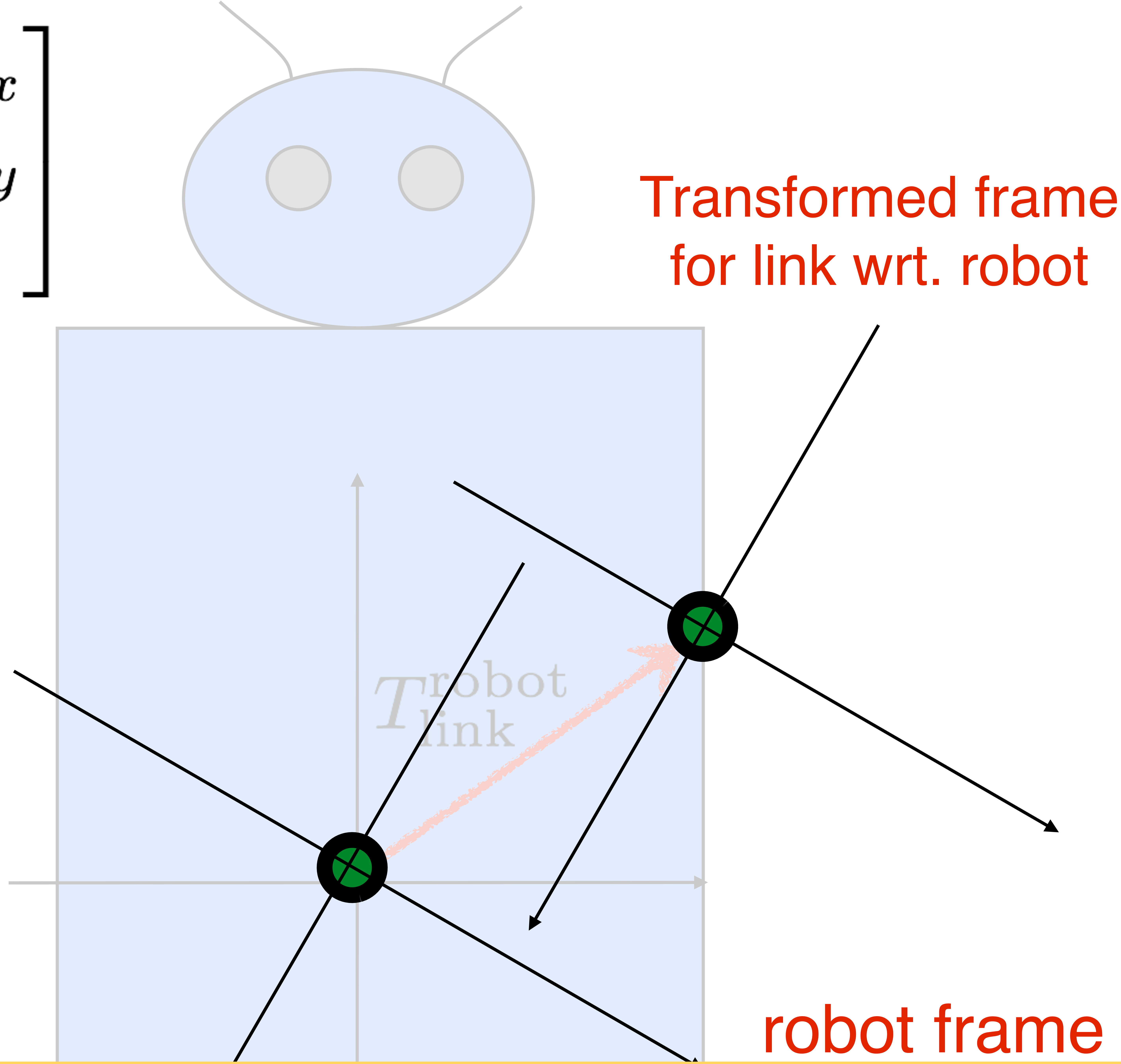


$$T_{\text{link}}^{\text{robot}} = \begin{bmatrix} R_{00} & R_{01} & d_x \\ R_{10} & R_{11} & d_y \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate link frame by *R*

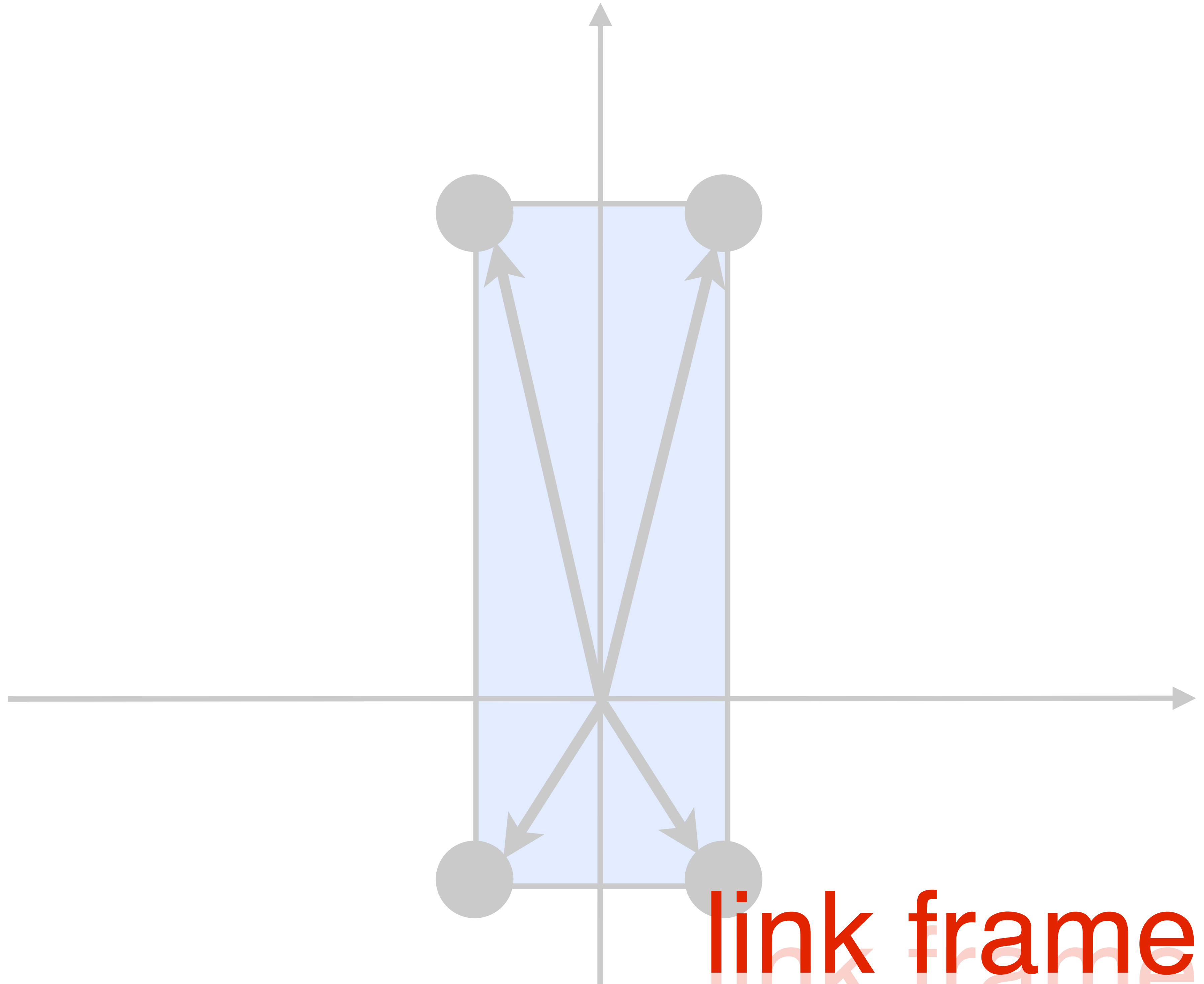
Transformed frame for link wrt. robot



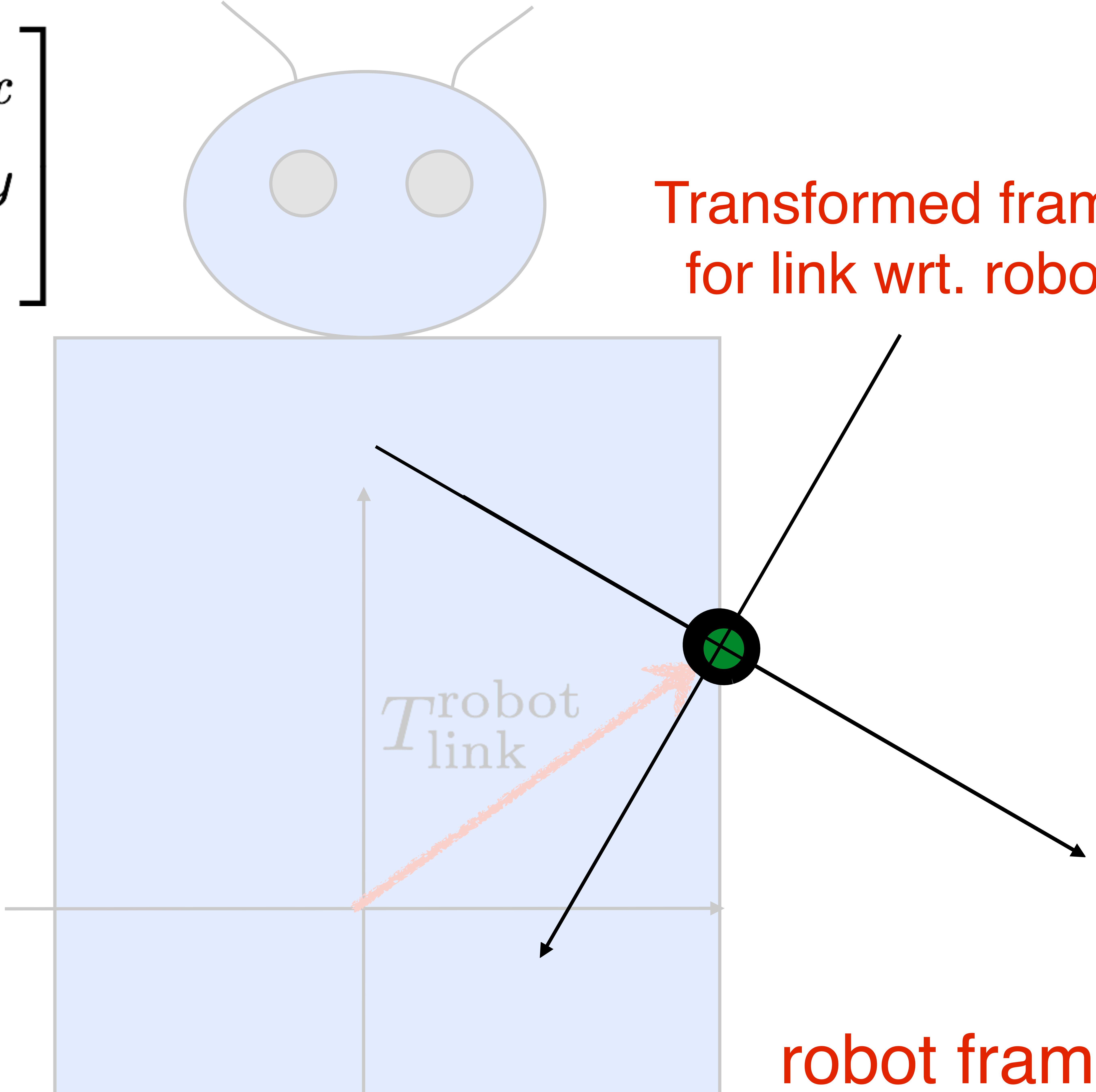


$$T_{\text{link}}^{\text{robot}} = \begin{bmatrix} R_{00} & R_{01} & d_x \\ R_{10} & R_{11} & d_y \\ 0 & 0 & 1 \end{bmatrix}$$

Translate link frame by *d*

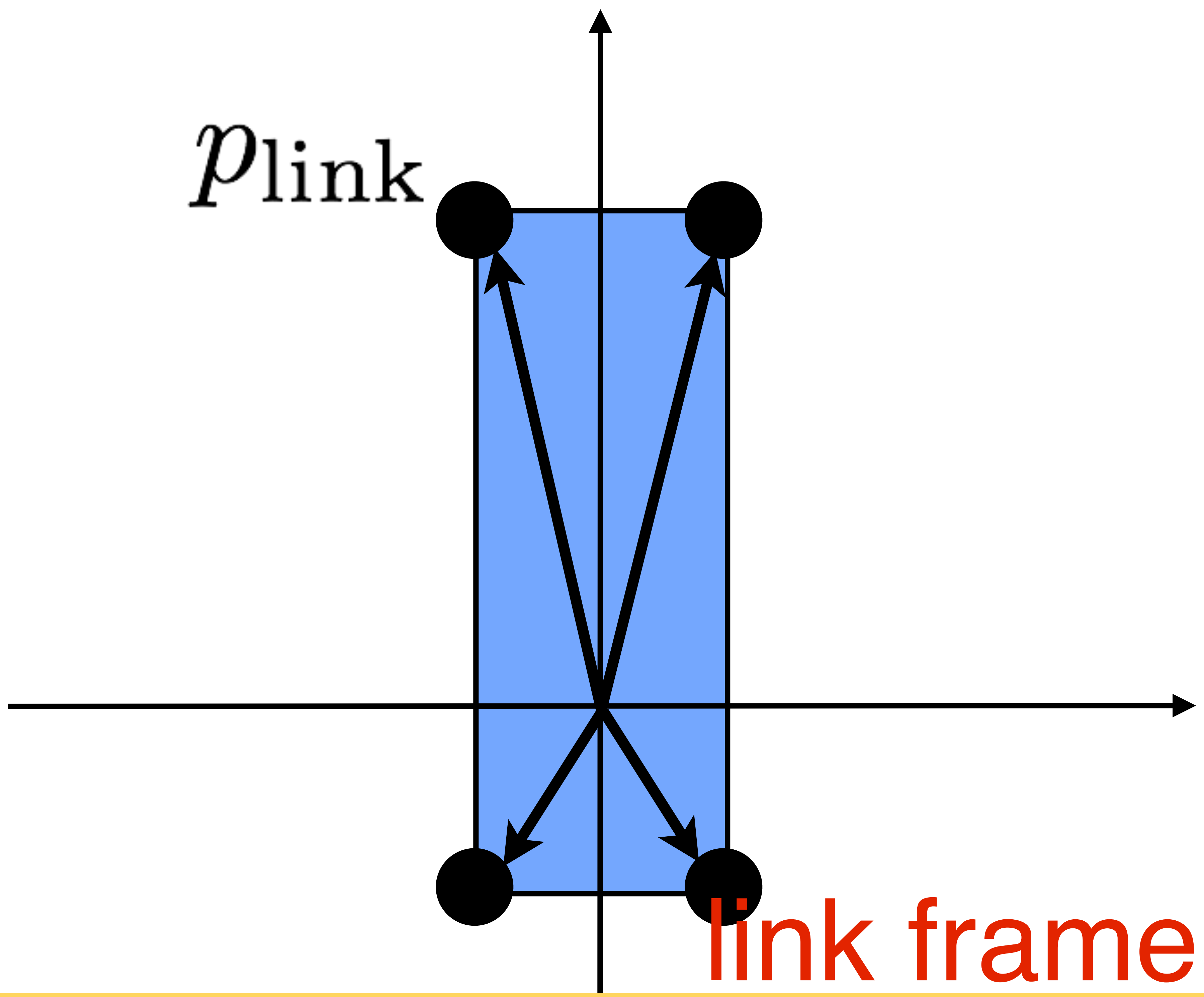


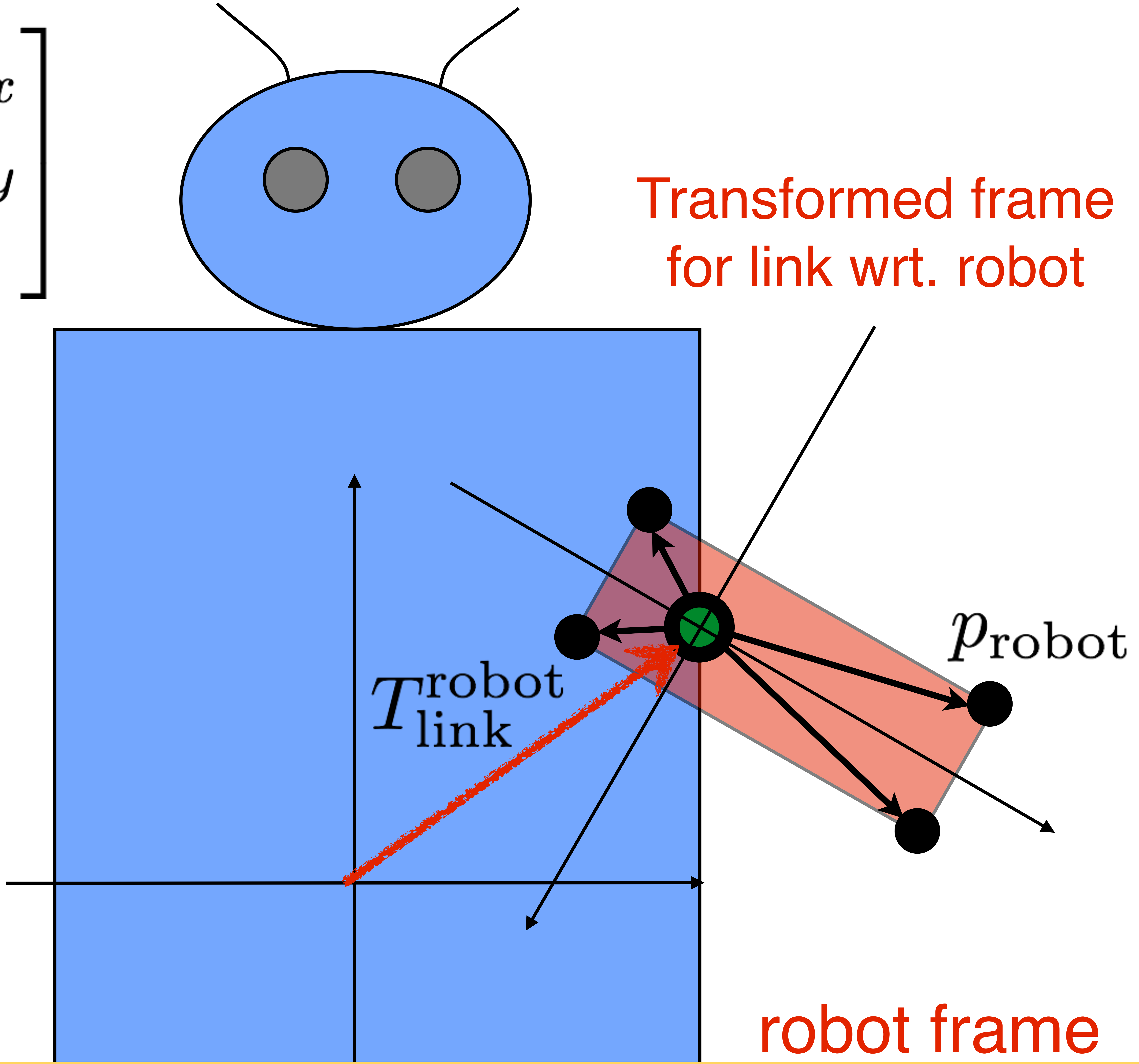
Transformed frame for link wrt. robot



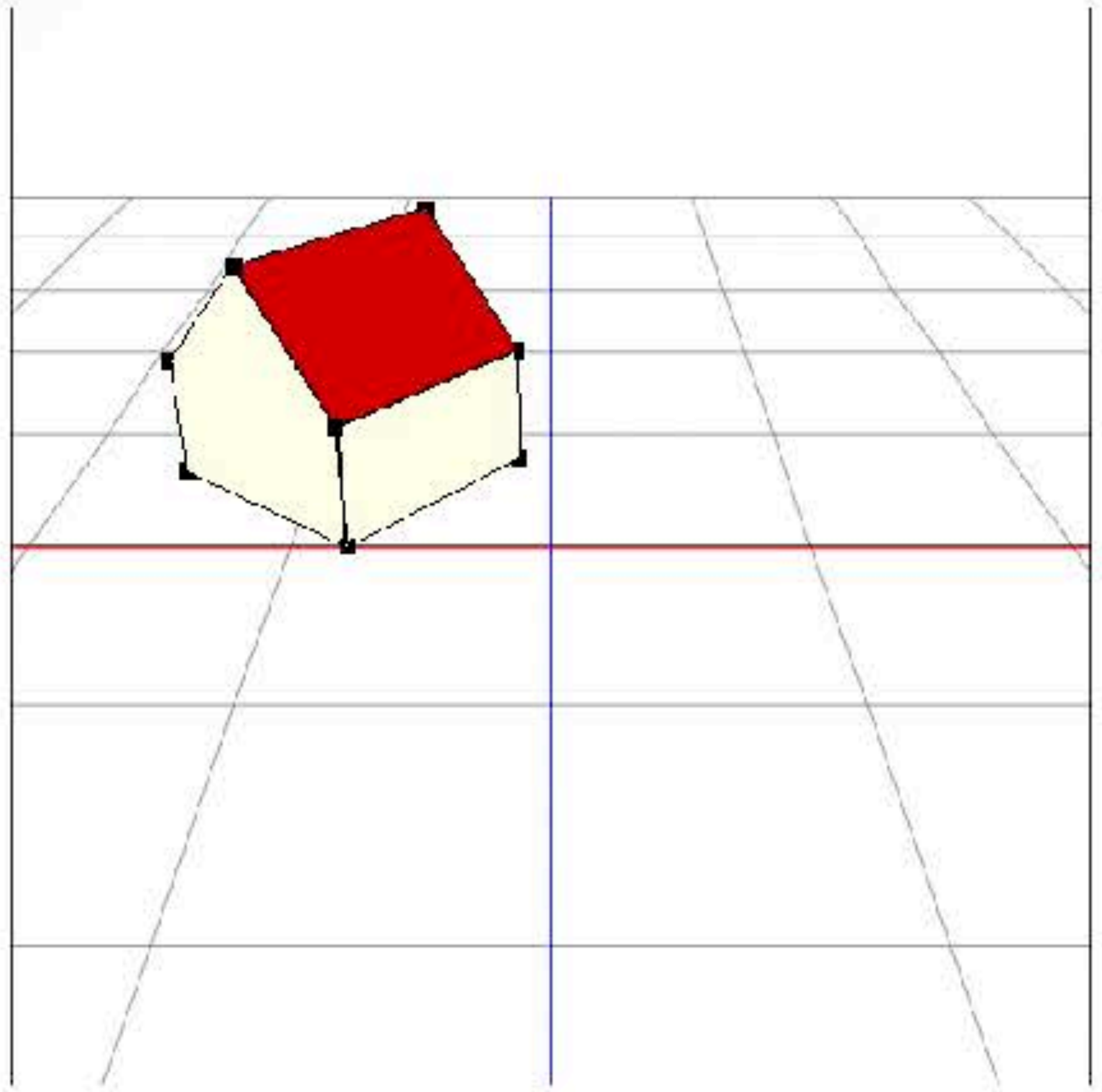
$$T_{\text{link}}^{\text{robot}} = \begin{bmatrix} R_{00} & R_{01} & d_x \\ R_{10} & R_{11} & d_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_{\text{robot}} = T_{\text{link}}^{\text{robot}} p_{\text{link}}$$

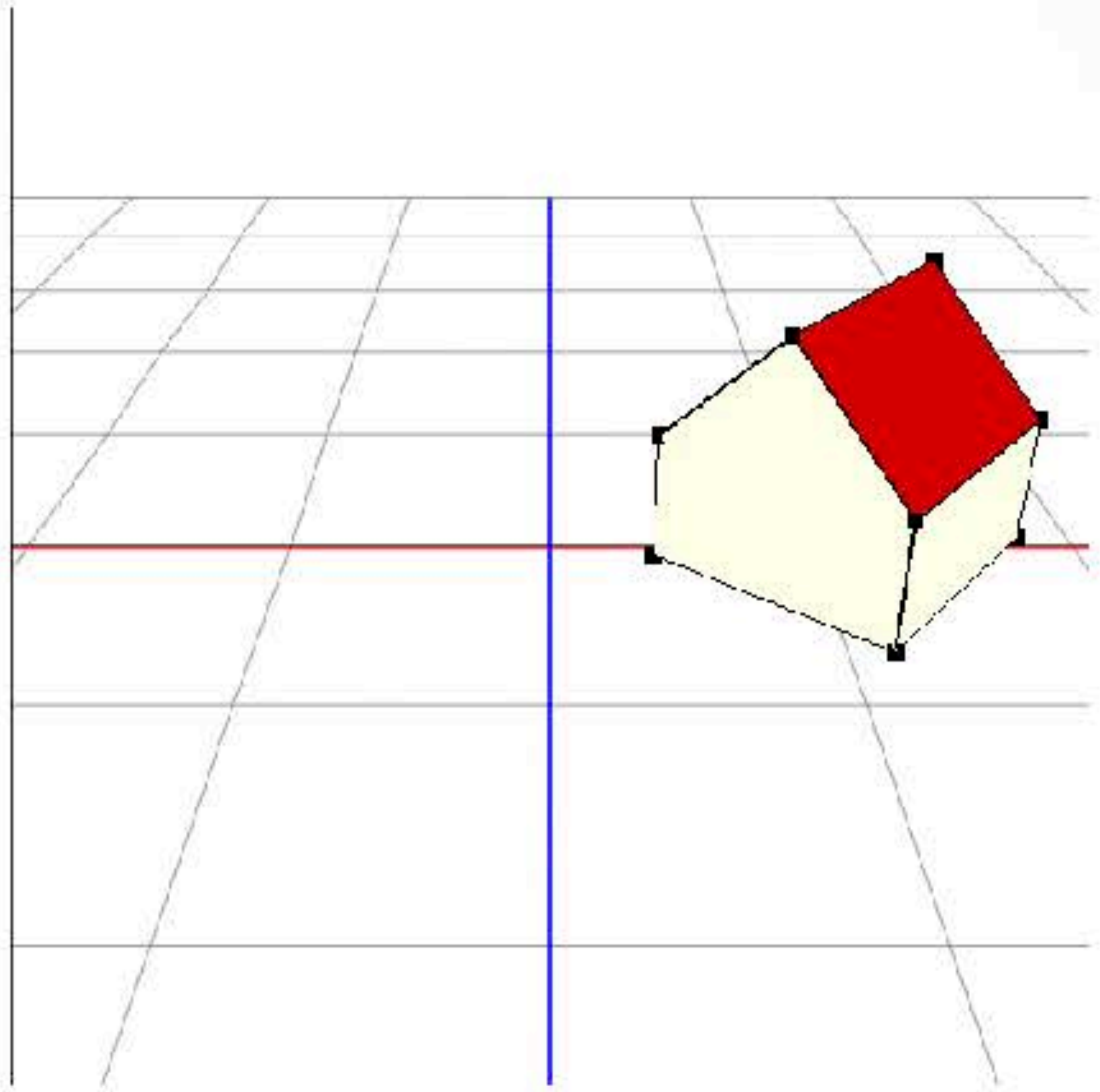




Why not translate then rotate?



$\mathbf{M} = \mathbf{R} \cdot \mathbf{T}$



$\mathbf{M} = \mathbf{T} \cdot \mathbf{R}$

Note the difference in behavior.

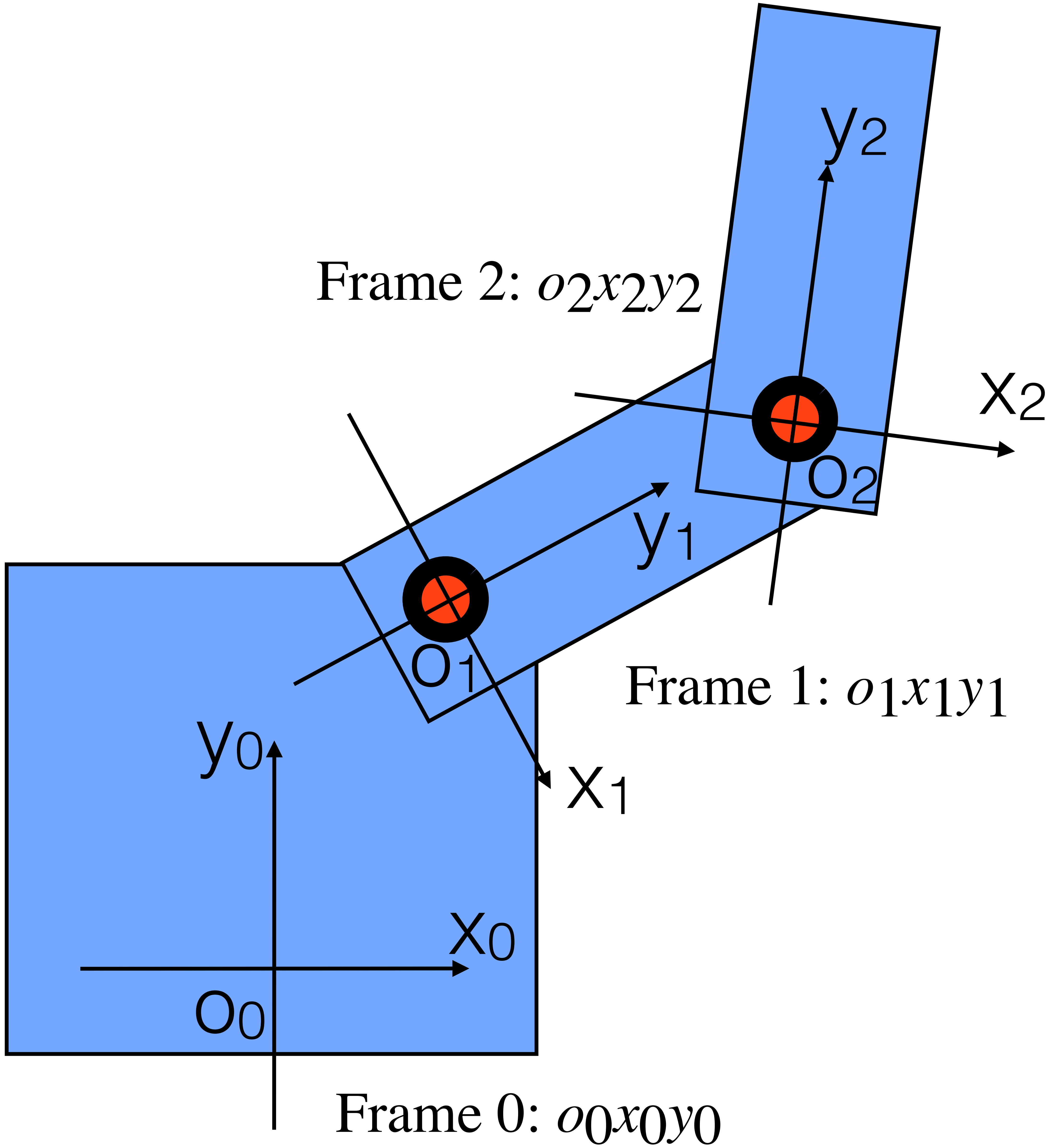
Translation along $x =$ 1.1

Rotation about $y =$ 140°

Can we compose multiple frame transforms?

Can we compose multiple frame transforms?

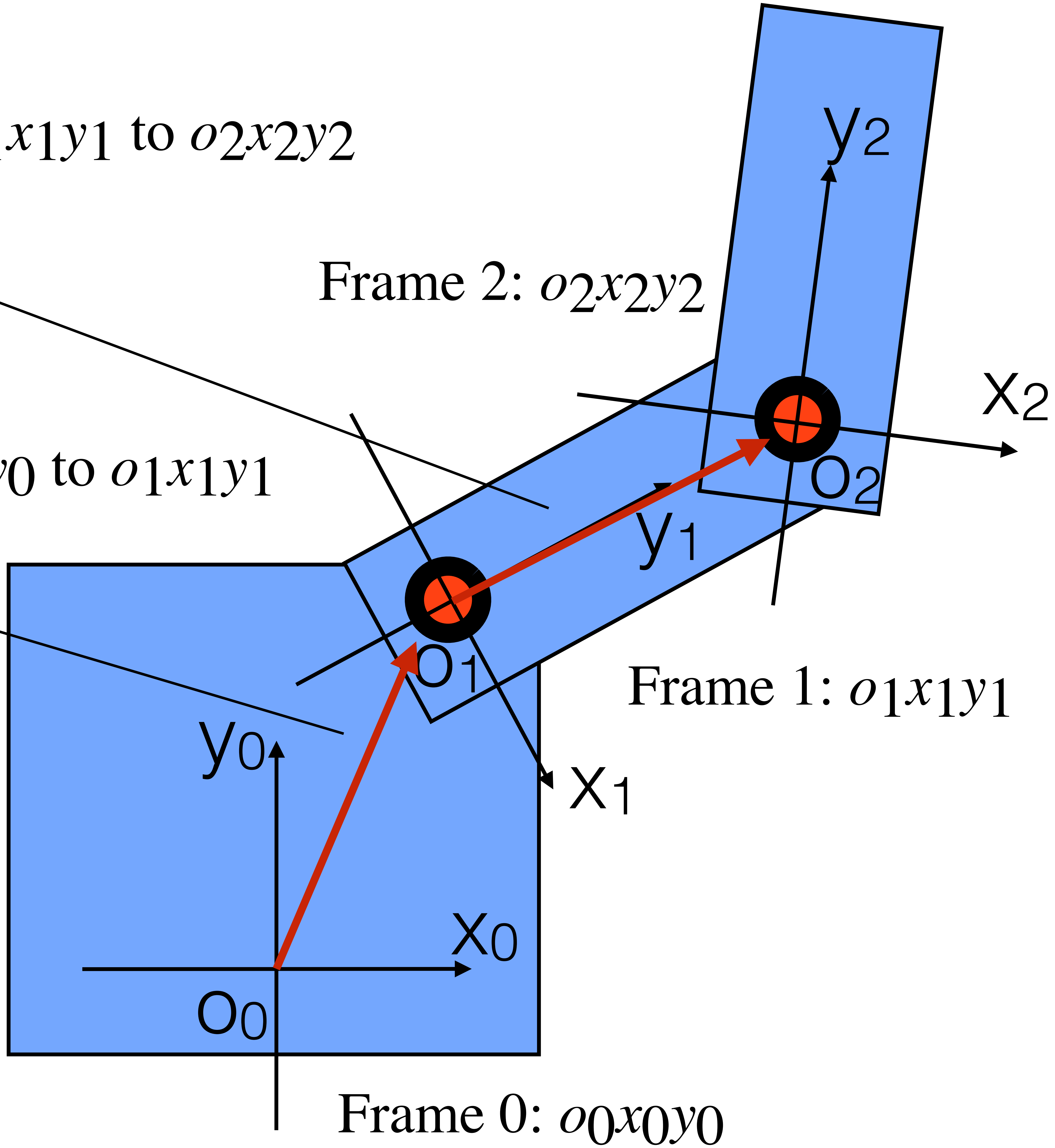
Consider the 3 frames of a planar 2-link robot



Frame 2: o2x2y2
y2
x2
o2
y1
o1
Frame 1: o1x1y1
y0
x1
x0
o0
Frame 0: o0x0y0

Rotation matrix $R_2{}^1$
Vector d_2 from origin $o_1x_1y_1$ to $o_2x_2y_2$

Rotation matrix $R_1{}^0$
Vector d_1 from origin $o_0x_0y_0$ to $o_1x_1y_1$



A point in frame 1 relates to a point in frame 0 by

$$p^0 = R_1^0 p^1 + d_1^0$$

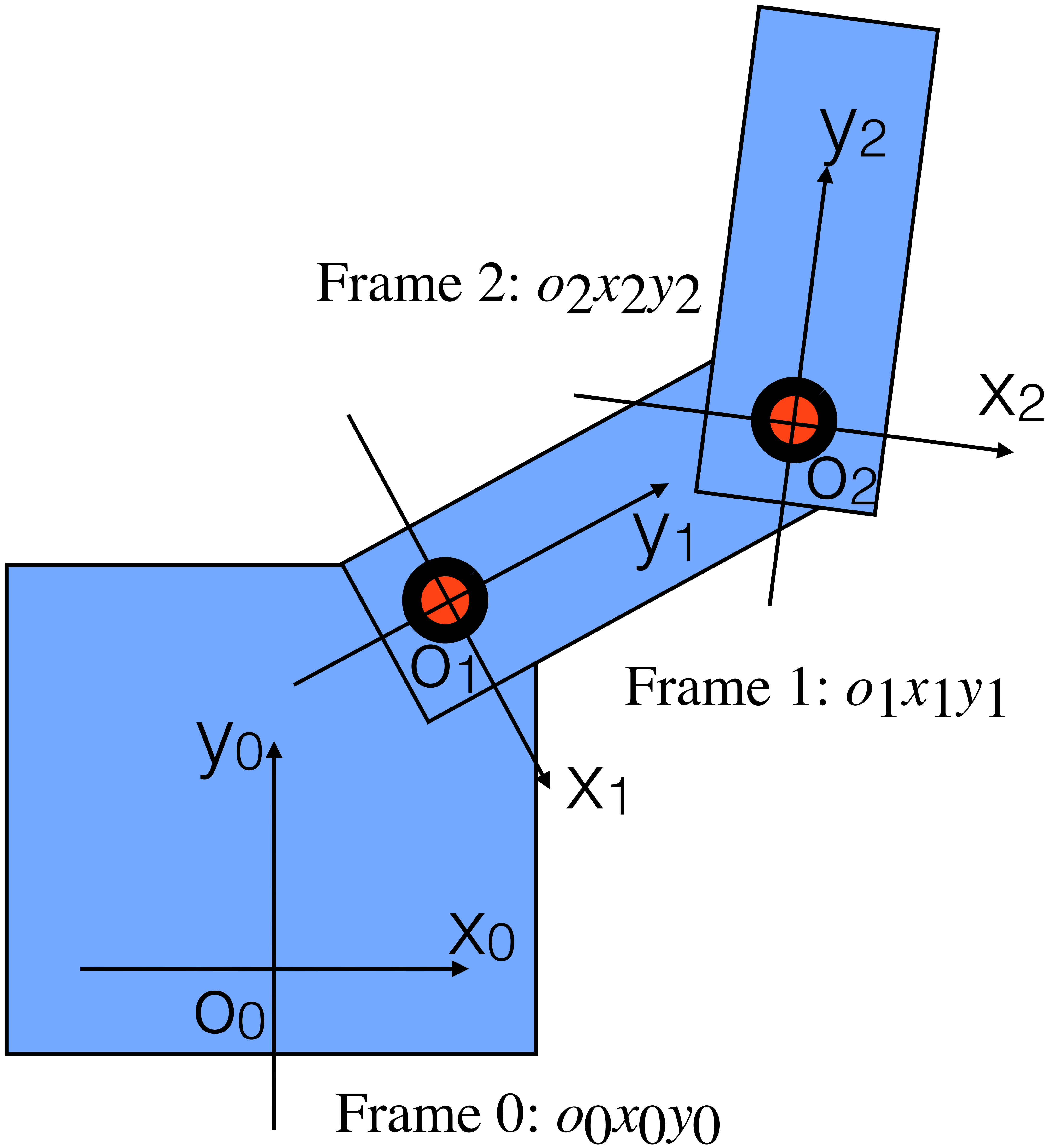
and point in frame 2 relates to point in frame 1 by

$$p^1 = R_2^1 p^2 + d_2^1$$

By substitution of p^1 into the expression for p^0,

a point in frame 2 relates to a point in frame 0 by

$$p^0 = \underbrace{R_1^0 R_2^1}_{R_2^0} p^2 + \underbrace{R_1^0 d_2^1 + d_1^0}_{d_2^0}$$



$$\begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^1 & d_2^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_1^0 R_2^1 & R_1^0 d_2^1 + d_1^0 \\ 0 & 1 \end{bmatrix}$$

Alternatively, relation expressed by composed transform from frame 2 to frame 0 as:

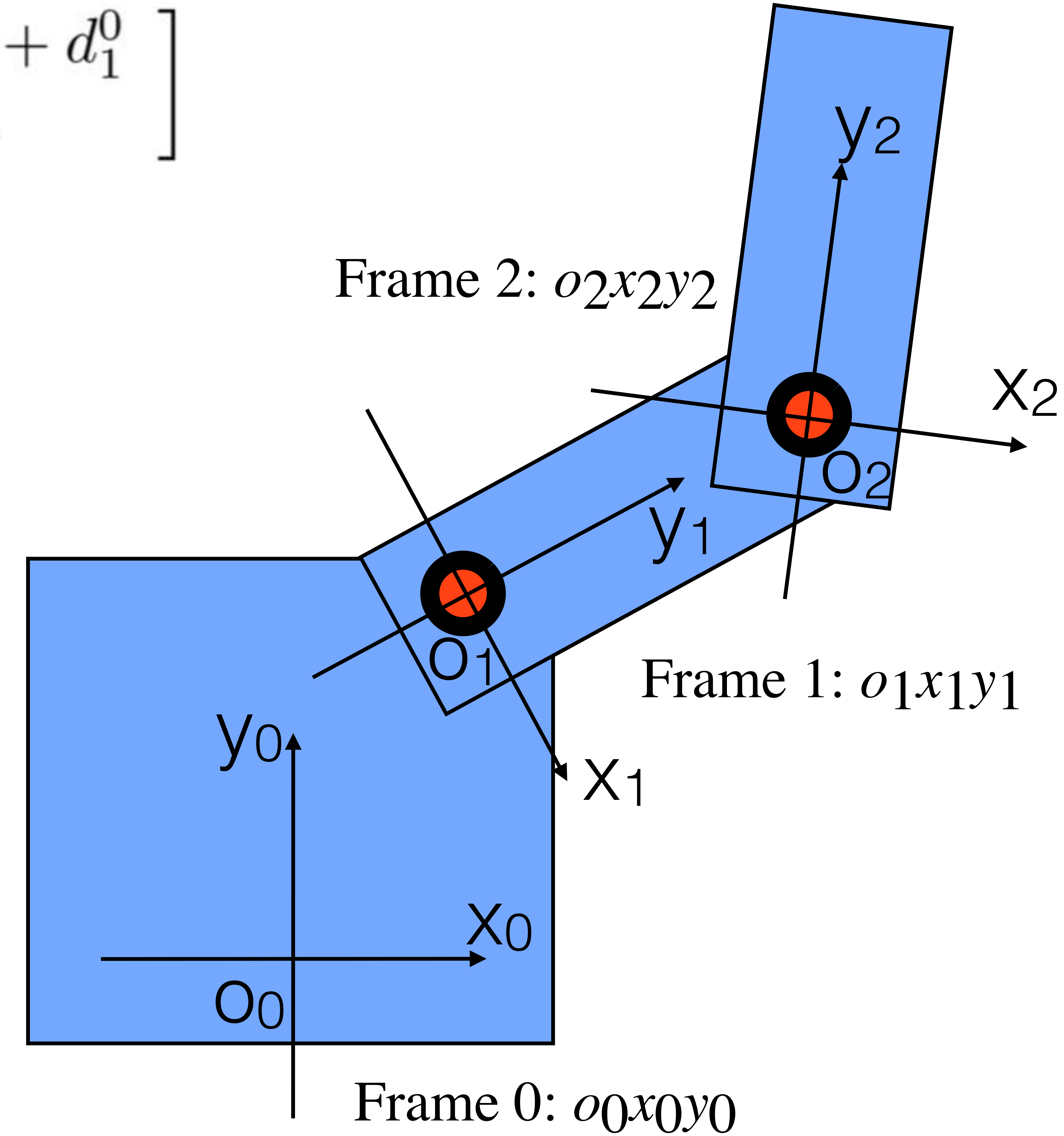
$$p^0 = R_2^0 p^2 + d_2^0$$

where

$$R_2^0 = R_1^0 R_2^1$$

$$d_2^0 = R_1^0 d_2^1 + d_1^0$$

which can be observed by block multiplying transforms



$$\begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^1 & d_2^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_1^0 R_2^1 & R_1^0 d_2^1 + d_1^0 \\ 0 & 1 \end{bmatrix}$$

Alternatively, relat[...]
transform from fra[...]

$p^0 = R_2{}^0 p^2 + d$[...]

where

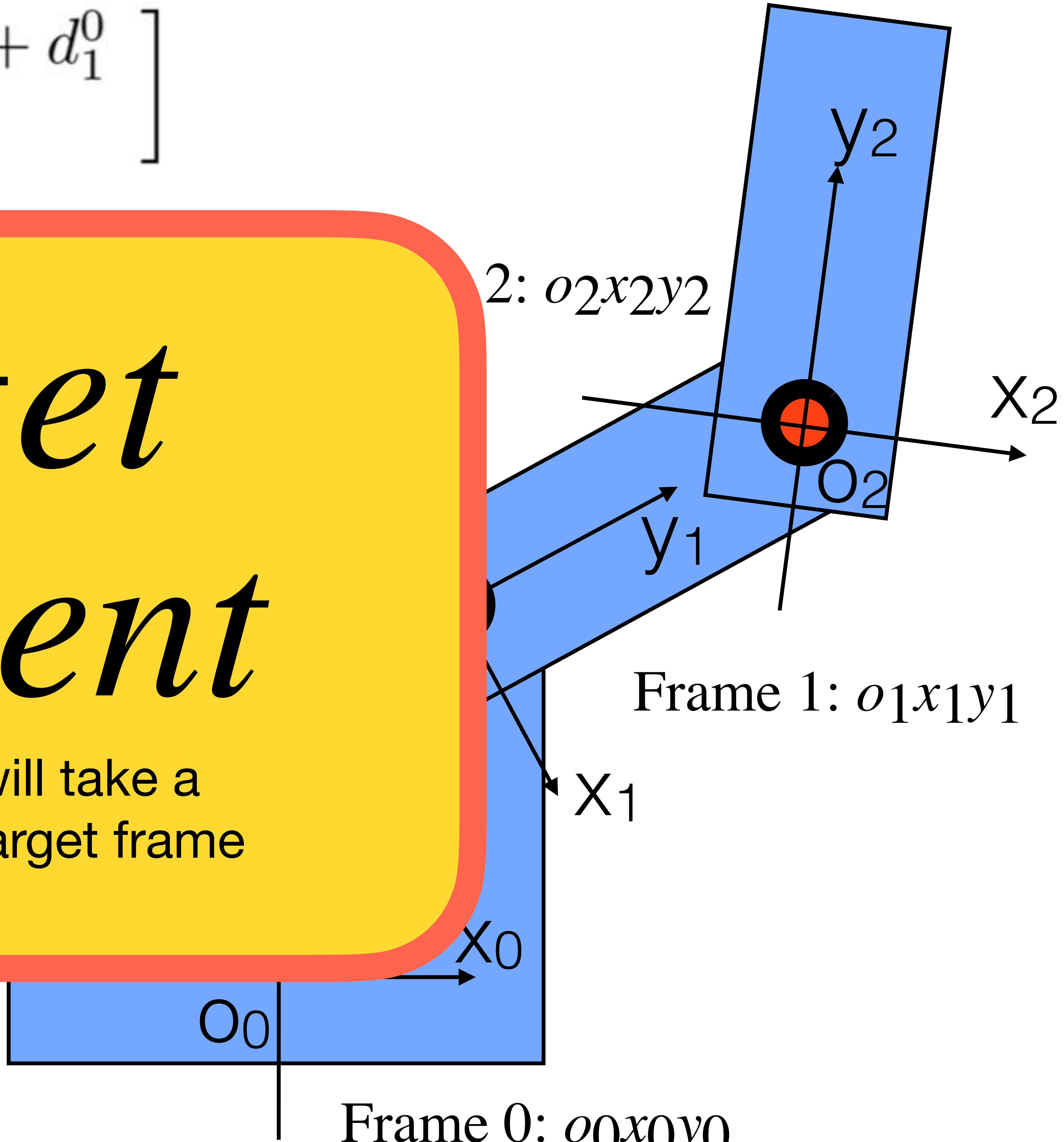
$R_2{}^0 = R_1{}^0 R_2{}^1$

$d_2{}^0 = R_1{}^0 d_2{}^1$[...]

$$R^{target}_{current}$$

Rotation Matrix that will take a point in current to the target frame

which can be observed by block multiplying transforms



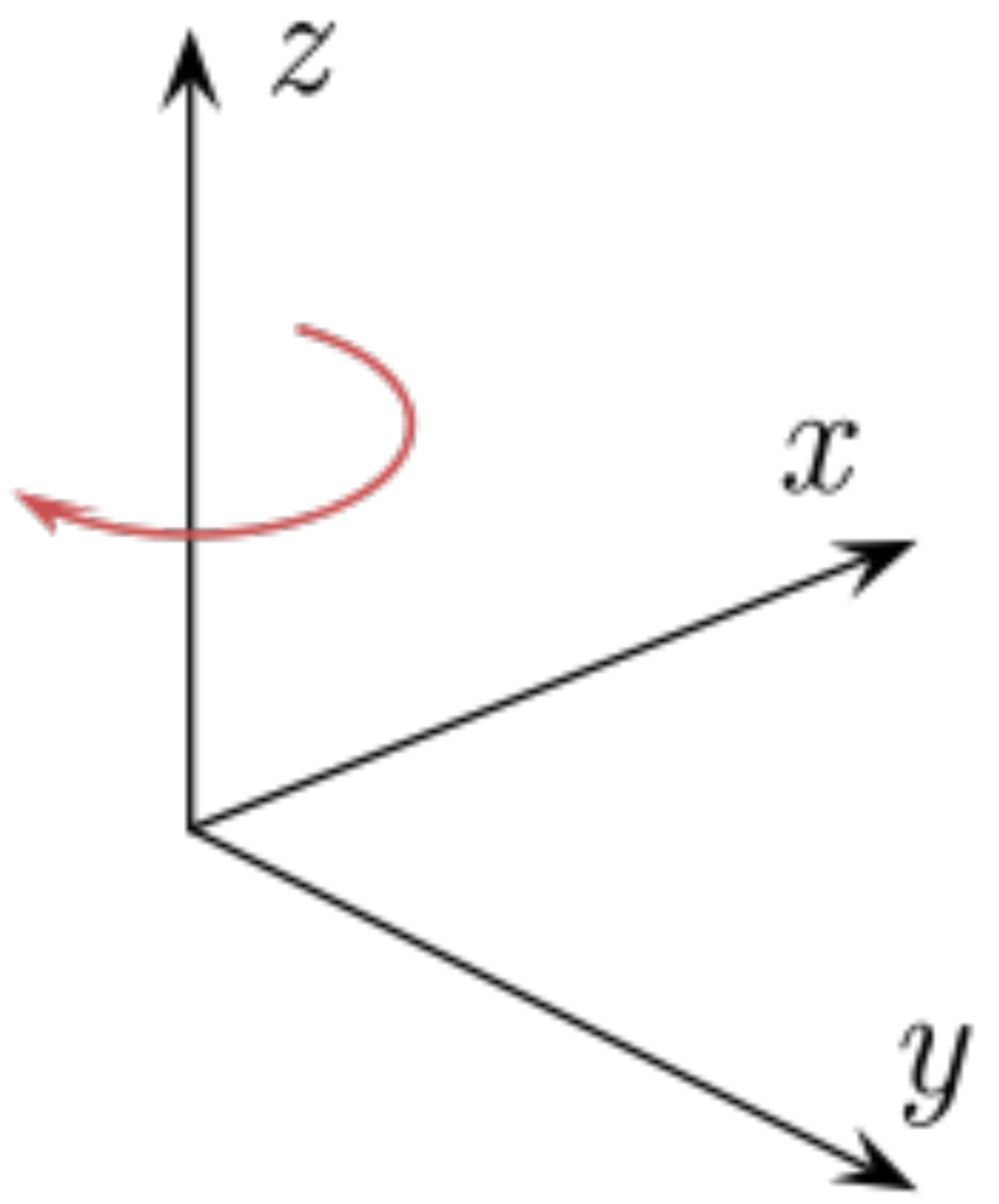
How do we extend this to 3D?

3D Translation and Rotation

$D(d_x, d_y, d_z)$
$$\begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_z(\theta)$
$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2D rotation in 3D is rotation about Z axis



3D Translation and Rotation

$$D(d_x, d_y, d_z) \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x(\theta) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Homogeneous Transform

Rotate about each axis in order R = $R_x(\Theta_x)$ $R_y(\Theta_y)$ $R_z(\Theta_z)$

$$\begin{bmatrix}1 & 0 & 0 & d_x\\0 & 1 & 0 & d_y\\0 & 0 & 1 & d_z\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\0 & \cos(\theta) & -\sin(\theta) & 0\\0 & \sin(\theta) & \cos(\theta) & 0\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos(\theta) & 0 & \sin(\theta) & 0\\0 & 1 & 0 & 0\\-\sin(\theta) & 0 & \cos(\theta) & 0\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos(\theta) & -\sin(\theta) & 0 & 0\\\sin(\theta) & \cos(\theta) & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

$D(d_x, d_y, d_z)$ $R_x(\theta)$ $R_y(\theta)$ $R_z(\theta)$

3D Homogeneous Transform

$$\begin{bmatrix}1 & 0 & 0 & d_x\\ 0 & 1 & 0 & d_y\\ 0 & 0 & 1 & d_z\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos(\theta) & -\sin(\theta) & 0\\ 0 & \sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos(\theta) & 0 & \sin(\theta) & 0\\ 0 & 1 & 0 & 0\\ -\sin(\theta) & 0 & \cos(\theta) & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos(\theta) & -\sin(\theta) & 0 & 0\\ \sin(\theta) & \cos(\theta) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$= H_3 = \begin{bmatrix}R_{00} & R_{01} & R_{02} & d_x\\ R_{10} & R_{11} & R_{12} & d_y\\ R_{20} & R_{21} & R_{22} & d_z\\ 0 & 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\mathbf{R}_{3\times3} & \mathbf{d}_{3\times1}\\ \mathbf{0}_{1\times3} & 1\end{bmatrix}$$

$$H_3 \in SE(3)$$
$$\mathbf{R}_{3\times3} \in SO(3)$$
$$\mathbf{d}_{3\times1} \in \Re^3$$

3D Homogeneous Transform

$$H_3 = \begin{bmatrix} R_{00} & R_{01} & R_{02} & d_x \\ R_{10} & R_{11} & R_{12} & d_y \\ R_{20} & R_{21} & R_{22} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{3\times 3} & \mathbf{d}_{3\times 1} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix} \in SE(3)$$

if $T_1^0 \in SE(3)$ and $T_2^1 \in SE(3)$ then composition holds:

$$\begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^1 & d_2^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_1^0 R_2^1 & R_1^0 d_2^1 + d_1^0 \\ 0 & 1 \end{bmatrix}$$

such that points in Frame 2 can be expressed in Frame 0 by:

$$p^0 = T_1^0 T_2^1 p^2$$

Next lecture:
Representations II: Rotations & Quaternions

PR2 Fetches Sandwich from Subway 11 years ago!

Autonomous Subway sandwich delivery
by a PR2 robot,
from the University of Tokyo and TUM

https://www.youtube.com/watch?v=RIYRQC2iBp0